ART. XLIII.—*Contributions from the Physical Laboratory of Harvard College.*—No. IV. *Induced currents and derived circuits*; by JOHN TROWBRIDGE.

THE expression for the intensity of an induced current, deduced by Neumann and Sir William Thomson, is as follows: $i = \frac{1}{k}\frac{dU}{dt}$, in which k is a coefficient depending upon the resistance of the complete wire in the secondary circuit, and U is a certain "force function" which depends solely upon the form and position of the wire at any instant, and on the magnetism of the influencing body. The expression, in general language, is as follows:

"When a current is induced in a closed wire by a magnet in relative motion, the intensity of the current produced is proportional to the actual rate of variation of the "force function" by the differential coefficients of which the mutual action between the magnet and the wire would be represented if the intensity of the current in the wire were unity."

This investigation was undertaken to ascertain if the laws of derived circuits apply to the currents of induction, which are represented by equations of which the above is a type. A reflecting galvanometer of large resistance was included in the secondary circuit, and connected by copper wires of very small resistance with the coil in which the secondary currents were produced: the resistance of these wires was infinitessimal in comparison with the resistance of the galvanometer. The galvanometer was then shunted. The first two columns of the following table show that, with an inappreciable resistance outside of the galvanometer coils, the shunts made no difference in the deflection of the galvanometer needle when the shunts were not less than three ohms. Below this the current divided. The resistance of the galvanometer was 5880 ohms, and the last numbers in the second and third columns show that an equal

Exterior Resistances, in ohms.	Shunts, in ohms.	Deflections.	Exterior Resistances, in ohms.	Deflections.
0	3	210	10	210
"	4	210	20	210
"	5	210	30	210
"	6	210	40	210
"	5880	210	100	190

impulse was transmitted through both the shunt and the galvanometer, for no reason can be assigned why it should take one course in preference to the other. Two galvanometers, therefore, of the same resistance, one forming the shunt to the other, will give the same deflection, which is equal to that given by the undivided circuit.

Resistances were then introduced in the circuit exterior to the galvanometer coils, and a shunt of 588 ohms was used.

The fifth column shows that no effect was produced by the shunt until the exterior resistance was appreciable in comparison with that of the galvanometer.

The following table exhibits the effect of resistances which were appreciable in comparison with the galvanometer resistance. The same shunt of 588 ohms was used. The second column is calculated on the assumption that

$i = \frac{dU}{dt}$, (where k' is a coefficient), is equivalent to $i_{1} = \frac{E}{R}$, and

$'R$

that the laws of Kirchoff hold. The third column is obtained from the experimental data. The fourth and fifth columns are also calculated on the assumption that $i = \frac{E}{R}$ = tangent of the deflection. Columns second and third are expressed in arbitrary scale divisions.

Exterior Resistances, in ohms.	Calculated value, of i.	Experimental value, of i.	Ratio of intensities.	Ratio of Tangents.
1500	1242	1375		
2000	1033	1055	1·06	1·03
2600	954	990	1·06	1·04
3000	767	825	1·06	1·04
3500	673	770	1·11	1·05
4000	613	660	1·05	1·03
4500	558	649	1·05	1·07
5000	514	550	1·04	1·03

It will be seen by comparison that, with large resistances exterior to the galvanometer resistance and appreciable in connection with it, the laws of the division of currents practically hold, and as the exterior resistance approaches that of the galvanometer, the coincidences with the laws is more marked.

From the above it appears that, under certain conditions, an induced current does not divide according to the laws of divided circuits, but approximates to these laws when there is a resistance exterior to the galvanometer which is appreciable in comparison with that of the galvanometer.

No. V.—*On a method of measuring induced currents;* by F. H. Bigelow.

If a Wheatstone's bridge be formed, in which the secondary coil of the inductorium is the resistance R_1 to be measured, the relation between the resistances will be expressed by the proportion $R_1 : R_2 = R_3 : R_4$ where R_3 and R_4 are in a fixed ratio. By passing a current from an independent battery through the primary coil, the strength of the current in R_2 will be increased or diminished on the breaking or making of the inducing current. Therefore, to preserve equilibrium in the bridge, R_1 must be changed, and we shall have $R_1 \pm C : R_2 \pm x = R_3 : R_4$, in which x is a resistance equivalent to the effect of the induced current. Let $R_1 \pm C$ be found by trial such that the addition or subtraction of the induced current will produce equilibrium in the bridge; that is, will be such as to bring the deflection of the galvanometer to the zero of the scale. We shall then have the strength of the induced currents expressed as resistances. One advantage of this method is this, that the readings are always reduced to the same point, the zero of the scale. This method also has a wider range than that of merely taking the swing of the galvanometer needle. For currents which would throw the spot of light in a reflecting galvanometer off the scale, can be readily kept on the scale in this method by merely altering the ratio of R_3 to R_4 in the Wheatstone's bridge. Since the shunting of induced currents is accompanied with difficulties, as will be seen by the paper of Mr. Trowbridge accompanying this, this method is especially advantageous. When the bridge was set up so that the smallest variation in the resistance of the branch containing the inductorium gave the greatest variation in the current going through the galvanometer, namely, when $\frac{dS_0}{dR_3} = 0$, S_0 being the current through the galvanometer, and the resulting value of R_3 being $R_3 = \sqrt{\frac{GR_1(2B+R_1)}{G+2R_1}}$, in which G is the resistance of the galvanometer, B that of the circuit exterior to the Wheatstone's bridge, it was found that the induced currents could be measured to one hundred-thousandth of an ohm.

The following table contains a comparison of the induced currents produced by making and breaking the circuit. The first two columns contain the variation in ohms of the variable resistance of the bridge; the third and fourth columns give the strength of the induced currents on making and breaking expressed in ohms.

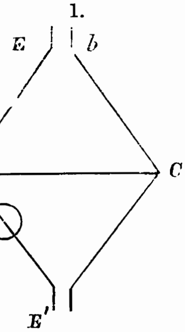
Change in the resistance on breaking.	Change in the resistance on making.	Strength of Induced Current on breaking.	Strength of Induced Current on making.
650	600	·00325	·00300
700	680	·00350	·00340
720	720	·00360	·00360
720	750	·00360	·00375
700	700	·00350	·00350
850	850	·00425	·00425

Care should be taken to send the induced currents to be compared in the same direction by means of a pole changer. It will be seen from the above table that the equality of the currents on making and breaking can readily be proved by this method.

No. VI.—*On methods of determining the resistance of a battery, deduced from Poggendorff's mode of measuring Electromotive Forces;* by N. D. C. Hodges.

In the process of obtaining the ratio between the electromotive forces of two cells, the expression $\frac{E'}{E}=\frac{B+R_0}{R_0}$ is obtained; in which $E>E'$, R_0 is the resistance necessary to bring the needle of the galvanometer G to zero, and B is the resistance of the battery to be measured. By introducing a new resistance R′ in the branch *ab*, we shall obtain the expression $\frac{E}{E'}=\frac{B+R'+R_0+R_1}{R_0+R_1}$, in which R_1 is the new value of R_0; hence $\frac{R_1}{R'+R_1}=\frac{R_0}{R+R_0}$ and $B=\frac{R_0R'}{R_1}$. If the ratio $\frac{E}{E'}$ becomes $\frac{m}{n}$ we shall have

1.



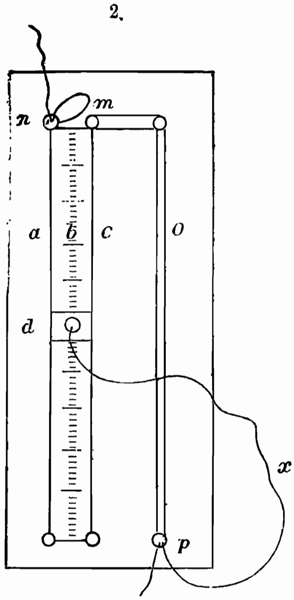
$$\frac{m}{n}=\frac{B+R_0}{R_0} \text{ or } B=R_0\left\{\frac{m}{n}-1\right\}$$

This expression was given by Mr. Mance in the Phil. Mag., vol. xli, p. 318, 1871, and is seen to follow directly from Poggendorff's method of measuring electromotive forces. The following are the results obtained from the expression $B=\frac{R_0R'}{R_1}$, in measuring the resistance of two Grove cells. One Grove cell was opposed to the two whose combined resistance was measured.

R_0	R′	R_1	B
·357	1	1·028	·347
"	2	2·057	·347
"	5	5·144	·347
"	7	7·140	·350

By Mr. Mance's method, with the same value of R_0, the value obtained was $B = \cdot 357$. The above numbers are expressed in ohms and fractions of an ohm. My method has the advantage that many determinations of the battery resistance can be made with the same arrangement of cells. In Mr. Mance's method but one determination can be made without altering the arrangement. I have found the following form of Rheocord useful in these determinations.

2.



In fig. 2 *a* and *c* are two German silver wires, with a slider at *d* which has a binding screw; between the wires is a mirror *b* with a scale upon it, or at its side, in order to avoid parallax in reading. Oρ is a German silver wire or rod of small resistance.

As the slider is now arranged, the current coming in at *n* will divide at *d*, pursuing the two paths $d\chi\rho$ and $dmo\rho$. By placing a box of resistance coils at χ, we can make the combined resistance of $d\chi\rho$ and $dmo\rho$ differ as little as we please from the standard resistance $o\rho$. It is thus possible, by making a large variation in χ, to measure small resistances a little larger than the small resistance $o\rho$; thus employing the correct principle of working from the greater to the less.

By disconnecting the resistance $o\rho$ at *m* and ρ and joining *mn*, it will be readily seen that the combined resistance of the circuit *dnmd* is one-half that of *nd* alone. It is often desirable to reduce the resistance in this manner.

The changes in the combined resistances can of course be readily calculated from the expression $\dfrac{\chi\eta}{\chi+\eta} = R$, where χ and η represent the two branches of the circuit. I have often found it advantageous in measuring large resistances to shunt them, so to speak, in the Wheatstone's balance, thus forming a combined resistance, one branch of which includes the large resistance and the other a carefully measured resistance much smaller than the resistance to be measured; the balance can in this manner be kept in its most sensitive arrangement, and a series of measurements can be made of the large resistance by varying the carefully measured portion of the combined resistance.