

ART. XXVII.—*On the Mean Motions of Jupiter, Saturn, Uranus, and Neptune*; by Professor DANIEL KIRKWOOD.

THE recent note of Professor Peirce,† announcing his discovery of some interesting relations between the mean motions of the four outer planets, has recalled my attention to a number of similar coincidences detected by myself several years since, while engaged in a somewhat laborious examination of the

* One arroba is equivalent to 25 lbs.

† For an article on this meteorite by Dr. J. Lawrence Smith, see this Journal, III, i, 335.

‡ This Journal for January, 1872. It will be observed that Professor Peirce's third equation is identical with that discovered by Professor Newcomb in 1857. See Gould's *Astr. Journ.*, vol. v, p. 101.

planetary elements. Of these, the following may be worth putting on record for future discussion:

$$2n^v - 3n^{vi} - 11n^{viii} = 0 \quad - \quad - \quad - \quad (1)$$

$$2n^{vi} - 21n^{vii} + 30n^{viii} = 0 \quad - \quad - \quad - \quad (2)$$

$$3n^v - 8n^{vi} - 2n^{vii} + 7n^{viii} = 0 \quad - \quad - \quad - \quad (3)$$

With the values of n^v , n^{vi} , and n^{vii} adopted in the American Ephemeris, the value of n^{viii} obtained from either of the above equations differs by less than one second from the latest determination.* The second equation was submitted some two years since to Professor Newcomb, of the U. S. Coast Survey. That distinguished astronomer was inclined, however, to regard the coincidence as merely accidental. Be this as it may, I have strong confidence in the accuracy of the third. The re-examination of this last has recently led to the discovery of two others, viz:

$$68n^{vi} - 325n^{vii} + 257n^{viii} = 0 \quad - \quad - \quad - \quad (4)$$

$$257n^v - 844n^{vi} + 587n^{vii} = 0 \quad - \quad - \quad - \quad (5)$$

both of which, I believe, are accurately true. The fifth, however, is not an independent equation, but is derived from the third and fourth. By means of these equations I have found the remarkable cycle of 11657.24 Julian years, which separates the epochs at which the planets Jupiter, Saturn, Uranus, and Neptune, return to the same relative positions. It is obvious, moreover, from the same equations, that *no three of the four outer planets can ever be in conjunction at the same time.*

Bloomington, Indiana, January, 1872.