ART. XLI.—*Brief Contributions from the Physical Laboratory of Harvard College.* No. 15.—*On the construction of Gaugain's Galvanometer*; by JOHN TROWBRIDGE.

IN this instrument the law of the proportionality of the strengths of the electric currents, circulating through the coil of the galvanometer, to the tangents of the deviations of the needle, is exact when the distance of the center of the needle from the center of the coil is equal to one-half the radius of the coil. The theoretical demonstration by Bravais of the truth of this law of the galvanometer leads to the expression

$$\text{Eq. (1.)} \quad i=\frac{T\rho^3}{2\pi R^2}\left[1-\frac{3l^2}{2\rho^2}+\frac{15l^2D^2}{\rho^4}+\frac{15l^2}{4\rho^4}(R^2-4D^2)\cos\delta^2\right]^{-1}\tan\delta$$

in which i=strength of current.
T=horizontal intensity of the earth's magnetism.
R=radius of coil or circular wire.
l=length of needle.
D=distance from center of needle to center of coil.
$\rho^2=l^2+D^2+R^2$.
δ=deviation of needle.

We see from eq. (1) that when $(R^2-4D^2)=0$, or $D=\frac{R}{2}$, that the terms in the parenthesis are of the order $\left(\frac{l}{\rho}\right)^4$ and when $\frac{l}{\rho}$ is small these can be neglected.

In constructing the galvanometer the serious question arises: how much will a small error in the measurement of D and of R affect the law? and which will have the greater effect, an error in the measurement of R or in that of D?

Let us suppose at first that R and D have been measured correctly. We shall then have

$$i=\frac{T}{2\pi R^2}(l^2+R^2+D^2)^3 \tan\delta, \text{ in which } D=\frac{R}{2}.$$

Suppose now that, while R remains constant D differs from its true value by αD, in which α is a very small quantity. We shall then have, neglecting small quantities in Eq. 1:

$$i=\frac{T}{2\pi R^2}(l^2+R^2+(D+\alpha D)^2)^3\tan(\delta-m)$$

in denoting the decrement of the angle of the needle. The difference in value of these two expressions for the strength of a current is

$$i-i_1=\frac{T}{2\pi R^2}\Big((l^2+R^2+D^2)^3\tan\delta-(l^2+R^2+(D+\alpha D)^2)^3\tan(\delta-m.\Big)$$

Suppose now that D remains constant or R varies from its true value by the small quantity βD, in order to produce the same decrement as in the angle; we shall have

$$i_2=\frac{T\ (l^2+D^2+(R+\beta R)^2)^3\ \tan\ (\delta-m.)}{2\pi\ (R+\beta R)^2}$$

$$i-i_2=\frac{T}{2\pi}\Big[\frac{(l^2+R^2+D^2)^3\tan\delta}{R^2}-\frac{(l^2+D^2+(R+\beta R)^2)^3\tan(\delta-m)}{(R+\beta R)^2}$$

The difference in the values of the strengths of the currents are indicated by the final terms in the values of $i-i_1$ and $i-i_2$ which are

$$\frac{(l^2+R^2+D^2\ (1+\alpha)^2)^3\tan\ (\delta-m)}{R^2}$$

$$\text{and}\ \frac{(l^2+D^2+R^2\ (1+\beta)^2)^3\tan\ (\delta-m)}{R^2(1+\beta)^2}$$

Since $D=\frac{R}{2}$ we have by developing and neglecting the squares of α and β,

$$\frac{\left(l^2+\frac{5}{4}R^2+\frac{R^2\alpha}{2}\right)^3}{R^2} \quad \text{and} \quad \frac{\left(l^2+\frac{5}{4}R^2+2\beta R^2\right)^3}{R^2(1+\beta)^2}$$

Neglecting α and β when they are additive to a large whole number, the differences in the strengths of the currents will be expressed by the terms $\frac{\alpha R^2}{2R^{\frac{2}{3}}}$ and by $\frac{2\beta\ R^2}{R^{\frac{2}{3}}}$, α is smaller from the nature of the case than β. Hence

$$\frac{\alpha R^{\frac{4}{3}}}{2}<2\beta R^{\frac{4}{3}}.$$

Therefore an error in the measurement of R will not affect the law of the galvanometer so much as one in the measurement of D.

In order to obtain an idea of the magnitude of errors resulting from changes in the value of D, I arranged a galvanometer so that the coil, consisting in this case of a circle of thick brass wire, the resistance of which could be neglected, could be moved away from its true position, after the manner of Wiedeman's galvanometer. The value of R was 12 cm. and that of D 6 cm.

Value of D+X	Value of δ.
6	20°
7	19
8	18
8·5	17·5
9	16·5

A variation of 1 cm. produced a change in the deviation of the magnetic needle of 1°. (The galvanometer needle was 25 mm. long and was suspended by a single fiber of silk.)

The apparatus was also so arranged that the radius of the circle of brass could be changed, while D remained constant. It was found that the changes in the indications of the needle were inappreciable until the increase of the radius was 5 mm. when the increase amounted to 1 cm.,—a variation of half a degree was produced in the indication of the needle. The increase in resistance of the circuit was inappreciable. An error, therefore, of five millimeters in the determination of the radius of the coil was inappreciable, while the same error in the measurement of the distance of the needle from the center of the coil resulted in, comparatively, a larger error. In approaching the subject from another point of view, it can be proved* that the expression for the attraction of a single circular current on a point situated on the line passing through the center of the coil and perpendicular to its plane is

$$\mathrm{X}=2\pi\mathrm{k}\left\{\frac{a^2}{r^3}-\frac{3a^4}{2r^5}+\frac{15}{8}\frac{a^6}{r^7}-\ .\ .\ .\ .\ .\ \right\}$$

in which k is a constant depending on the strength of the current, a is distance of point from center of coil, and r is the radius of the coil. By inspection it will be perceived that a small variation in the value of a will affect the value of X more than a similar slight variation in the value of r.

* James Stuart, M. A., Phil. Mag., 46, 1873, p. 231.