ART. XVI.—*Researches in Acoustics;* by ALFRED M. MAYER. Paper No. 5.

(Continued from page 109.)

5. *Six Experimental Methods of Sonorous Analysis described and discussed.*

THE remarkable discoveries in sound made in these later times by Helmholtz were owing, in great part, to his having early seen the necessity of obtaining precise knowledge of the composition of sounds,—by means of the methods of sonorous analysis which he had devised,—before one could attempt to give an explanation of the causes of timbre, of the mechanism of audition, and of the physiological causes of musical harmony; and furthermore, he reduced all of the analytic methods and explanations, contained in his classical work, to a harmonious system, by showing how they naturally flowed from the fertile theorem of Fourier. As Helmholtz distinctly states: "In letzter Instanz ist also der Grund der von Pythagoras aufgefundenen rationellen Verhältnisse in dem Satze von Fourier zu finden, und in gewissem Sinne ist diese Satz als die Urquelle des Generalbasses zu betrachten." (Tonempfindungen, p. 346.)

The evident importance of the subject of sonorous analysis will probably render interesting the few remarks I here venture to offer on this subject. I will describe in order six methods of sonorous analysis. Methods 1, 2 and 5 have been used by Helmholtz and König. Methods 3, 4 and 6, as far as I know, originated with me. To render comparable all of the experiments which I shall describe, I shall always use one composite sound of uniform intensity by sounding with a blast of constant pressure an Ut_2 Grenié free-reed pipe, from which has been removed its reinforcing pyramidal pipe.

(1.) *Analysis by means of Resonators applied directly to the ear.*

This method of Helmholtz is so well known that it need not here be described, but I will give some experiments which I have made on its degree of precision under the head of the sixth method of analysis, to be described.

(2.) *Analysis by means of Resonators connected with König's manometric flames.*

This is the least delicate and accurate of all methods of sonorous analysis, but it has a value in giving an objective, ocular illustration, which is sometimes of use. I do not, however, here refer to the use of a manometric flame placed in connection with a conical pipe, into which one sings, and which instrument in the hands of König has done such admirable work in the analysis of the vowel sounds; but I refer to the harmonic series of resonators connected with as many flames which burn near a long revolving mirror.

The following experiments will show the degree of delicacy of the above apparatus. I sounded the reed pipe with its maximum intensity when it was about one foot from the center of the series of resonators, and produced well marked serrations in the Ut_3 and Ut_4 flames, but the other flames showed only slightly indented top borders. It was only when I sang loudly Ut_3 or sounded this same note on a French horn that the edges of the flames became deeply serrated.

(3.) *Analysis by means of Resonators which are successively brought near the source of origin of a composite sound and thus successively reinforce all of its sonorous elements.*

If we take any two resonators separated by a known interval, and vibrate before them the forks of this interval, we can carry these sounds in the memory. Now close the nipples of the resonators with wax and successively bring their mouths near the origin of the composite sound. If the simple sounds to which the resonators are tuned exist as components of the sound, we shall hear them singing out clearly above the general chorus of the other harmonics. Thus I have often successfully shown to an audience the composition of a composite sound.

(4.) *Analysis by means of Resonant boxes carrying solid bodies tuned in unison with the sounds to which the air in these boxes resounds.*

This method is an excellent one when the composite sound can be obtained with intensity, when the boxes are accurately in tune with the solid bodies (forks or strings) attached to them, and when the boxes are in unison with the sonorous elements of the composite sound which we would analyze. The last mentioned condition is not always fulfilled in the boxes on which forks have been sometime mounted, for the former are apt to change their interior capacity by warping. This fact can readily be ascertained by partly introducing the hand into the mouth of the box, and noting the effect on the intensity of the sound.

This method of analysis is similar to the one previously described; for the resonant box of a fork acts like a resonator and can be used to intensify any harmonic of a composite sound; but there is an important difference in the methods, for the fork or the string being in unison with the proper note of the mass of air in the box, is set in vibration by the latter, so that after the box has been removed from the vicinity of the origin of the composite sound, and the latter has ceased, we find that the fork sings out alone, and thus shows that it has selected from a chorus of harmonics that one which is in unison with its own tone. I have thus been able, by placing one fork after another, of the series of the harmonics of the Ut_2 reed, to show the composition of its sound to a large audience with entire satisfaction. I have also succeeded with the following experiment. Forcibly sound the reed, and place around the opening of its "stump" all of the eight forks, of the harmonic series of Ut_2, with the mouths of their resonant boxes toward the reed. After the reed has sounded for a few seconds, stop it, and we shall find that all of the forks are in vibration; and thus singing together, they approximately reproduce the sound of the reed. This experiment to succeed requires the resonant boxes, the forks, and the harmonics of the reed to be in exquisite unison.

(5.) *Analysis by means of the beating of simple sounds of known pitch with the harmonics of the composite sound to be analyzed.*

If we use forks for the simple sounds, it will be better slightly to flatten or sharpen the note of the sound to be analyzed. Then knowing the number of beats that the fundamental of the note gives with its corresponding fork, we can designate the number of any other harmonic by the number of beats it makes with a fork of known pitch; for the number of beats observed, when referred to the number of beats of the fundamental as unity, will be directly as the number of the harmonic in the series. If we cannot well alter the pitch of the composite sound, then it will be well to use forks with sliding weights on graduated prongs, or loaded strings accurately tuned and provided with meter scales. In this system of analysis it is very important to guard against being led astray by the beating of resultant tones; therefore, the forks or strings should be gently vibrated and resonators used to assist the ear.

(6.) *Analysis by means of a loose membrane which receives the composite sonorous wave and transmits its vibrations through filaments or light rods to a series of forks mounted on their resonant cases.*

This method of analysis is the one we devised in our experimental confirmation of Fourier's theorem and described in sec. 1 (pp. 82–84). We here wish to call attention to the precision of this method of analysis, while at the same time acknowledging the delicate instrumental conditions required to use it successfully. The principal interest attached to it is that it shows in a vivid manner the operation of the instantaneous decomposition of a composite wave into its elementary pendulum-vibrations. The method has also peculiar interest as showing, in the most striking manner, the exaltation of the action of very feeble periodic impulses to such degrees of intensity as to set into synchronous vibration very large masses of matter; and it may be well before we discuss the subject proper of this section to call attention to this very interesting result, as set forth in the following experiment with our apparatus. To the membrane, covering the hole in the box of the reed-pipe, I attached one end of a fiber of silk-worm cocoon, one meter long and weighing one milligram. The other end of the fiber was cemented to the face of a prong of an Ut_2 fork. This fork weighed 1,500 grams, while the top of its resonant box and the air in the latter weighed respectively 102 and 22 grams. Therefore, the fiber set in motion 1,624,000 times its own weight by only a fraction of the force which traversed it, and even this force was a yet more minute fraction of the whole energy of the aerial vibrations produced by the reed.

An experiment like the above is instructive as an analogical illustration of the manner in which we may imagine an ætherial vibration to produce chemical decomposition by causing such powerful synchronous vibrations in the molecules of compounds as to shake their atoms asunder; and we have already seen how very feeble impulses sent through a medium of great tenuity can, when rapidly recurring and of the proper period, produce mechanical effects which at first sight appear incredible. Time is required in both cases to produce an appreciable action. The time required in the case of the sonorous vibrations decreases as their number per second increases, and in the case of the ætherial vibrations we have analogical phenomena. In the acoustical experiment, if the fork be much out of tune with the pulses transmitted by the fiber, no motion is produced in the fork; likewise, we may imagine that when the period of vibration of one or more of the constituent atoms of a certain molecule is far removed from unison with any of the ætherial vibrations falling upon it, no motion, or chemical decomposition, will ensue.

The analogy between the two classes of phenomena is yet more striking when we remember that the fork *selects*, from the composite vibratory motion which traverses the fiber, only that vibration which is in unison (or only slightly removed from unison) with its own proper periodic motion; so, likewise the molecule, or atoms of the molecule, select only those vibrations from the ray which are in unison with their own atomic periods; and on the tuning of the atom depends whether the result of the action of the ray will evince itself as heat, phosphorescence, chemical action, or fluorescence.

The following experiments show that the method of analysis we are now discussing surpasses in delicacy and sharpness of definition any other method in which sympathetically vibrating bodies are employed. As already shown, the forks select from the composite vibratory motion which strikes them only those simple vibrations which are in unison with their own vibratory periods. This remark, however, requires some modification, though the qualification necessary is less than is required when other similar methods of analysis are used. In all cases of co-vibration there is a certain range of pitch, above and below the sound which is in unison with the existing vibration, through which the co-vibrating body responds. The farther the remove from unison the weaker the response.* But in some cases a slight remove of the pitch from unison will cause a great diminution in the intensity of the co-vibrations, while in others the same departure from unison causes only a slight or even inappreciable change in their intensity. For example, I connected the Ut_3 fork—the second in the harmonic series of Ut_2—to the membrane on the Ut_2 reed, and sounded the latter during a few seconds. After the reed was silent, I heard the fork sounding with intensity.† But, on loading the fork with a

* See Helmholtz's Tonempfindungen, p. 217, for the law connecting the variation of intensity of co-vibration with the variation of pitch in the existing body.

† It is perhaps hardly necessary to state that in all of these experiments I first ascertained that when the fiber was detached from the fork, the latter did not emit a sound caused by the action of the intervening vibrating air. This condition is readily attained by screening the mouths of the resonant boxes, or by turning these mouths away from the sounding reed. Also, I should here state that the reed was tuned to the fork after the fiber was stretched between the latter and the membrane on the reed-pipe.

piece of wax, so that it gave *five* beats per second with the note of the fork when unloaded, I could not, by any variation in the tension of the fiber or of any other circumstances of the experiment, set in vibration the fork by means of the pulses sent from the reed through the fiber; yet, on placing the nipple of an Ut$_3$ resonator in my ear, I perceived that this flattened note of the fork produced a decided resonance, thus showing that although the fork could not respond to its own note flattened five beats per second, yet the resonator, under the same circumstances, did enter into sympathetic vibration. When the fork gave *four* beats per second it responded to the reed, but this response was only audible on placing the ear close to the mouth of the fork's resonant box. With *three* beats per second the sound of the fork was readily perceptible, while the resonator reinforced it very decidedly. When the fork was out of unison *two* beats per second, its sound was slightly increased; and with a departure of *one* beat per second, the response of the fork was yet stronger, but greatly inferior in intensity to that produced when the fork was in unison with its proper sound—the second harmonic of the reed; yet the resonator reinforces this flattened sound as forcibly as it does that which emanates from the unloaded fork. These facts concerning the want of sharpness in the detection of pitch by means of resonators are not in accordance with the statements made in recent popular works on sound, where the resonator is described as remaining dumb until the exact pitch to which it is tuned is reached, when it responds with a suddenness which has been compared to an explosion!

The fork, the stretched fiber and the intensity of the sound of the reed remaining in the same conditions as in the above experiments, I gradually unloaded the fork until it made only *one beat in eight seconds*, and yet even this slight departure from unison with the second harmonic of the reed was evident in the difference in the intensities of the fork's responses when thus loaded and when the wax was removed. This fact I have repeatedly confirmed by testing the intensities of the two sounds by different hearers, who were placed so that they could not see when the fork was loaded or unloaded. Now E. H. Weber has found that only the most accomplished musical ears can distinguish between the pitch of two notes whose vibrations are as 1000 to 1001, but by the above method we can readily detect a departure from unison in the two notes amounting to the interval of 2000 to 2001, or to the $\frac{1}{120}$th of a semitone.

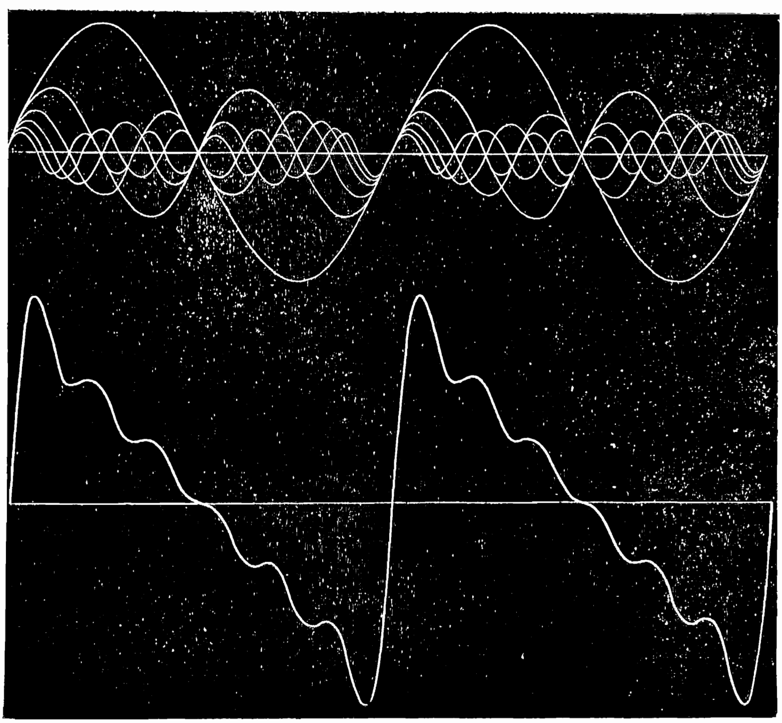
In connection with the preceding observations, the following experiments on resonators and sympathetic vibrations may be of interest. I substituted for the ear the manometric flames of König, viewed in a revolving mirror, and tested the response of the resonators to sounds not in accord with their proper notes. The results agreed with those previously obtained on placing the resonator to the ear. I now mounted an Ut_3 fork on its resonant case, and sounding it *strongly* before all the resonators of the harmonic series of Ut_2, I caused all of the manometric flames connected with these resonators to vibrate, each giving the same number of serrations as when the Ut_3 fork was brought near its own resonator. The same result was obtained when the fork was separated from its case, with this important difference, however, that when the face of fork Ut_3 was brought near the mouth of the Ut_4 resonator, the flame connected with this resonator gave at the same time serrations belonging to both Ut_3 and to Ut_4, and this result accords with the following experiment. If one sounds the Ut_3 fork on its box, and brings the mouth of the latter near the mouth of the box of the Ut_4 fork, the Ut_4 fork will co-vibrate, and after Ut_3 has ceased to vibrate, Ut_4 will sound out quite clearly. If, however, Ut_4 be lowered in tone, by weighting it with a piece of wax, so that it gives two beats per second with its proper tone, then the Ut_4 fork cannot be set into sympathetic vibration by impulses from Ut_3. Also, if forks Ut_3 and Ut_4 be detached from their boxes, and fork Ut_3 be strongly vibrated while the face of one of its prongs remains quite close and parallel to the face of a prong of fork Ut_4, the latter is set in vibration by the impulses of Ut_3 sent through the intervening air. Fork Ut_3 was now loaded so that it successively made 1, 2, 3, and 4 beats per second with the true Ut_3 pitch. When it made 3 beats per second, it caused Ut_4 to vibrate so feebly as to be barely audible, while 4 beats per second departure of Ut_3 from unison produced no effect on Ut_4.

The following experiment was now made to show the want of precision in the determination of the exact pitch of sonorous elements by means of resonators. We have just seen that the Ut_3 fork could not vibrate the Ut_4 fork when both forks were on their cases, and when Ut_3 was flattened by two beats per second; and also that a departure of four beats per second prevented Ut_3 from setting Ut_4 in sympathetic vibration, when both forks were off their resonant cases, and with their prongs close and parallel to each other. But, when the Ut_3 fork was loaded so that it made from 15 to 20 beats with its proper tone, it caused the serrations of the Ut_3 resonator to appear, accompanied by the serrations of its octave. The Mi_3 fork, although it developed the serrations belonging to its own pitch when brought opposite the Ut_3 resonator, yet did not,—as might have been expected,—develop its own tone and the octave when brought near the mouth of the Ut_4 resonator. It is probably not necessary to add that the above effects of simultaneously developing in the flame the serrations of the proper note of a resonator and those of its octave are only produced when the fundamental sound is intense.

(7.) *The Curve of a Musical Note, formed by combining the sinusoids of its first six harmonics; and the curves formed by combining the curves of musical notes corresponding to various consonant intervals.*

We have already seen that any composite vibration, which produces in us the sensation of a musical note, can always be reproduced by the simultaneous production of a certain number of the simple sounds of a harmonic series, provided these simple sounds have the proper relative intensities. Therefore to obtain the resultant curve corresponding to a musical note, we draw on one axis its harmonic components with their proper wave-lengths and amplitudes, and the algebraic sums of their corresponding ordinates are the ordinates of the required resultant curve.

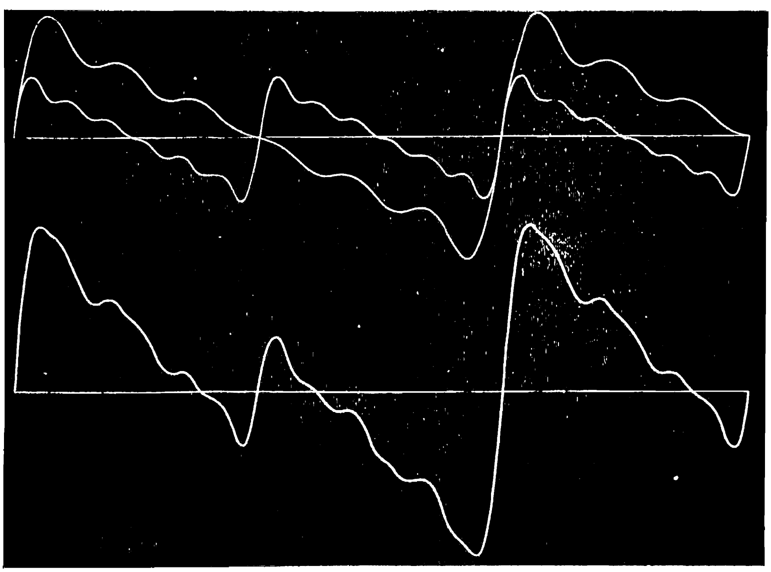
3.



Curve of a Musical Note; being the resultant of the simple vibrations of its first six harmonics.

Fig. 3 is the curve of a musical note, being the resultant of the simple vibrations of its first six harmonics. The first six harmonics having been drawn on a common axis, I erected 500 equidistant ordinates, and extended these ordinates some distance below the axis on which I desired to construct the resultant. The algebraic sum of the ordinates, passing through the harmonic curves, were transferred to the corresponding ordinates of the lower axis, and by drawing a continuous line through these points, I formed the resultant curve. The first six harmonics are alone used in the combinations which I have given, because the 7th, 9th, 11th harmonics, and the major number of those above the 12th, form dissonant combinations with the lower and more powerful harmonics. Indeed, the harmonics above the 6th are purposely eliminated from the notes of the piano by striking the string in the neighborhood of its 7th nodal point. The amplitudes of the harmonics of fig. 3 are made to vary as the wave-length; not that this variation represents the general relative intensities in such a composite sound, but they were so made to bring out strongly the characteristic flexures of the resultant. To simplify the consideration of the curves, they are all represented with the same phase of initial vibration. Of course the resultants have an infinite variety of form, depending on the difference in the initial phase, and on the amplitudes of the harmonic elements.

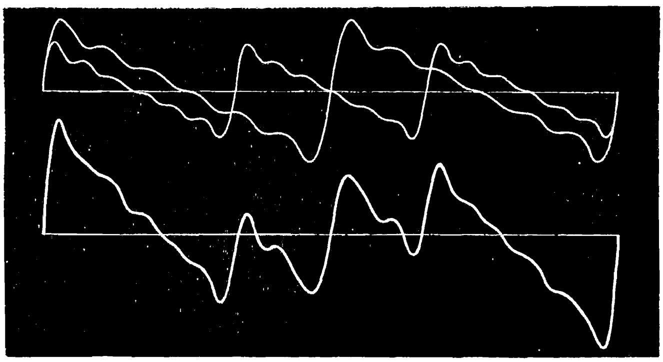
4.



Resultant curve of a musical note combined with its octave. $\lambda : \lambda' :: 1 : \frac{1}{2}$.

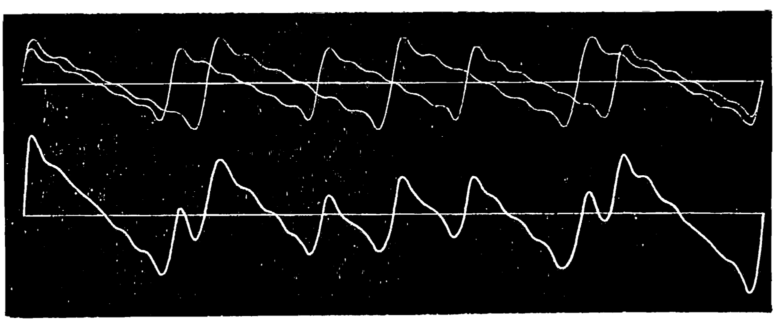
In figs. 4, 5 and 6, I have drawn the resultant curves formed by combining the curves of musical notes corresponding to the various consonant intervals indicated below the curves. As these curves are the resultants formed by the combination of the composite vibrations of musical notes, it follows that the components of these curves are not simple harmonics, as in the case of fig. 3, but are derived from the *resultant* of fig. 3, by reducing to one-fourth the amplitude of that curve and by taking wave-lengths corresponding to the intervals indicated below the figures.

5.



Resultant curve of a musical note combined with its fifth. $\lambda : \lambda' :: 1 : \frac{2}{3}$.

6.



Resultant curve of a musical note combined with its major third. $\lambda : \lambda' :: 1 : \frac{4}{5}$.

All of the curves which I have given in this paper were first drawn on a large scale and then reduced by photography to a size suitable to be transferred to the engraver's block.

(8.) *Experiments in which are produced from the above curves (sec. 6) the Motions of a Molecule of Air when it is animated with the resultant action of the six elementary vibrations forming a musical note; or is set in motion by the combined action of sonorous vibrations forming various musical intervals.*

We may imagine the curve corresponding to a musical note, represented in fig. 3, as formed by the trace of a vibrating molecule of air, or of a point of the tympanic membrane, on a surface which moves near these points. Therefore if we slide this curve along its axis, under a slit in a screen which allows only one point of the curve to appear at once, we shall reproduce in this slit the vibratory motion of the aerial molecule and

7.

of the point on the tympanic membrane. I have exhibited this motion in a continuous, or rather, recurring manner, as follows: On a piece of Bristol board I drew a circle, and in one quadrant of this circle I drew 500 equidistant radii. On these radii, as ordinates, I transferred the corresponding values of the same ordinates of the resultant of fig. 3, diminished to one-fourth of their lengths. I thus deflected the axis of curve fig. 3 into one-fourth of a circle curve; and this repeated four times on the Bristol board, rendered the curve continuous and four times recurring, as shown in fig. 7. I now cut this curved figure out of the board and used it as a template. I placed the latter centered on a glass disc of 20 inches in diameter. The disc was covered on one side with opaque, black varnish, and with the template and the separated points of a pair of spring-dividers, I removed from the glass disc a sinuous band, as shown in fig. 7. The glass disc was now mounted on a horizontal axis and placed in front of a lantern the diameter of whose condensing lens was somewhat greater than the amplitude of the curve. The image of that portion of the curve which was in front of the condenser was now projected on a screen, and then a piece of card board having a narrow slit cut in it was placed close to the disc, with the slit in the direction of one of its radii. On now revolving the disc I reproduced on the screen the vibratory motion of a molecule of air, or of a point on the tympanic membrane, when these are acted on by the joint impulses of the first six harmonic or pendulum vibrations, forming a musical note. On slowly rotating the disc one can readily follow the compound vibratory motion of the spot of light; but on a rapid revolution of the disc, persistence of visual impressions causes the spot to appear lengthened into a band; but this band is not equally illuminated—it has six distinct bright spots in it, beautifully showing the six inflections in the curve.

By sticking a pin in the center of fig. 7, as an axis about which revolves a piece of paper with a fine slit, the reader can gain some idea of the complex motion which I have described.

From the curves of figures 4, 5 and 6 can similarly be reproduced their generating motions. Of course it is understood that in all these cases the amplitude of these vibrations are enormously magnified when compared with the wave-lengths, and that it is really only when the amplitudes of the elementary pendulum vibrations are infinitely small that the resultant curves we have given can be rigorously taken as representing what they purport to; for the law of the "superposition of displacements" depends on the condition that the force with which a molecule returns to its position of equilibrium is directly proportional to the amount of displacement, and this condition only exists in the case of infinitely small displacements; yet the law holds good for the majority of the phenomena of sound.

As a periodic vibration can alone produce in the ear the sensation of sound, and as the duration of the period is always equal to the least common multiple of the periods of the pendulum vibrations of the components, it follows that in the case of a simple sound, or of a sound formed of a harmonic series, that the period equals the time of one vibration of the fundamental; but in the case of any other combinations the duration of the period of the recurring vibration increases with the complexity of the ratio of the times of vibration of the components; thus, the durations of the following combinations are placed after them in fractions of a second.

$$C_3+C_4=\tfrac{1}{256};\ C_3+G_3=\tfrac{1}{128};\ C_3+E_3=\tfrac{1}{64};\ C_3+E_3+G_3=\tfrac{1}{256};$$
$$C_3+E_3+G_3+C_4=\tfrac{1}{17}\text{th of a second.}$$

The above mentioned facts suggest a curious physiological inquiry, viz: Does it require a combination of sounds, simple or composite, to remain on the ear the duration of an entire period, in order that it shall give the same sensation as is produced when the same combination is sounded continuously? In other words, will a portion of the recurring composite vibration produce the same sensation as an entire period or several periods? The solution of this problem has been the object of a prolonged experimental research, but up to this time the results have been so difficult of interpretation that I have not arrived at any certain knowledge on the subject. I shall, however, return to this interesting but very difficult work.