ART. V.—*The motions of Fluids and Solids relative to the Earth's Surface;* by W. FERREL, Assistant in the Nautical Almanac Office.

1. UNDER the above title I have published a paper in the Mathematical Monthly, in which I endeavored, by a complete analytical investigation of the motions of fluids surrounding the earth, arising from the different circumstances of a difference of pressure caused by a difference of temperature, combined with the modifying forces arising from the earth's rotation, to give a satisfactory explanation of all the general motions of the atmosphere and the ocean, the cause of the greater barometric pressure of the atmosphere near the tropics than at the equator and the poles, and of the greater pressure generally in the northern hemisphere than in the southern, to account for the motions of revolving storms in both hemispheres, from the equator towards the poles in the parabolic paths, and to establish completely their gyratory character; none of which phenomena had ever been satisfactorily accounted for by any of the theories which do not take into account the effect of the earth's rotation. It has been suggested by several that a paper more popular in its character, although less complete, which should contain only the more essential parts of the analysis, and in which familiar illustrations should supply in some measure the more difficult parts of the analysis, would be more satisfactory to many readers. It is proposed, therefore, in this paper, in consequence of the general interest taken in the subject, to treat it acccording to this suggestion, and to give only the most essential part of the analysis, showing the influence of the earth's rotation, which being based upon well known principles, instead of being deduced from general fundamental equations, is very simple, but is sufficient for a general understanding of the subject.

I. *The effect of the earth's rotation upon moving bodies at its surface.*

2. If a body were set in motion upon the surface of the earth, supposed to be entirely without friction, it would not in general move in the circumference of a great circle around the earth, but would be continually deflected to one side by a force arising from the earth's rotation.

Let r be the radius of the earth,

θ the polar distance in arc,

φ the longitude, and

n the angular velocity of the earth's rotation.

Then $r \sin \theta$ is the distance of a body on the earth's surface from the axis of rotation, and $r \sin \theta n^2$ the centrifugal force in the direction of a perpendicular to the earth's axis, arising from the earth's rotation. If in addition to the angular motion common to all bodies at rest on the surface of the earth, the body has an angular motion $D_t\varphi$ relative to the earth, then the centrifugal force becomes $r \sin\theta\,(n+D_t\varphi)^2$. Now if we resolve the preceding force in the directions of the meridian and a perpendicular to it, the part acting in the direction of the meridian, neglecting the small effect of the earth's ellipticity, is $r \sin\theta \cos\theta\,(n+D_t\varphi)^2$. The part of this force which gives ellipticity to the earth's surface, and which is necessary to keep a body at rest on the elliptical surface, and prevent it from sliding toward the pole, is $r \sin\theta \cos\theta\, n^2$. Hence the difference of these two forces, when the body has a motion eastward relative to the earth, is a deflecting force which has a tendency to cause the body to move from the pole toward the equator. The difference of these forces is $r \sin\theta \cos\theta\,(2n+D_t\varphi)\,D_t\varphi$, and hence when the body is entirely free to move in any direction, we have

$$(1.)\qquad r\,D_t^2\theta = r \sin\theta \cos\varphi\,(2n + D_t\varphi)\,D_t\varphi.$$

3. Again, if the body has a motion toward or from the pole, it must satisfy the well known principle of the preservation of areas, so that as it approaches the pole, and consequently the axis of rotation, the angular motion must be increased, that is, it must acquire a motion eastward relative to the earth, but if it recedes from the pole, it must acquire a relative westward motion. In order to satisfy the preceding principle, the motion must satisfy the following equation:

$$(2.)\qquad r^2 \sin^2\theta\,(n + D_t\varphi) = \text{constant.}$$

Taking the derivative with regard to t we get

$$(3.)\qquad r \sin\theta\,D_t^2\varphi = -2r \cos.\theta\,(n + D_t\varphi)\,D_t\theta.$$

4. Equations (1) and (3) determine the motions of a free body on the earth's surface. If the body is constrained to move either in the direction of a meridian or a parallel of latitude only, the deflecting force instead of causing a deflection, causes only a pressure. If we put P for the part of the force or pressure acting in the direction of the meridian, which depends upon the earth's rotation only, and Q for the part depending upon the earth's rotation, which acts in the direction of a parallel of latitude, we have

$$(4.)\qquad \begin{cases} P = 2rn \sin\theta \cos\theta\,D_t\varphi \\ Q = -2rn \cos\theta\,D_t\varphi. \end{cases}$$

In the preceding equations $rD_t\theta$ represents the lineal velocity of the body in the direction of the meridian, and $r \sin\varphi D_t\varphi$ the lineal velocity relative to the earth in the direction of the parallel of latitude. Hence the deflecting force is the same in both directions in the same lineal velocity.

5. If v is the velocity of a body moving in any direction whatever, and F the deflecting force perpendicular to this direction, by resolving the preceding forces and velocities in the direction of v and the perpendicular to it on the right, we get

$$(5.) \qquad F = 2nv \cos\theta.$$

In the northern hemisphere $\cos\theta$ is positive, but in the southern negative. Hence, we have established this important principle, *in whatever direction a body moves, it is always deflected to the right in the northern hemisphere, and the contrary in the southern hemisphere.*

The forces resolved in the direction of v cancel each other, and hence the velocity is never accelerated or retarded.

6. Since rn^2 represents the centrifugal force at the equator arising from the earth's rotation, and is known to be $\frac{1}{289}$ of g or gravity, the preceding equation may be reduced to

$$(6.) \qquad F = \frac{2v\cos\theta}{289\,n}\,g.$$

This form of the equation is convenient for comparing the force F with gravity.

7. In the preceding equations rn is the lineal velocity of the equator, and is equal to 1523·2 feet, the second being the unit of time. Hence $n = {\cdot}000072924$. Also $g = 32{\cdot}2$ feet.

It may be remarked here that the preceding deflecting force is not an absolute force, such as would be required to deflect a moving body from a fixed direction in space, but is only relative, being somewhat of the nature of a centrifugal force, and arises from the fact that the direction relative to the earth to which the moving body is referred, is continually changing its direction with regard to fixed directions in space.

II. *The general Motions and Pressure of the Atmosphere.*

8. By the general motions of the atmosphere are meant all those motions produced by a difference of density between the equatorial and polar regions arising principally from a difference of temperature. If the atmosphere in all parts of the earth had the same density, every part would be in a state of statical equilibrium, and its surface and the strata of equal density would assume the elliptical figure of the earth's surface, and consequently the pressure of the atmosphere at the earth's surface would be everywhere the same. But the temperature of the atmosphere being less and consequently its density greater in the polar than the equatorial regions, the greater pressure of the polar regions causes the surface of the atmosphere and the strata of equal density in the equatorial regions to rise a little above the level of equilibrium, and hence the atmosphere in the upper regions flows toward the poles, while the greater pressure of the

polar regions causes a counter current toward the equator in the parts nearer the earth's surface, which would extend down to the earth's surface, if it were not for the modifying causes arising from the friction of the earth's surface, which will be explained. If the earth had no rotation on its axis, this interchanging motion between the equatorial and polar regions would be exactly in the directions of the meridians; but having a rotation, from what has been demonstrated in the preceding section, the atmosphere above in flowing toward the poles acquires an eastward motion relative to the earth's surface, and after descending in the polar regions and flowing back nearer the earth's surface toward the equator, it tends toward the west, and on arriving in the equatorial regions it has a westward motion. If it were not for the resistance of the earth's surface, the mutual actions of the strata upon one another, whatever the initial state of the atmosphere, would cause them to have finally the same east or west motion at all heights in the same latitude, and this motion would be such as to satisfy equation (2), and also, since the mutual actions of the strata upon one another could not affect the sum of the moments, it would be such that the sum of the moments of all the particles would be the same as that of the initial state arising from the earth's rotation and from any initial motion relative to the earth, which it might have had. This latter condition determines the constant in equation (2), which was shown in my paper in the Mathematical Monthly, to which I must refer for the method, to be, on the hypothesis of an initial state of rest relative to the earth, equal to $\frac{2}{3} r^2 n$. With this value of the constant the equation gives

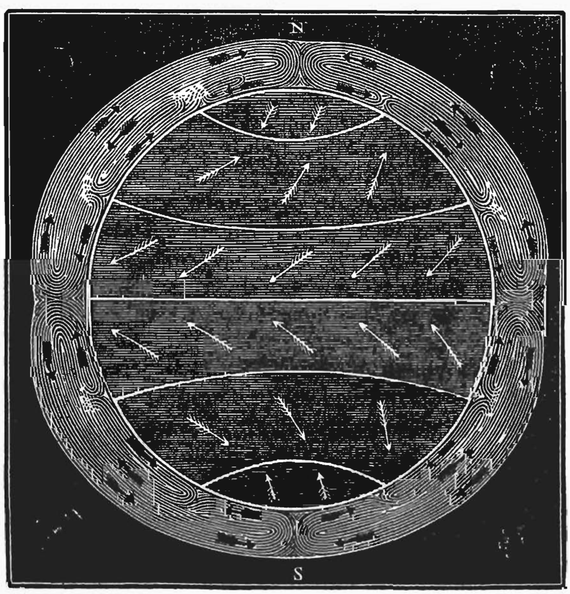
$$(7.) \qquad D_t\varphi = \left(\frac{2}{3 \sin^2\theta} - 1\right) n.$$

9. Near the poles, where $\sin^2\theta$ is very small, $D_t\varphi$ and consequently the lineal velocity $r \sin\theta D_t\varphi$ must be very great. Toward the equator $D_t\varphi$ is negative, and hence the motion there is westward. Putting $D_t\varphi = 0$, the equation gives $\sin^2\theta = \frac{2}{3}$, and hence the sine of the latitude where there is no motion east or west is equal to $\sqrt{\frac{1}{3}}$, which answers nearly to the parallel of 35°. Hence between this parallel and the poles the motion is eastward, but between it and the equator, toward the west. From what has been shown therefore in § 2, the eastward motion toward the poles, and the westward toward the equator, since $D_t\varphi$ is negative there, must both cause a pressure toward the parallels of 35°, and consequently an accumulation of atmosphere there, and a depression at the poles and at the equator. The amount of the pressure is represented by the term $r \sin\theta \cos\theta (2n + D_t\theta) D_t\varphi$, and hence toward the pole, where $D_t\varphi$ is very large, this pressure is very great, and at the poles it would

be infinite. Hence at the poles and at some distance from them, on the hypothesis of no resistances from the earth's surface, the atmosphere cannot exist. Between the parallels of 35° and the equator $D_t\varphi$ is less and also $\cos\theta$, and consequently the pressure from the equator and the depression there are comparatively small. It was shown in the Mathematical Monthly that in the case of a fluid surrounding the earth five miles high, the fluid would recede from the poles about 28° and be depressed at the equator about 4000 feet. This, however, is upon the hypothesis that the upward expansion of the atmosphere arising from a greater temperature is insensible.

10. The preceding results, it must be remembered, are all upon the hypothesis that the atmosphere is not resisted in its motions by the friction of the earth's surface. Although these results are much modified by the resistance of the earth's surface, yet they will be of great advantage in explaining its general motions; for as there can be no resistance until there is motion, the atmosphere must have a tendency to assume, in some measure, the same motions and figure as in the case of no resistances. Hence, towards the poles the general motions of the atmosphere must be towards the east, and in the torrid zone towards the west;

1.



but as these motions, in consequence of the resistances, are small in comparison with those in the case of no resistances, instead of the atmosphere's receding entirely from the poles, there must be only a comparatively small depression there, as represented in the figure, and instead of its being about 4,000 feet lower at the equator than at the place of its maximum height near the tropics, there must be only a very slight depression there, both on account of the small pressure from the equator, and also on account of the upward expansion arising from a greater temperature.

11. That the atmosphere must assume thc preceding figure in consequence of the eastward motion toward the poles and westward motion near the equator, will be readily understood from the following illustration. It is well known that if the atmosphere had the same motion of rotation with the earth, that it would assume the same figure. Now, if the whole atmosphere had a greater angular motion, that is, if it had an eastward motion relative to the earth, the increased centrifugal force would evidently cause it to accumulate at the equator and to be depressed at the poles. On the other hand, if it had a less angular motion, that is, a motion westward relative to the earth, it would accumulate at the poles and be depressed at the equator. Hence, since the motion is eastward in the higher latitudes and westward near the equator, it is plain that there must be a depression at both the poles and the equator, and consequently an accumulation having its maximum height at some parallel between.

12. The force which overcomes the resistance of the earth's surface to the east and the west motions of the atmosphere depends upon the term in equation (3) containing $D_t \theta$ as a factor, which depends upon the interchanging motion of the fluid between the equatorial and the polar regions, and hence the term must vanish at the equator and the poles. All the east or west motion of the atmosphere is consequently destroyed by the resistances at these places, and hence as $D_t \theta$ vanishes there also, there is a belt of calms at the equator, called the equatorial calm belt, and there must be also a region of calms about the poles.

13. As the motion of the atmosphere is east towards the poles and west near the equator, somewhere between the equator and the poles there must be a parallel of no motion east or west, which, in the case of no resistance, was determined upon the hypothesis of an initial state of rest, and found to be at the parallel of 35°, (§ 9). In the case of the atmosphere this parallel is entirely independent of the initial state of the atmosphere, and depends in a great measure upon the law of resistance, and hence it cannot be accurately determined. It is evident, however, that the east and west motions of the atmosphere at the earth's surface must be such that the sum of the resistances of each part of the earth's surface multiplied into its distance from the axis of rotation, must be equal to 0, else the velocity of the earth's rotation would

be continually accelerated or retarded, which cannot arise from any mutual action between the surface of the earth and the surrounding atmosphere. Now, as the part of the earth's surface where the motion of the atmosphere is west is much farther from the axis than the part where it is east, the latter part must comprise more than half of the earth's surface, unless the velocity of the eastern motion towards the poles is much greater than that of the western motion near the equator. Therefore, since one-half of the earth's surface is contained between the parallels of 30°, the parallels of no east or west motion at the earth's surface must fall within these parallels, and they are accordingly found on the ocean to be near the tropics. Hence the maximum height of the atmosphere, as represented in the figure, must also be near the same parallels.

14. The increase of pressure arising from the accumulation of atmosphere near the tropics, caused principally by the deflecting forces (§ 5) arising from the more rapid east and west motions of the atmosphere in the upper regions, where there is least resistance, gives the atmosphere a tendency to flow from beneath this accumulation both towards the equator and the poles, since the motions, and consequently the forces, which cause this accumulation, are much less near the surface. But on account of the greater density of the atmosphere towards the poles, it has a tendency also to flow, at the earth's surface, from the poles towards the equator. Between the parallels of greatest pressure and the equator, these tendencies combine, and produce a strong surface current, which, combining with the westward motion there, gives rise to the well-known northeast wind in the northern hemisphere, and the southeast wind in the southern hemisphere, called the trade winds. But between the parallels of greatest pressure and the poles, these tendencies are opposed to each other, and the one arising from the accumulation of atmosphere near the tropics being the greater in the middle latitudes, causes the atmosphere to flow at the earth's surface towards the poles; and this motion, combining with the general eastward motion of the atmosphere in those latitudes, gives rise to the southwest wind in the northern hemisphere and the northwest wind in the southern hemisphere, called the passage winds.

15. Near the poles, the tendency to flow towards the equator seems to be the greater, and causes a current there *from* the poles, which, being deflected westward (§ 5,) causes a slight northeast wind in the north frigid zone, and a southeast wind in the south frigid zone. But this is only near the earth's surface; and the general tendency of the atmosphere in the upper regions must be towards the east, as will be seen.

16. Since the atmosphere near the tropics can have no motion in any direction at the earth's surface, there are calm belts there,

called the tropical calm belts. Near the polar circles, where the polar and passage winds meet, there must also be calm belts, which may be called polar calm belts. The motions of the atmosphere, therefore, at the earth's surface, if they were not modified by the influence of continents, would be as represented in the interior of the figure, in which the heavy lines represent the calm belts. On account of the influence of the continents, these belts are somewhat displaced and irregular, and on account of the varying position of the Sun, they change their positions a little in different seasons of the year.

The southern limit of the polar winds in the northern hemisphere, and also the limit between the trade and passage winds, has been determined by Prof. J. H. Coffin, from the discussion of a great number of observations at different points, and given in a chart, in his treatise on the winds, published in the seventh volume of the Smithsonian Contributions.

17. That the atmosphere is depressed at the equator and the poles, and has its maximum height near the tropics, as has been represented, is indicated by barometrical pressure. It was formerly thought that this pressure, at the level of the ocean, was very nearly 30 inches in all latitudes ; but it is now well established that it is much less towards the poles than near the tropics, and also a little less at the equator. Says Captain Wilkes: " The most remarkable phenomenon which our observations have shown is the irregular outline of the atmosphere surrounding the earth as indicated by the pressure upon the measured column at different parts of the surface. Our barometrical observations show a depression within the tropics, a bulging in the temperate zone, again undergoing a depression on advancing towards the arctic and antarctic circles."

Says Sir James Ross :* "Our barometrical experiments appear to prove that the atmospheric pressure is considerably less at the equator than near the tropics ; and to the south of the tropic of Capricorn, where it is greatest, a gradual diminution occurs as the latitude is increased, as will be shown from the following Table, derived from hourly observations of the height of the column of mercury between the 20th of November, 1839, and the 31st of July, 1843."

Extract from Ross's Table.

Latitude.	Pressure.	Latitude.	Pressure.	Latitude.	Pressure.
	inches.		inches.		inches.
Equator,	29·974	42° 53′	29·950	55° 52′	29·360
13° 0′ S.	30·016	45 0	29·664	60 0	29·114
22 17	30·085	49 8	29·467	66 0	29·078
34 48	30·023	51 33	29·497	74 0	28·928
		54 26	29·347		

* Voyage to the Southern Seas, vol. ii, p. 383.

18. The following table, first published by M. Schouw, and reduced here from millimetres to English inches, shows that there is a similar bulging of the atmosphere in the middle latitudes and depression at the pole in the northern hemisphere, as has been observed in the southern hemisphere.

Place.	Latitude.	Pressure.	Place.	Latitude.	Pressure.
		inches.			inches.
Cape,	33° 0′ S.	30·040	London,	51° 30′	29·961
Rio Janeiro,	23 S.	30·073	Altona,	53 30	29·937
Christianburg,	5 30 N.	29·925	Dantzic,	54 30	29·925
La Guayra,	10	29·928	Konigsberg,	54 30	29·941
St. Thomas,	19	29·941	Apenrade,	55	29·905
Macao,	23	30·039	Edinburgh,	56	29·851
Teneriffe,	28	30·087	Christiana,	60	29·866
Madeira,	32 30	30·126	Bergen,	60	29·703
Tripoli,	33	30·213	Hardanger,	60	29·700
Palermo,	38	30·036	Reikiavig,	64	29·607
Naples,	41	30·012	Godthaab,	64	29·603
Florence,	43 30	29·996	Eyafiord,	66	29·669
Avignon,	44	30·000	Godhaven,	69	29·674
Bologna,	44 30	30·008	Upernavik,	73	29·732
Padua,	45	38·008	Mellville Isle,	74 30	29·807
Paris,	49	29·976	Spitzbergen,	75 30	29·795

19. From the preceding tables, it is seen that the barometric pressure is much less, especially in the southern hemisphere, towards the poles than at the equator, although the density towards the poles is much greater, and hence the depression there must be considerable.

20. It has been seen (§ 1), that, in consequence of the earth's rotation, the interchanging motion of the atmosphere between the equator and the poles give rise to a force, by which this motion itself is counteracted. For instance, the motion toward the poles in the upper regions causes an eastward motion which gives rise to a force toward the equator, and which, consequently, counteracts the motion toward the poles, and the motion toward the equator produces a westward motion which gives rise to a force acting in the direction of the poles, which counteracts the motion toward the equator. The motion of the atmosphere, therefore, between the equator and the poles, is not produced by the whole force arising from the difference of density between the equator and the poles, but by a small difference only between the two forces. Hence if the earth had no rotatory motion, the force which produces this motion would be very much greater, and there would be a sweeping hurricane from the pole to the equator.

21. It is evident, where the motions of the atmosphere are resisted by the earth's surface, that all the conditions cannot be satisfied by a motion at the surface from the poles towards the equator, and by a counter motion in the upper regions. For we have seen (§ 13,) that the atmosphere at the surface of the earth

must have an eastern motion in the middle latitudes; but it cannot have such a motion, unless it also have a motion toward the poles, in order that the deflecting force (§ 5) arising from this motion may overcome the resistance to the eastern motion. But it is evident there cannot be a complete reversal of the motions in the middle latitudes, but some portion of it must flow toward the poles in the upper regions, else the eastern motion there could not be greater than at the surface, which the conditions require. The motions, therefore, must be somewhat as represented in the figure. The part of the atmosphere next the earth's surface in the middle latitudes having a motion towards the poles, extends to a considerable height, since it generally embraces the region of fair weather clouds, as may be seen by observation.

22. Since the force arising from the eastward motion of the atmosphere above must resist in a great measure the tendency to flow from the equator to the poles, caused by the difference of level arising from a greater upward expansion of the atmosphere near the equator on account of a greater temperature, and since this difference of level, and consequently the tendency to flow toward the poles, must increase with the height, the eastward motion in the middle and higher latitudes must be much greater above than below, and in the trade wind region, near the tropical calm-belts, where the motion is westward below it must be toward the east above. This is also evident from the general consideration, that the whole amount of deflecting force eastward arising from the motion of the atmosphere towards the poles is equal to the deflecting force westward arising from its motion back towards the equator, and that the deflecting force eastward is principally above where there is less resistance than near the surface. Hence, at the top of Mauna Loa in the Sandwich Islands, and on the peak of Teneriffe, both of which places are near the tropical calm-belt at the surface, a strong southwest wind prevails. Hence, also, "on the eruption of St. Vincent, in 1812, ashes were deposited at Barbadoes, sixty or seventy miles eastward, and also on the decks of vessels one hundred miles still further east, whilst the trade wind at the surface was blowing in its usual direction." The eastward motion of the atmosphere above, in the latitudes of the trade winds, is also confirmed by observations made on the directions of the clouds at Colonia Tovar, Venezuela, latitude 10° 26′, as given in the Report of the Smithsonian Institution for 1857 (p. 254). While the motion of the lower clouds was in general from some point towards the east, the observed motion of nearly all the higher clouds was from some point towards the west.

23. From what precedes, the limit between the atmosphere which moves eastward in the middle latitudes and westward nearer the equator, which at the earth's surface is at the tropical

calm belt, must be a plane inclining toward the equator above. And since, according to (§ 21), the atmosphere near the earth's surface cannot have an eastward motion, unless it also has a motion toward the poles, this plane near the earth's surface must nearly coincide with the one which separates the atmosphere moving towards the poles from that moving towards the equator, in the trade wind regions, and hence the latter must also incline above towards the equator. This explains the winds at the peak of Teneriffe, which at the top always blow from the southwest while at the base they blow alternately from the northwest and northeast, changing with the seasons. As the tropical calm belt together with this dividing plane changes its position with the seasons, as will be explained, in the latter part of summer when this plane is farthest north, it still leaves the top of the peak north of it while the base is south of it; and hence the wind at the top always blows from the southwest, even when at the base it blows from the northeast. As this plane moves south in the fall, more of the peak gradually becomes north of it; and hence the southwest wind, which always prevails at the top, gradually descends lower on the sides of the peak until it reaches the base. Hence, when this plane reaches its most southern position, in the latter part of winter, the southwest wind prevails at both the base and the top.

24. The depression of the atmosphere at the poles and at the equator, and the accumulation near the tropics, may be explained in a general manner by means of the principle in (§ 5) that when a body moves in any direction in the northern hemisphere, it is deflected to the right, and the contrary in the southern. The atmosphere towards the poles having an eastward motion, the deflecting force arising from it causes a pressure towards the equator, and the motion near the equator being westward, the pressure is towards the poles; and hence there must be a depression at the poles and at the equator, and an accumulation near the tropics. Since this deflecting force is as $\cos\theta$, it is small near the equator; and, consequently the depression there is small.

25. According to the preceding tables of barometric pressure, there is more atmosphere in the northern than in the southern hemisphere. Says Sir James Ross, "the cause of the atmosphere being so very much less in the southern than in the northern hemisphere remains to be determined." This is very satisfactorily accounted for by the preceding principle; for as there is much more land, with high mountain ranges, in the northern hemisphere, than in the southern, the resistances are greater, and consequently the eastern motions, upon which the deflecting force depends, is much less; and the consequence is, that the more rapid motions of the southern hemisphere cause a greater

depression there, and a greater part of the atmosphere to be thrown into the northern hemisphere.

This also accounts for the mean position of the equatorial calm belt being, in general, a little north of the equator. But in the Pacific Ocean, where there is nearly as much water north of the equator as south, its position nearly coincides with the equator.

For the same reason the tropical calm belt of the northern hemisphere is farther from the equator than that of the southern hemisphere; and, on account of the irregular distribution of the land and water of the two hemispheres in different longitudes, it does not coincide with any parallel of latitude. In the longitude of Asia, where there is all land in the northern hemisphere and the Indian Ocean in the southern, this belt, which is also the dividing line which separates the winds which blow east from those which blow west, is farther from the equator than at any other place, as shown by Professor Coffin's chart.

26. In winter, the difference of temperature between the equator and the poles, upon which the disturbance of the atmosphere depends, is much greater than in summer; this causes the eastward motion of the atmosphere in either hemisphere during its winter to be greater, while in the other hemisphere it is less. Hence a portion of the volume of the atmosphere in winter is thrown into the other hemisphere; but, although the volume or height of the atmosphere is then less, yet, being more dense, the barometric pressure remains nearly the same. The difference at Paris, and in the middle latitudes generally, between winter and summer, is only about $\frac{1}{10}$ of an inch.

On account of this alternate change with the seasons of the velocity of the eastward motion of the atmosphere in the two hemispheres, the equatorial and tropical calm belts change their positions a little, moving north during our spring, and south in the fall.

III. *The Motions of the Atmosphere arising from local disturbances.*

27. Besides the general disturbance of equilibrium arising from a difference of specific gravity between the equator and the poles, which causes the general motions of the atmosphere, treated in the last section, there are also more local disturbances, arising from a greater rarefaction of the atmosphere over limited portions of the earth's surface, which give rise to the various irregularities in its motions, including cyclones or revolving storms, tornadoes, and water-spouts. When, on account of greater heat, or a greater amount of aqueous vapor, the atmosphere at any place becomes more rare than the surrounding portions, it ascends, and the surrounding heavier atmosphere flows in below, to supply its place, while a counter current is consequently pro-

duced above. As the lower strata of atmosphere generally contain a certain quantity of aqueous vapor, which is condensed after arising to a certain height, and forms clouds and rain, the caloric given out in the condensation, in accordance with Espy's theory, produces a still greater rarefaction, and doubtless adds very much to the disturbance of equilibrium, and to the motive power of storms. So long, then, as the ascending atmosphere over the area of greater rarefaction is supplied with aqueous vapor by the current flowing in from all sides below, the disturbance of equilibrium must continue, and consequently the local disturbances of the atmosphere to which it gives rise, whether those of an ordinary rain storm, or a cyclone, may continue many days, while the general motions of the atmosphere may carry this disturbed area several thousands of miles.

28. When the area of rarefaction is such as to cause the atmosphere to flow in below from all sides toward a centre and the reverse above, thus establishing a constantly interchanging motion between the internal and external part, the case becomes very similar to that of the general hemispherical motions of the atmosphere in which the motion is between the polar and equatorial parts. For if the earth's rotation on its axis is analyzed with reference to any other axis, the pole of which is at the distance of θ from the pole of the earth, it is found to have a rotation around this latter axis equal to $n \cos \theta$ (Peirce's *Analytical Mechanics*, § 25). Hence the interchanging motion between the internal and external part in this case must cause the internal part to gyrate around the centre from right to left in the northern hemisphere, and the external part the contrary way, and thus give rise to a cyclone or revolving storm just as in the case of the hemispherical motions the part nearest the pole acquires an eastward motion, and that near the equator a westward motion. This is also evident from the principle demonstrated in § 5, according to which the atmosphere in flowing toward the center below, must be always deflected in the northern hemisphere to the right, and consequently give it a gyratory motion around the center from right to left. In moving out above toward the external part, the deflection tends to give the atmosphere a gyratory motion the contrary way, and hence in flowing out above the gyratory motion which it has while rising to the upper strata in the interior, is not only destroyed but on arriving at the external part it has a gyratory motion the contrary way. In the southern hemisphere, since the deflections there are all to the left of the direction of motion, the gyrations are all reversed, which is the observed law of storms in all parts of the world, as shown by Redfield, and also by Reid, in his *Law of Storms*. It is also evident that at the equator, where $\cos \theta$ vanishes, upon which the deflecting force depends, there cannot be a cyclone, and hence,

of all those which Redfield has investigated, and given in his charts of their routes, none have been traced within 10° of the equator. The typhoons or cyclones, also, of the China sea, have never been observed within 9° of the equator.

29. In the case of the general hemispherical motions the internal or polar part is most dense, but in the case of cyclones the external part. Hence the motions between the centre and the external part are different, in the former case the motion toward the centre being above, but in the latter below. In the case of no resistances from the earth's surface this does not affect the gyrations, since on account of the action of the different strata upon one another, they all eventually have the same gyratory motion. But where there are resistances, they cause the gyrations in the external part to be most rapid above just as in the system of hemispherical motions, they are most rapid above in the internal or polar part.

30. If the gyrations were not resisted by the earth's surface it is evident that the rapid gyrations of the cyclone would cause the atmosphere to recede entirely from the centre, but on account of the resistance the very rapid gyrations are in general in a great measure prevented, so that instead of a complete vacuum the strata of the atmosphere are only somewhat depressed in the interior, as represented in fig. 2. For since the force which produces the gyrations depends upon the velocity of the flow to and from the centre, it is evident, that, at the centre and at the external part of the disturbed portion of atmosphere, where this velocity must vanish, the resistances destroy all gyratory motion. Hence, instead of very rapid gyrations near the centre, as in the case of no resistances, there must be a calm there, and the most rapid gyrations be at some distance from the centre, in accordance with observation. The diameter of the comparatively calm portion, in the centre of the large cyclones, is sometimes about 30 miles. The velocity of gyration of the external part, which, in the case of no resistances, is small, is in a great measure destroyed by the resistances of the surrounding atmosphere, so that it is, for the most part, insensible to observation, and only the more rapid gyrations of the internal part are observed. The motion of gyration combined with the motion at the earth's surface towards the centre, gives rise to a spiral motion towards the centre, exactly in accordance with the observed motions of the atmosphere in great storms or hurricanes, as has been shown by Redfield, in a number of papers on the subject, published in this Journal.

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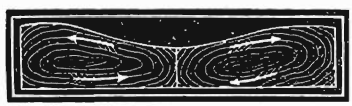

31. Since the atmosphere is depressed in the middle of cyclones, they must sensibly affect the barometer; and this is the true cause of all the great barometrical oscillations, as was first suggested by Redfield.* As the cyclone approaches, there is generally a very slight rise of the barometrical column, which is at its maximum at the greatest accumulation near the external part of the cyclone, after which it is gradually depressed, until the middle of the cyclone arrives, where the atmosphere is most depressed, when the barometer is at its minimum, and then it returns in a reverse manner to its former height, when the cyclone has passed. In great storms the mercury sometimes falls more than two inches. In oblong storms, and all imperfectly developed cyclones, the same phenomena must take place in some measure, as in a complete cyclone. We have reason to conclude, therefore, that nearly all the oscillations of the barometer are caused by a cyclonic motion of the atmosphere, by which it is depressed in the middle of the cyclones. The cyclones may be very irregular and imperfectly developed, and not of sufficient violence to produce a strong wind, and several may frequently interfere with one another, so that the oscillations may frequently be very slight ones only, and very irregular.

Since the gyratory motion of a cyclone, and the consequent depression at the centre, depend upon a term containing as a factor, cos θ, (§ 28), which is the sine of the latitude, according to the preceding theory of barometrical oscillation, the oscillations should be small near the equator, and increase towards the poles, somewhat as the sine of the latitude. Accordingly, at the equator, the mean monthly range of oscillation is only two millimetres, or less than $\frac{1}{10}$ of an inch, while there is a gradual increase with the latitude; so that at Paris it is $23{\cdot}66^{mm}$, and at Iceland, $35{\cdot}91^{mm}$. (Kaemtz's *Meteorology*, by C. Walker, page 297.)

32. The greater rarefaction of the atmosphere at some times than at others, without doubt, has considerable effect upon the barometer; but the theory which attributes the whole of the barometrical oscillations to the rarefaction of the atmosphere produced by the condensation of vapor in the formation of clouds and rain, can not be maintained; for according to that theory, in the rainy belt near the equator, where there are always copious rains during the day, which are succeeded by a clear atmosphere during the night, the oscillations of the barometer should be greatest, and towards the poles, where there is little condensation of vapor into rain, they should be the least; but we have seen that just the reverse of this is true. The atmosphere is extremely mobile, so that if there were much difference

* This Journal, [1], xx, 45.

of pressure between one part and the surrounding parts, the equilibrium would be at once restored if the motions toward the centre produced by this difference of pressure, did not give rise to a cyclonic motion, and thus to a centrifugal force combined with the deflecting force arising from the earth's rotation which in a great measure counteracts the force arising from a difference of pressure between the external and internal parts. If therefore the earth had no rotation, the oscillations of the barometer would be no greater in any part of the earth than they are at the equator.

33. When the disturbance of equilibrium is great, but extends over a small area only, the centripetal force is much greater than in the case of large cyclones, and the gyrations are then very rapid and very near the centre, as in the case of tornadoes. Tornadoes generally occur when the surface of the earth is very warm, and the atmosphere calm. For then the strata near the surface becomes very much rarefied, and are consequently in a kind of unstable equilibrium for a while, when from some slight cause, the rarefied atmosphere rushes up at some point through the strata above, and consequently flows in rapidly from all sides below, and then, unless the sum of all the initial moments of gyration around the centre is exactly equal 0, which can rarely ever be the case, it must run into rapid gyrations near the centre, and a tornado is the consequence. This may be exemplified by the flowing of water through a hole in the bottom of a vessel. If the fluid at the beginning is entirely at rest, it runs out without any gyrations; but if there is the least perceptible initial gyratory motion, it runs into very rapid gyrations near the centre.

34. In the case of tornadoes, which are always of small extent, the influence of the earth's rotation in producing gyrations is generally very small in comparison with that of the initial state of the atmosphere, so that the gyration depends principally upon the initial gyratory state of the atmosphere with regard to the centre of the tornado, and may be either from right to left, or the contrary. Hence there may be tornadoes at the equator, although there cannot be large cyclones. In large cyclones the effect of the initial state, except at the equator, is insignificant in comparison with the influence of the earth's rotation; and the latter, moreover, is a constant influence, while the former is soon destroyed by resistances. Hence large cyclones are of long duration, while small tornadoes, depending principally upon the initial gyratory state for their violence, are soon overcome by the resistances.

35. On account of the centrifugal force arising from the rapid gyrations near the centre of a tornado, it must frequently be nearly a vacuum. Hence, when a tornado passes over a build-

ing, the external pressure, in a great measure, is suddenly removed, when the atmosphere within, not being able to escape at once, exerts a pressure upon the interior of perhaps nearly fifteen pounds to the square inch, which causes the parts to be thrown in every direction to a great distance. For the same reason, also, the corks fly from empty bottles, and every thing with air confined within, explodes.

36. When a tornado happens at sea, it generally produces a water-spout. This is generally first formed above, in the form of a cloud, shaped like a funnel or inverted cone. As there is less resistance to the motions in the upper strata than near the earth's surface, the rapid gyratory motion commences there first, when the upper strata of the agitated portion of atmosphere have a tendency to assume somewhat the form of the strata in the case of no resistance. This draws down the strata of cold air above, which, coming in contact with the warm and moist atmosphere ascending in the middle of the tornado, condenses the vapor and forms the funnel-shaped cloud. As the gyratory motion becomes more violent, it gradually overcomes the resistances nearer the surface of the sea, and the vertex of the funnel-shaped cloud gradually descends lower, and the imperfect vacuum of the centre of the tornado reaches the sea, up which the water has a tendency to ascend to a certain height, and thence the rapidly ascending spiral motion of the atmosphere carries the spray upward, until it joins the cloud above, when the water spout is complete. The upper part of a water-spout is frequently formed in tornadoes on land.

When tornadoes happen on sandy plains, instead of water-spouts they produce the moving pillars of sand which are often seen on sandy deserts.

37. The routes of cyclones in all parts of the world, which have been traced throughout their whole extent, have been found by Redfield* to be somewhat of the form of a parabola, as represented in fig. 3. Commencing generally near the equator, the cyclone at first moves in a direction only a little north or south of west, according to the hemisphere, when its route is gradually recurvated towards the east, having its vertex in the latitude of the tropical calm belt. The motion of a cyclone toward the poles may be accounted for by the principle in § 5. The motion of the equatorial side of a cyclone in either hemisphere is always toward the east, and hence the deflecting force causes a pressure toward the equator, but that of the polar side being always toward the west, the deflecting force causes a pressure toward the pole. Now these deflecting forces being as the sine of the latitude, as may be seen from (§ 5), the pressure on the polar side toward

* See this Journal, Second Series, vol. i, Chart i.

the pole is greater than on the other side toward the equator, and hence the cyclone moves in the direction of the greatest pressure. It is not to be supposed, however, that there is an actual transfer of all the atmosphere of a cyclone from the equator to the polar regions. For the motions and pressure of the cyclone being greater on the polar side, where the deflecting forces which cause it are greatest, its action upon the atmosphere in advance of it is greater than on the equatorial side, where these forces are much

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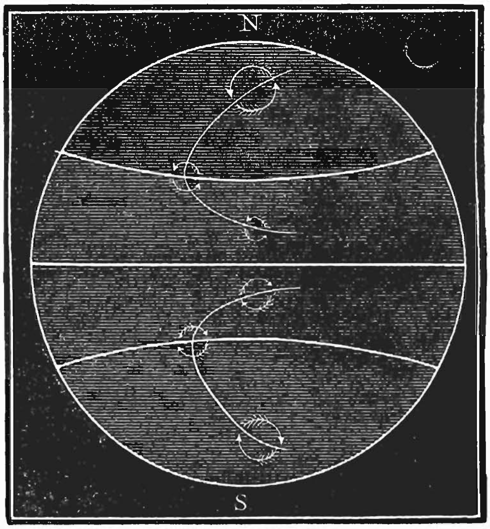

less, and hence new portions of the atmosphere are being continually brought into action on the one side, while the resistance of the earth's surface, and the adjacent portions of atmosphere on the other side, are continually overcoming the comparatively weak forces there, and destroying the gyratory motion of the cyclone; so that the centre of the cyclone is being continually formed in advanced portions of the atmosphere. Since many cyclones are more than one thousand miles in diameter, the difference in the violence of its action on the two sides is very considerable. Hence the interior and most violent portion of a cyclone, always gyrating from right to left in the northern hemisphere, and the contrary in the southern, must always gradually move towards the pole of the hemisphere in which it is. While between the equator and the tropical calm belt, it is carried westward by the general westward motion of the atmosphere there, but after passing the tropical calm belt, the general motion of the atmosphere carries it eastward, and hence the parabolic form of its route is the resultant of the general motions of the atmosphere, and of its gradual motion toward the pole.

IV. *The Motions of the Ocean.*

38. Besides the actions of the sun and moon which give rise to the tides, there are only two causes which can produce any sensible motions of the waters of the ocean. One of these is the action of the atmosphere upon the surface of the ocean, and the other, the difference of density between the water near the equator and that towards the poles, arising from a difference of temperature. The general motions of the atmosphere at the surface of the ocean have a tendency to cause a westward motion of the water in the torrid zone, and an eastward motion in the middle and higher latitudes; and from what we know of the effects of strong winds upon the ocean, we have reason to think that these general motions of the atmosphere are adequate to produce *sensible* motions, since, after the inertia of the water is once overcome, which, however small the force, is only a question of time, the only force necessary is that which is adequate to overcome the resistance of friction, which is very small where the velocity is small. The difference of density between the equator and the poles causes a slight interchanging motion of the water between them, and consequently, where not interrupted by continents, it produces a system of motions in the ocean similar to those of the atmosphere. Hence these two causes of oceanic disturbance, whatever their relative weight, both act in the same directions, and conjointly cause the observed westward motion of the ocean near the equator, and eastward motion towards the poles.

39. The westward motion of the water of the ocean in the torrid zone was first observed by Columbus, and is now well established; and observations also show that there is a motion towards the east in higher latitudes. A bottle thrown into the ocean near Cape Horn was picked up three and a half years afterward at port Phillip, Australia, a distance of 9000 miles, which makes the eastward velocity in that latitude more than 7 miles per day. And Sir James Ross, when sailing eastward near Prince Edward's Island, found himself every day from 12 to 16 miles by observation in advance of his reckoning. (*Voyage to the Antarctic Seas*, vol. ii, p. 96.) But a westward motion being established in the torrid zone, an eastward motion in the higher latitudes must be admitted; for, as was shown in the case of the atmosphere (§ 13), the one cannot exist without the other.

40. It has generally been supposed that the equatorial westward current of the ocean is caused principally by the action of the westward winds there; but Professor Guyot thinks that "it is too deep and rapid to admit of being explained by their action alone," and that "the difference of temperature betwen the regions near the equator and those near the poles controls all other causes by its power and the constancy of its action." (*Earth and*

Man, pp. 189, 190.) The torsive or deflecting force which causes the westward motion of the atmosphere and the ocean in the equatorial regions, and the eastward motion in the higher latitudes, has been shown to be as the velocity of the interchanging motion between the equatorial and the polar regions; and hence if this motion in both were similar, the relative amount of this force in each must be as the whole mass multiplied into the velocity of this motion between the equator and the poles. If we suppose the ocean to be 3 miles in depth, its mass is about 500 times that of the atmosphere, and hence if the motion between the equator and the poles were only $\frac{1}{500}$ of that of the atmosphere, the part of the force which gives it a westward motion near the equator, and an eastward motion towards the poles, arising from this cause, must be greater than that of the action of the atmosphere upon it, since the whole amount of this force in the atmosphere is not spent upon the ocean, but only that part which overcomes the resistances to its motions. Although the effect of temperature in producing a difference of density, and consequently of disturbing the equilibrium, is very much less in the ocean than in the atmosphere, yet since the amount of motion which a given disturbing force will produce where time is not considered, depends as has been stated, upon the amount of the resistances, and not upon the amount of inertia to be overcome; and since the resistances diminish as the square of the velocity, a very small amount of disturbing force arising from a difference of density must be adequate to cause an interchanging motion in the ocean between the equatorial and the polar regions equal to $\frac{1}{500}$ of that of the atmosphere; and hence we have reason to think that a greater part of the motions of the ocean is due to this cause than to the action of the atmosphere upon it.

41. The motions of the ocean being similar to those of the atmosphere, they must cause a slight elevation of the surface about the parallels of 30°, and a depression at the equator and the poles, just as in the case of the atmosphere, except that it will be less in the ratio of the relative velocities of the motions of the ocean and of the atmosphere. If we suppose the east and west motions of the ocean to be $\frac{1}{60}$ of those of the atmosphere at the earth's surface, which would require the maximum eastward velocity in the southern hemisphere to be about 10 miles per day, it would cause the surface of the ocean in the southern hemisphere to be about 15 feet higher at the parallel of 30° than at the pole, and also a little higher than at the equator. Now if the motions which cause this accumulation of water were the same at the bottom of the ocean as at the surface, there would be no tendency of the water to flow out at the bottom from beneath this accumulation; but since the motions there must be much less, it must flow out both towards the equator and the pole, especially

toward the latter, as the depression there is much the greater. Since the density of sea-water does not increase below the temperature of 28°, the density of the ocean does not increase beyond a certain latitude, and hence there is no flow of the water at the bottom from the poles toward the equator, arising from the maximum density at the pole, as seems to be the case in a very slight degree in the atmosphere, but the under current at the bottom, arising from the greater pressure about the parallel of 30°, must extend entirely to the poles; so that there must be a slight tendency of the water to rise at the poles, and flow at the surface some distance towards the middle latitudes. As the water toward the bottom of the ocean is always about the same as the mean temperature of the earth, when it first rises to the surface at the pole, it must be much warmer than it is after it has flowed some distance from it, and hence we have reason to think that there may be open polar seas, surrounded by barriers of ice at some distance from the pole, where there is the maximum temperature of the surface water. A surface current from the poles is indicated by the motions of icebergs in both hemispheres from the polar regions towards a lower latitude.

42. Where the east and west motions of the ocean are entirely intercepted by continents, as in the northern hemisphere, the water receives a slight gyratory motion from left to right. The westward motion of the waters of the Atlantic in the torrid zone, impinging against the continent of America, causes the surface of the water of the Caribbean Sea and the Gulf of Mexico to be a little above the general level, while the eastward motion of the northern part of the Atlantic causes the surface of the water adjacent to the eastern coast of North America, in that latitude, to be a little lower. Hence there is a flow of warm water from the Gulf of Mexico along the coast of the United States toward the lower level about Newfoundland, which, on account of the peculiar configuration of the coast about the Gulf of Mexico, and the peninsula of Florida, gives rise to the Gulf Stream. The eastward motion also of the northern part of the Atlantic causes the surface of the water on the western coast of Europe to be a little *higher* than the general level, while the westward motion in the torrid zone causes it to be depressed, on the western Coast of Africa, a little *below* this level, and hence the water of the eastern side of the Atlantic, flowing from a higher to a lower level, has a motion toward the the equator. The whole of the North Atlantic has therefore a very slight gyratory motion from left to right, and is supposed to make a complete gyration in about three years.

43. A portion of the equatorial current flowing from the higher level of the Caribbean Sea toward Cape Horn causes the Brazil current, which is deflected eastward by the general east-

ward motion of the Southern Ocean. The east side of the South Atlantic, as well as that of the North Atlantic, seems to have a motion toward the equator. Says Sir James Ross, "There is a current from the Cape of Good Hope along the west coast of Africa 60 miles wide, 200 fathoms deep, with a velocity of one mile per hour, of the mean temperature of the ocean." (*Voyage to the Southern Seas*, vol. ii, p. 35.) This cannot be a portion of the Mozambique current from the warm waters of the Indian Ocean, passing around the Cape of Good Hope, and giving rise to the equatorial current of the Atlantic, as has been supposed, but must come from the colder waters of the Southern Ocean. Hence the South Atlantic also has a tendency to assume a gyratory motion, and the equatorial current of the Atlantic is merely the equatorial portion of these two gyrations, with perhaps a small part of the Mozambique current passing around the Cape.

44. The general eastward motion of the water of the northern part of the Atlantic, and the consequent depression of the water next the coast of North America, is the cause of the cold current of water flowing from Baffin's Bay and the east coast of Greenland, between the Gulf Stream and the coast of the United States called the Greenland current. Since the warm water of the Gulf Stream, in flowing northward, is deflected toward the east (§ 5), and that of the Greenland current, in flowing south, tends toward the west, there is no intermingling of the waters of the two currents, but they are kept entirely separate as if divided by a wall, as has been established by the Coast Survey.

45. There must be a motion of the waters somewhat similar to the Gulf Stream and the Greenland current, wherever the great equatorial current impinges against a continent, and the eastward motion toward the poles is interrupted. Hence, on the eastern coast of South America there is the warm Brazil current which has been mentioned, and on the eastern coast of Asia there is the warm China current, flowing toward the north, similar to the Gulf Stream, and the cold Asiatic current insinuating itself between it and the coast, like the Greenland current. On the east coast of Africa, also, there is the Mozambique current flowing south like the Brazil current, and it is also now well established that, east of the Cape of Good Hope, the general tendency of the water is toward the south. This water must mingle with the general eastward current of the South Sea, and hence there is a slight tendency to a gyratory motion in the Indian Ocean also.

46. On the western sides of the continents there is a motion somewhat the reverse of this, and instead of a warm current flowing north, there is a cold one flowing toward the equator, as has been shown to be the case in the Atlantic. Hence, on the west coast of North America there is a flow of colder water

along the coast from the north, and on the west coast of South America is Humboldt's current, much colder than the rest of the ocean in the same latitude, both tending toward the equator to join the great westward current there across the Pacific, and to fill up, as it were, the vacuum which this current has a tendency to leave about the equator, on the west coast of America.

47. With regard to the gyratory motion of the oceans, it may be further added here, that such gyrations are clearly demonstrated by the positions of the isothermal lines, as has been shown by Professor Dana, in a paper read at the twelfth meeting of the American Association for the Advancement of Science (*Proceedings*, vol. xii, p. 77). According to this paper, the isothermal line of 68° F., in winter, extends, in the North Atlantic, from 56° N. on the American side, to 12° N. on the African, and in the South Atlantic, from latitude 31° S. on the South American coast, to 7° S. on the African side. Similar evidences are given of gyratory motions, in a less degree, in both the North and South Pacific, and also in the Indian Ocean.

48. When a portion of fluid on the earth's surface gyrates from left to right, the deflecting force arising from the earth's rotation being in this case toward the interior, the surface assumes a slightly convex form. The water of the North Atlantic having a very small gyratory velocity in comparison with that of the earth's rotation, the interior is a little elevated above the general level, and consequently the pressure upon the bottom increased. Now the gyrations which cause this elevation in the middle being principally toward the top, the increased pressure upon the bottom causes the fluid there to flow out on all sides with a very small velocity, towards the circumference, and hence the water at the surface has a slight tendency to flow in from all sides towards the interior to supply its place. This completely accounts for that vast accumulation of drift and sea-weed, covering a large portion of the interior of the North Atlantic, called the Sargasso Sea. From what has been stated, the North Pacific must also have a slight gyratory motion from left to right, and hence it likewise has its Sargasso Sea.

V. *The Motions of Solid Bodies at the Earth's Surface.*

49. It has been shown that if a body were set in motion upon the earth's surface it would move with uniform velocity, but would be continually deflected to one side. When the range of motion is small, cos θ (equations (5)) may be regarded as constant, and hence the deflecting force in this case is constant, and must cause the body set in motion to describe the circumference of a circle. If we put ϱ for the radius of curvature, and m for

the angular velocity about the center of curvature, the centrifugal force of the body is ϱm^2 which must be put equal to the deflecting force, (equation 5). Hence we have

$$\varrho m^2 = 2 n v \cos\theta$$

Also, since v is the lineal velocity of the moving body,

$$\varrho m = v.$$

From these two equations we get

$$\varrho = \frac{v}{2 n \cos\theta},$$

$$m = 2 n \cos\theta$$

50. When the range of motion is so small that $\cos\theta$ may be regarded as constant, ϱ and m are constant, and hence the body then moves with a uniform angular velocity in the circumference of a circle. If we put T for the time of a revolution, we shall have

$$T = \frac{2\pi}{m} = \frac{\pi}{n\cos\theta} = \tfrac{1}{2} \text{ day} \times \sec\theta.*$$

Hence, since v disappears in the result, T is independent of the initial velocity.

51. If a body is forced to move in a straight line F (equation (6)) is the lateral pressure of that body. If we put $v{=}60$, which is a velocity of about 40 miles per hour, the equation gives $F = \frac{1}{5188} g$, at the parallel of 45°. Hence if a railroad train moves in a straight line 40 miles per hour at the parallel of 45°, the lateral pressure is $\frac{1}{5188}$ of its weight, and this is precisely the same in all directions, and not in the direction of the meridian only as has been generally supposed.

52. The deflecting force (§ 5) also causes the gyration of a vibrating pendulum. If the pendulum were suspended at the pole it would evidently vibrate in the same plane in space, and consequently perform a gyration in one day. Since the velocity of the earth's revolution around any other point of the earth's surface is $n\cos\theta$ (§ 28), the time of gyration there is $\frac{1 \text{ day}}{\cos\theta} =$ 1 day $\times$ sec. θ.

53. This same deflecting force may be used to explain some of the motions of a rotating body. Suppose such a body be placed with its axis of rotation in any direction parallel with the horizon. Then the motions of the upper and lower parts being in contrary directions relative to the earth, and the deflecting force in both being either to the right or the left, according to the hemisphere in which it is, gives the axis of rotation a tendency to assume a perpendicular position. But there are other forces beside these horizontally deflecting forces, so that all the forces

* This result was erroneously given in the Mathematical Monthly, 1 day $\times$ sec. θ.

which tend to change the position of the axis would not be in equilibrium with the axis in that position. For the equatorial side then would have a motion coinciding in direction with the motion of the earth's rotation, while the other side would have a motion the contrary way, and consequently the centrifugal force arising from the motion of the earth's rotation, combined with that of the rotating body, would be greater on the equatorial than on the polar side, and give the axis of the rotating body a tendency to move in the plane of meridian. It might be easily shown, when the axis has a position parallel with the axis of the earth, that the forces which tend to change its direction are then in equilibrium, and consequently the axis, if free to turn in any direction, does not change its position.

54. These deductions from theory are in exact accordance with some very delicate experiments made by Foucault with a peculiar form of gyroscope, an account of which is given in this Journal, Second Series, vol. xv, p. 263. See also vol. xix, p. 141.