

ART. LIX.—*The Solar Parallax as derived from the American Photographs of the Transit of Venus, 1874, December 8–9; by D. P. TODD, M.A., Assistant in the Office of the American Ephemeris and Nautical Almanac.*

HITHERTO no value of the solar parallax has been derived from the observations of the transit of Venus made at the American stations in 1874.

In the volume of observations recently issued, Part the First, *General Discussion of Results*, are given most of the data which are necessary for the derivation of the solar parallax from (1) the photographs of the transit, (2) the optic observations of the transit. We shall concern ourselves only with the photographic results: these are presented in pages 104–117, in very nearly the form of equations of condition involving the correction of the difference of right ascension of the sun and Venus, the correction of the difference of declination of the sun and Venus, and the correction of the assumed value of the solar parallax. The residual differences, (O.—C.), being given in distance, s , and in position-angle, p , every photograph furnishes two distinct equations of condition. The total number of photographs is two hundred and thirteen, distributed among the several stations as follows:—

THE NORTHERN STATIONS.

Wladiwostok	13
Nagasaki	45
Peking	26

THE SOUTHERN STATIONS.

Kerguelen	8
Hobart Town	37
Campbelltown	32
Queenstown	45
Chatham Island	7

These equations cannot be regarded as definitive: in addition to the corrections for longitude of the stations (provisional values having been adopted in several cases), three small corrections arising from the absorption of the solar and the terrestrial atmospheres have still to be applied, whose general effect on the solar parallax has been considered immediately preceding the equations of condition themselves, pages 102–103. The first and the second effects being supposably small and acting contrariwise on the solar parallax, we may, without very great uncertainty, disregard their combined action. The third correction should be investigated independently from the equations of condition themselves; and this again cannot advantageously be done until the definitive longitudes have been obtained: all the effects of absorption of the solar and the terrestrial atmospheres may then be most opportunely considered.

THE EQUATIONS OF CONDITION IN s .

Putting, then, $\delta\lambda_1, \delta\lambda_2 \dots \delta\lambda_s$ equal to zero, every photograph gives one equation of condition in s of the form

$$0 = a \delta A + b \delta D + c \delta \varpi - (O.-C.)$$

The normal equations in s are as follow :—

$$\begin{aligned} + 23.99 \delta A + 24.71 \delta D - 28.72 \delta \varpi - 82.17 &= 0 \\ + 24.71 \delta A + 184.46 \delta D - 3.16 \delta \varpi - 439.51 &= 0 \\ - 28.72 \delta A - 3.16 \delta D + 484.51 \delta \varpi + 21.72 &= 0 \end{aligned}$$

whose solution gives—

$$\begin{aligned} \delta A &= + 1''.181 \pm 0''.202 \\ \delta D &= + 2''.225 \pm 0''.070 \\ \delta \varpi &= + 0''.0397 \pm 0''.0418 \end{aligned}$$

The probable error of a single photograph is $0''.88$.

The sums of the squares of the absolute terms in the equations of condition are as follow :—

$$\begin{array}{ll} [nn]_W = 52''.48 & [nn]_K = 37''.02 \\ [nn]_N = 318.25 & [nn]_H = 115.65 \\ [nn]_P = 130.18 & [nn]_C = 414.46 \\ & [nn]_Q = 337.63 \\ & [nn]_{Ch} = 27.83 \\ & [nn] = 1433.50 \end{array}$$

The sums of the squares of the outstanding residuals are as follow :—

$$\begin{array}{ll} [vv]_W = 7''.14 & [vv]_K = 20''.44 \\ [vv]_N = 52.47 & [vv]_H = 59.74 \\ [vv]_P = 29.28 & [vv]_C = 95.16 \\ & [vv]_Q = 85.67 \\ & [vv]_{Ch} = 8.91 \\ & [vv] = 358.81 \end{array}$$

THE EQUATIONS OF CONDITION IN p .

Putting, as before, $\delta\lambda_1, \delta\lambda_2 \dots \delta\lambda_s$ equal to zero, every photograph gives one equation of condition in p of the form

$$0 = a' \delta A + b' \delta D + c' \delta \varpi - (O'.-C.)$$

The normal equations in p are as follow :—

$$\begin{aligned} + 8682117 \delta A - 1404261 \delta D - 138999.20 \delta \varpi - 142109.4 &= 0 \\ - 1404261 \delta A + 1521370 \delta D - 25093.11 \delta \varpi + 10442.1 &= 0 \\ - 138999.20 \delta A - 25093.11 \delta D + 7326.76 \delta \varpi + 2651.6 &= 0 \end{aligned}$$

whose solution gives—

$$\begin{aligned} \delta A &= + 1''.109 \pm 0''.109 \\ \delta D &= + 0''.637 \pm 0''.224 \\ \delta \varpi &= + 0''.0252 \pm 0''.0595 \end{aligned}$$

The probable error of a single photograph is $3'.447$.

The sums of the squares of the absolute terms in the equations of condition are as follow :—

$$\begin{array}{ll} [nn]_w = 358'65 & [nn]_k = 381'20 \\ [nn]_N = 1271'49 & [nn]_H = 1924'85 \\ [nn]_P = 269'08 & [nn]_C = 2044'68 \\ & [nn]_Q = 1462'97 \\ & [nn]_{Ch} = 220'70 \\ & [nn] = 7933'62 \end{array}$$

The sums of the squares of the outstanding residuals are as follow :—

$$\begin{array}{ll} [vv]_w = 323'65 & [vv]_k = 174'34 \\ [vv]_N = 972'02 & [vv]_H = 840'69 \\ [vv]_P = 519'77 & [vv]_C = 1236'90 \\ & [vv]_Q = 1232'09 \\ & [vv]_{Ch} = 176'04 \\ & [vv] = 5475'50 \end{array}$$

FINAL VALUES OF δA , δD , $\delta \varpi$.

It seems likely that some of the probable errors of these quantities which have been derived are illusory. The best course, however, which we can now pursue will be to combine the values of δA , δD , and $\delta \varpi$ obtained from the two solutions in accordance with the weights depending upon these probable errors.

We thus have, for δA ,

$$\begin{array}{ll} \text{From solution in } s, & \delta A = + 1''181 \pm 0''202 \\ \text{From solution in } p, & \delta A = + 1''109 \pm 0''109 \\ \text{Final value,} & \delta A = + 0''075 \pm 0''006 \end{array}$$

And for δD ,

$$\begin{array}{ll} \text{From solution in } s, & \delta D = + 2''225 \pm 0''070 \\ \text{From solution in } p, & \delta D = + 0''637 \pm 0''224 \\ \text{Final value,} & \delta D = + 2''083 \pm 0''067 \end{array}$$

And for $\delta \varpi$,

$$\begin{array}{ll} \text{From solution in } s, & \delta \varpi = + 0''0397 \pm 0''0418 \\ \text{From solution in } p, & \delta \varpi = + 0''0252 \pm 0''0595 \\ \text{Final value,} & \delta \varpi = + 0''035 \pm 0''034 \end{array}$$

The assumed value of ϖ being $8''848$, we have, finally, for the mean equatorial horizontal parallax of the sun,

$$8''883 \pm 0''034$$

corresponding (if we adopt the dimensions of the earth given by Colonel A. R. Clarke*) to a distance between the centres of the sun and earth equal to

$$148,103,000 \text{ kilometers} = 92,028,000 \text{ miles.}$$

Washington, April 27, 1881.

* Geodesy . . . Oxford, Clarendon Press, 1880, page 319.