ART. X.—*On some Phenomena of Binocular Vision;* by JOSEPH LECONTE.

[Read before the National Academy of Sciences, April 20, 1880.]

XI.—*Laws of Ocular Motion.**

IN March, 1869, I published a paper "on the rotation of the eyes on their optic axes, in convergence." The results reached in that paper were briefly as follows:

1. In optic convergence in the *primary* visual plane there is a rotation of both eyes on their optic axes *outward*, and this rotation increases with the degree of convergence.

2. In inclining the visual plane downward the rotation for the same degree of convergence *decreases* until when the inclination is 45° below the primary position, the rotation becomes zero for all degrees of convergence. Below 45° the rotation becomes *inward.*

3. In elevating the visual plane the rotation, for strong convergence, *increases.*

There can be no doubt that the 1st law is plainly in violation of the *Law of Listing* which is supposed to govern all the movements of the eye: for that law requires that all movements of the eye in the primary plane are effected without any rotation on the optic axes (torsion). But it seems not impossible and perhaps not improbable, that the modifications of the effect of convergence in elevating and depressing the visual plane may be the result of the operation of that law; for by that law oblique position upward or downward and to one side or the other does produce rotation. Furthermore, according to Helmholtz, oblique position upward and to one side produces rotation (torsion) to the *opposite* side, and oblique position *downward* and to one side produces torsion to the *same* side. If this be true, then supposing the eyes under the influence of two laws, viz: a law of torsion by convergence, and a law of torsion by oblique position, in elevating and converging the eyes the two would coöperate and produce greater torsion, as indeed we find; and in depressing and converging the eyes, the two would antagonize and neutralize each other, and thus decrease the rotation, as we also find. This seems a simple and satisfactory mode of explaining the whole phenomena of torsion in convergence.

It was in this spirit and the expectation of this result, that I recently undertook a re-investigation of the whole subject of the laws of ocular motion. My first effort was directed to a thorough mastering of the law of Listing; for the statements

* For the other papers on this subject, see this Journal, II, vol. xlvii, pp. 68 and 153, III, vol. i, p. 33, vol. ii, pp. 1, 314, 417, and vol. ix, p. 159.

concerning this law had seemed to me inconsistent with each other. I therefore read again carefully and thoughtfully Helmholtz's great work on Physiological Optics, the acknowledged standard on this subject. I read and re-read several times his chapter on the laws of ocular motion, and pondered upon his results. I repeated all his experiments, and made many more of my own. But so difficult and delusive are experiments of this kind, so beset on every side with sources of fallacy, that the more I experimented and pondered, the more I became bewildered. But now at last the whole subject has become clear, and all my experiments consistent with each other. I now see also, that the true cause of my bewilderment was not so much the delusiveness of the phenomena, but the too ardent desire to verify the results of others, rather than to determine the law for myself. I have been driven almost against my will to the conclusion that there are some strange and apparently inadvertent mistakes in Helmholtz's interpretation of Listing's law, and that this law governs the motions of the eyes only when they *move* parallel to each other, but cannot in any way account for the torsions of the eyes in convergence.

I will now detail the experiments upon which these conclusions are based.

It is well known that spectral images (accidental images of Helmholtz) are the most accurate means of determining the torsions of the eye. They are so because being the result of changes in the retina lasting sometimes a minute or more—being in fact the outward manifestation of images as it were burned into the retina—they must of necessity follow with the greatest exactness all the motions of the eye. There is no other mode of detecting torsions of the eyes, in *parallel motion.* All my experiments, therefore, were made with these images.

Experiment 1.—I darken the experimental room by closing the shutters, but allow the light to enter through a narrow vertical slit between two shutters. I now gaze steadily with head erect on the vertical slit for a minute or so. On turning to the blank white wall I see distinctly a colored vertical spectral image of the slit. I arrange my head if necessary, so that the image is perfectly vertical. If I now turn my eyes (without moving the head) horizontally right and left, the image remains vertical; if I turn my eyes directly up or down, by elevating or depressing the visual plane the image still maintains its vertical position. But if I elevate again the visual plane to the extremest degree, say 40°, and then move the point of sight horizontally as far as possible, say 40° to *the right*, the image is no longer vertical but inclines very decidedly to the right, thus / . If I move my eyes horizontally to the left, the image inclines equally to the left, thus \ . If, after renewing the spectrum, I now depress the visual plane 40°, and then cause the point of sight to travel 40° to the *right*, the image inclines to the *left*, thus \ ; if the point of sight moves to the extreme left, the image turns to the right, thus / .

In all cases the degree of inclination or torsion of the image increases with the degree of elevation or depression of the visual plane and the amount of lateral excursion of the point of sight, right or left. Also the degree and direction of the torsion of the image will be the same for the same position of the line of sight, however that position may have been reached, whether by two motions along rectangular coördinates, as in the preceding experiments, or by oblique motion from the first or primary position.

Experiment 2.—I next made similar experiments, using a *horizontal*, instead of a vertical image. Such an image may be made in the same way, by means of a horizontal slit in the window. When such an image is thrown on a perpendicular wall with the eyes in the primary position (i. e. with face perpendicular and the eyes looking horizontally) its position is of course horizontal. When the eyes move from side to side horizontally, or up and down vertically, it retains its perfect horizontality. But if the eyes be turned obliquely upward and *to the right*, the image inclines to the left, thus —; if upward and to the left, the torsion is to the right, thus —. In depressing the plane of sight, movement of the eyes to the right makes the image incline to the right, thus —, while movement to the left, makes it incline to the left, thus —.

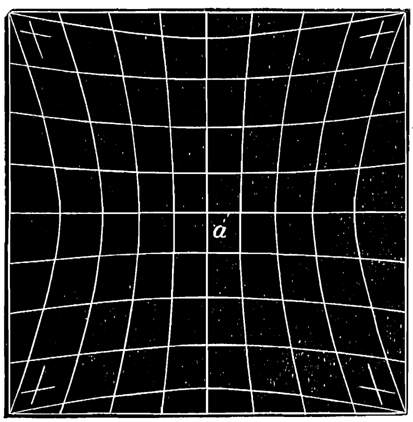
The *fact* and the *direction* of the torsion of the images, both vertical and horizontal, are very easily established by the somewhat rough method just described. But if we desire to *measure* the amount of torsion of the image, the wall or other experimental plane must be covered with rectangular coördinates vertical and horizontal. By this means I find that extreme. oblique positions produce an inclination of the vertical image on the true vertical of the wall of about 15°, but of the horizontal image on the true horizontals of the wall of only about 5°. There is a reason for this difference, which we explain farther on.

Putting now all these results together, the following diagram (Fig. 1) shows the direction and the degree of inclination of the image for all positions of the point of sight—the center representing the primary position, and the corners extreme oblique positions of the point of sight.

Helmholtz's results are exactly the same as my own, except that he makes the inclination of the vertical and the horizontal image exactly equal, while I find the inclination of the horizontal image much less than that of the vertical image. He embodies his results, therefore, in a diagram like the above, except that the curves of the vertical and the horizontal lines are *exactly* equal each to each in every part of the diagram, while in mine the vertical curves are much greater.

The following diagram plainly shows that the apparent torsion of the vertical and horizontal images are in *opposite* directions. If the inclination or torsion of the images show a corresponding torsion of the eye, the evidence of the two images is contradictory. There must, therefore, be a fallacy somewhere. They both cannot be right; for when one indicates torsion of the eye to the right, the other indicates torsion to the left, and *vice versa*. The vertical and horizontal curves in the diagram are not everywhere at right angles to each other, as they ought to be, if they were both true representatives of ocular torsion. This is best shown by using an image in the form of a rectangular cross.

1.

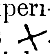
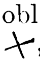
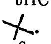


Experiment 3.—If such an image, made by gazing on a cross slit in the window, be used in the experiments already described, then on turning the eyes obliquely upward and to the right, the cross by the turning of the two parts in opposite directions is distorted, thus +, so that the angles are not all right angles, but 70° and 110°. On turning the eyes upward and to the left, the cross becomes thus +, downward and to the right, thus +, to the left, thus +. The same mode of crossing is observed in the lines of the diagram, and in the crosses in the corners.

It is perfectly evident that this distortion is produced by *projection* of the image on a plane inclined to the line of sight. Helmholtz also attributes this distortion to projection, but he gives no experimental method of eliminating this source of fallacy. If he had done so he would have escaped what I conceive to be the error into which he has inadvertently fallen.

The method which I use to eliminate this source of fallacy is the obvious one of *projecting the image on a plane in every case perpendicular to the line of sight.**

* With *one* eye "*line of sight*" is the proper term—but with two eyes "*median line of sight.*" But except in very near objects the difference is so small that I shall neglect it.

Experiment 4.—For this purpose I prepare a plane a yard square, and cover it with vertical and horizontal lines. In the center I place an upright rod and make it accurately perpendicular to the plane. I place the plane inclined from the perpendicular 30°–40°, and so set that with the face looking straight to the experimental window the plane is 30° or 40° above and to the right; and when I turn my eyes obliquely upward and to the right, I look directly on the end of the rod, so that it is projected as a spot on the plane. I thus know that the line of sight is perpendicular to the plane. Having arranged the plane and my own position to my satisfaction, I gaze on a cross slit in the window until the impression is as it were branded upon the retina, and then turning the eyes obliquely upward and to the right, I throw the image on the center of the plane. The cross image retains perfectly its rectangular symmetry, but is rotated *both parts alike, to the right*, thus,  plainly showing a torsion of the eyes in the same direction. I then make a similar experiment on the left side: the cross turns to the left thus . I now arrange the plane below the head and to the right, but perpendicular to the line of sight when the eyes are turned in that direction. When the image is thrown upon the plane, by turning the eyes obliquely downward and to the right, the cross rotates thus , or when placed on the left, thus . In every case the rectangular symmetry of the cross is perfectly preserved, a sure sign that there is no error by projection.

Experiment 5.—Determined to neglect no means of testing the correctness of these results, I next made experiments in the open air, using the sky as the plane upon which to cast the image. This spatial concave is of course everywhere perpendicular to the line of sight, and therefore eliminates every source of error from projection. Standing with head erect, I gaze on a perpendicular flag-staff until a strong impression is made on the retina. If now holding the head steady, I cast the image on the sky obliquely upward and to the right, the image inclines decidedly to the *right;* if thrown similarly to the left, it inclines to the *left.* With the head in this position, of course the ground prevents making the same experiments with the visual plane depressed. I therefore varied the conditions a little. Sitting on the ground in front of the college building, with the morning sun shining obliquely on its face, the perpendicular light-colored pilasters gleaming in the sunshine, contrast strongly with the shadows which border their northern margin. Gazing steadily on the building, I easily get a strong spectral image of the whole building, with its distinctly-marked vertical and horizontal lines. Now throwing myself flat on my back, I see the image perfectly erect, in the zenith. Turning now the eyes upward (toward the brows) and to the right and left; and then downward (toward the feet), and to the right and left, the whole image of the building rotates without distortion, precisely as indicated in my previous experiments.

I am perfectly confident then that I am justified in formulating the torsions of the eyes when moving together with their optic axes parallel, thus:

1. When the visual plane is *elevated* and the eyes move to the *right*, they rotate on their optic axes to the *right;* when they move to the *left*, they rotate to the *left.*

2. When the visual plane is *depressed*, then motion to the *right* is accompanied by rotation to the *left*, and motion to the *left* by rotation to the *right.*

3. The degree of rotation increases with the amount of elevation or depression of the visual plane, and of the lateral excursion of the point of sight.

Now, the above laws (1 and 2) concerning the direction of torsion, are precisely the reverse of those given by Helmholtz, and therefore of what I expected to find when I commenced this investigation. I quote from his work on Physiological Optics, French edition, 1867.* This edition was revised, corrected and added to by Helmholtz himself, and by his own statement is not only later but more authoritative than the German.

"When the plane of regard is directed upward, lateral displacement to the *right* makes the eye turn to the *left*, and displacement to the *left* makes it turn to the *right.*

"When the plane of regard is depressed, lateral displacements to the *right* are accompanied with torsion to the *right*, and *vice versa.*

"In other words, when the vertical and lateral angles are both of the *same sign*, the torsion is *negative;* when they are of *contrary signs* the torsion is *positive.*"

The very reverse of every one of these propositions is demonstrably true.

I next set myself to find out how the mistake arose. I find its origin evidently contained in the following statement:

"If we throw a *vertical* image on the wall (supposed to be covered with rectangular coördinates vertical and horizontal), we obtain a rotation in direction contrary to that which we have just seen (in the case of the *horizontal* image). In fact, if one looks upward and to the right, the image *does not turn to the left*, but *to the right in relation to the vertical lines of the wall. But one cannot conclude* from this that there is a *rotation* of the eye

* Optique Physiologique, p. 602 and 603.

to the right, for *in this case the vertical lines of the wall do not coincide with a projection, on the wall, of a perpendicular to the plane of regard. The latter would, on the contrary, appear turned in the same direction as the image, and at an angle much greater than that of the image.*"*

In other words, (since the plane perpendicular to the line of sight is the only true plane of projection, and verticals on that plane are perpendicular to the plane of regard), according to Helmholtz the horizontal lines on the wall are true terms of comparison for determining the rotation of spectral images, because they coincide with the horizontals on the plane perpendicular to the line of sight; but the vertical lines on the wall are not true terms of comparison, because they do not coincide with the verticals in the plane perpendicular to the line of sight. Now, the very reverse is true. It is the *verticals* on the wall which coincide with the verticals in the plane perpendicular to the line of sight or perpendiculars to the plane of regard, and the horizontals on the wall which do not coincide with the horizontals on that plane. Therefore, it is the verticals on the wall which are the true terms of comparison, by which to determine the direction of torsion of the eye, and horizontals which give deceptive results by projection.

As this is a fundamental point, I must pause to make it clear. Suppose, then, one stands in a room before a wall covered with rectangular coördinates, vertical and horizontal. Suppose, farther, such an one surrounded by a spherical wire-cage, constructed of rectangular spherical coördinates, or meridians and parallels, with pole above the head, and eye in the center. Evidently the surface of this spherical concave is everywhere perpendicular to the line of sight, and therefore like the sky, is a proper surface for testing the true direction of rotation of images in every position of the eye. Evidently, also, the meridians and parallels, everywhere at right angles to each other, are the true coördinates with which to compare the spectral images, in order to determine the direction and degree of their rotation. Now the simple question is: how do these meridians and parallels project themselves on the wall, to an observer at the center? How would their shadows be cast by a light at the center? Evidently the meridians would be cast as straight vertical lines, and therefore coincident with the verticals on the wall. But the parallels would be projected not as straight horizontal lines, and therefore not coincident with the horizontals on the wall, but *as hyperbolic curves inclined in the same way as the horizontal image in Helmholtz's diagram, but at much greater angle.* I repeat, then, that the inclination of the vertical image on the vertical lines of the wall gives the true

† Ibid, p. 605.

torsion of the eye, but the inclination of the horizontal image on the horizontal lines of the wall does not give the true torsion of the eye.

There are many other ways of testing the truth of this last proposition and the falsity of the reverse statement of Helmholtz. If we make, as before, a vertical image, and instead of turning the *eyes* upward and to the right, turn the *body* to the right and the *face* upward and cast the image on the extreme right and upper portion of the wall; the vertical image will be projected vertically on the wall, but a horizontal image cast to the same place, in the same way, will be inclined in the same way as in Helmholtz's diagram, but *at much greater angle.* In this case, the eyes are in the primary position, and therefore there is no rotation at all, the inclination of the horizontal image is the result of projection alone.

Without any attempt at mathematical accuracy, the diagram, figure 2, shows the manner in which spherical coördinates would project on a plane perpendicular wall. The crosses in the corners show how a rectangular cross image

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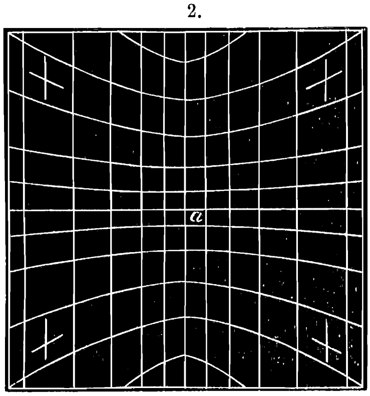

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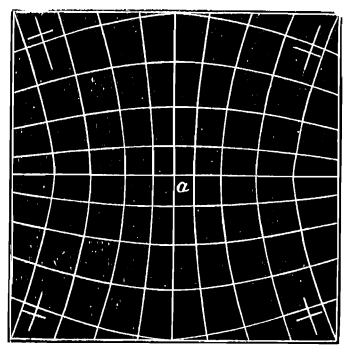
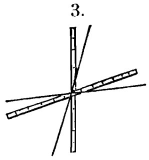


would be distorted by *projection alone.* Now by careful plotting, I have found that at a point 40° upward or downward, and 40° to one side right or left, the inclination of the hyperbolic curves with the true horizontals of the wall is about 20°—which makes the angles of the projected cross 70° and 110°. The rotation of such a cross 15°, would give exactly the results obtained by experiment. In figure 3, the heavy cross shows the position of the image when distorted by projection only, and the lighter lines the same as rotated 15° to the right. As the result of this rotation, the vertical line is inclined 15° *to the right,* while the horizontal line is inclined only 5° *to the left,* as we found by experiment.

3.



Therefore, the diagram which expresses correctly the real torsions of the eyes, in every position of the line of sight, or the diagram which shows the inclination of the vertical and horizontal images, when referred to the meridians and parallels of a spatial concave everywhere perpendicular to the line of sight, is shown in figure 4. In this diagram the lines representing the inclination of the vertical and horizontal images are everywhere at right angles to each other, and always turned in the same direction. By simple inspection of this figure the law of torsion of the cross images and therefore of the eyes in various positions is seen at a glance.

But again, and finally, Helmholtz's statements in regard to the direction of torsion are, it seems to me, in direct contradiction to his own general formulation of Listing's law, taken from Listing himself. This general formula is as follows: "*When the line of regard passes from the primary position to any other position, the angle of torsion of the eye in its second position, is the same as if the eye had come to this second position by turning about a fixed axis perpendicular both to the first and to the second position of the line of regard.*"* Now an axis which satisfies these conditions can be none other than an equatorial axis, i. e., an axis at right angles to the polar or optic axis. In turning from side to side horizontally, it is a *vertical* equatorial axis. In turning up and down, it is a *horizontal* equatorial axis. In turning obliquely, as in the experiments on torsion, it is an *oblique* equatorial axis. Now let any one take a globe, and placing the equator in a vertical plane, make a distinct vertical and horizontal mark across the pole. If now the globe be turned on an oblique equatorial axis so that the pole shall look upward and to the right, it will be seen that the polar cross is no longer vertical and horizontal, but has rotated *to the right*, not the left, as Helmholtz's statements would indicate. Turning of the globe on a fixed axis so that the pole looks upward and to the left, will cause the cross to rotate to the left. So turning downward and to the right, produces rotation to the left, and to the left rotation to the right. If the globe turns thus on an axis inclined 45° to the vertical, and through an arc of 90°, the rotation of the polar cross will be exactly 45°. Thus Listing's law, as understood by himself, is in exact accordance with my results.

I have now, I believe, established on the firm basis of experiment, the true law of torsion of the eyes, when moving parallel to each other. I have also shown that it is identical with Listing's law, properly understood, and I shall therefore continue to call it by that name. Misled by Helmholtz's very positive statements, I commenced this investigation with the expec-

* Op. cit., p. 606.

tation that the operation of Listing's law, when combined with that of convergent motion, would completely explain all the phenomena described in my previous paper. In this expectation I have been disappointed. On the contrary, the investigation brings out in stronger relief than ever before, the *complete contrast* between the two laws. We would thus formulate the contrast:

1. When the eyes move in the *same direction* parallel to each other, in the *primary plane*, there is *no torsion* or rotation on the optic axis; but when they move in the primary plane in *opposite* directions as in convergence, they rotate outward, i. e., toward the temples, thus 

2. When the plane of sight is elevated, and the eyes move together parallel to each other, then if the lateral motion is to the right, the rotation is to the right, if to the left, the rotation is to the left; but when in the same position of the visual plane, the eyes move in opposite directions, as in convergence, then as the right eye moves to the left (toward the nose) it rotates to the right, and as the left eye moves to the right (i. e., toward the nose), it rotates to the left. If Listing's law operated at all in convergence, it would tend to neutralize the contrary effect of convergence; but such is not the fact.

3. When the visual plane is *depressed*, the direction of rotation is the same for parallel motion and convergent motion; in both cases the rotation is contrary to the direction of motion. But there is this great difference between the two; by the law of parallel motion, the rotation *increases* with the angle of depression, while by the law of convergent motion, it *decreases* to zero at 45°. If Listing's law operated at all in convergence, it would in this case coöperate and *increase* the motion, but the reverse is the fact, the rotation decreases.

4. There can be no doubt that in oblique motion the vertical and horizontal meridians of the eye become actually inclined on the true vertical and true horizontal, and that if we observed the iris of another person we would see it apparently rotated like a wheel. But although in deference to usage of other writers and to appearance, I have called this change a rotation on the optic axis, yet it seems to me it cannot be properly so called. For all parallel motions of the eyes are rotations on equatorial axes, and therefore on axes in a *plane perpendicular to the polar* or optic axis, and therefore cannot be resolved into rotations on the latter. In parallel rotation, therefore, the so-called torsion is only *apparent and the result of position*, or in other words the result of *reference to a new spatial meridian.* Turning the eye from side to side in the *primary* plane produces no torsion because all the spatial meridians are there *parallel*, but turning from side to side in an *elevated* plane pro-

duces apparent torsion, because the spatial meridians are there *convergent.* But in convergent motion, on the contrary, there is a *real* rotation on the polar or optic axis. This is shown by the fact proven in my previous paper* that one eye, without changing its position, will rotate through the influence of the convergent motion of the other eye.

5. It would seem at first sight, that spectral images might be used also for determining rotations of the eyes in convergence, but they cannot be used for this purpose at all. This brings into view another point of contrast, between convergent and parallel motion. In parallel motion spectral images follow all the motions of the eyes up and down, or to right and left, and all their rotations to one side or the other, with the utmost exactness. In convergent motion, on the contrary, though the eyes may each move through an angle of 45° or more, the position of the spectral image is the same, viz: in front; and though the eyes in extreme convergence may rotate in opposite directions each 10°, yet the spectral image retains its vertical position. The reason of this is that, although there are two retinal brandings, and therefore two spectral images, the external representatives of these brandings, yet the brandings being on corresponding points of the two retinæ, their external representatives, the two spectral images, are indissolubly united. Their separation, either wholly or partially, would be a violation of the law of corresponding points, a law which is never violated under any circumstances whatever.

In conclusion, then, it is evident that when the eyes move in the same direction, parallel to each other, as in ordinary vision of objects, all their motions are governed by the Law of Listing. But when, on the contrary, they move in *opposite* directions, as in strong convergence, then the law of Listing is entirely abrogated or overborne, and another law reigns in its place.