ART. XLV.— *Value of the force exerted by a current of Electricity in a circular conductor on a unit magnetic pole at its center;* by S. T. MORELAND.

THE following method of finding this value is new so far as I know.

Let i=strength of current
r=mean radius of the conductor supposed to make one turn.

So far as magnetic effects are concerned it is known that such a current produces the same result as a *magnetic shell* having its boundary in the circle and satisfying the condition

$$i=It$$

where I is the uniform surface density of the magnetism and t is the thickness of the shell. Substitute a *hemispherical magnetic shell* for the current in this case having plus magnetism on the inner surface and minus on the outer, the total quantity on each of the two surfaces being equal.

Let $\sigma_{\prime}$ =surface density on inner surface.
$\sigma_{\prime\prime}$= " " " outer "
$r_{\prime}$ =radius of inner surface.
$r_{\prime\prime}$= " " outer "

Then $r_{\prime\prime}-r_{\prime}=t$, a very small quantity, and $r=\sqrt{r_{\prime}r_{\prime\prime}}=\frac{r_{\prime}+r_{\prime\prime}}{2}$ very nearly, and $I=\sqrt{\sigma_{\prime}\sigma_{\prime\prime}}=\frac{\sigma_{\prime}+\sigma_{\prime\prime}}{2}$ very nearly.

Also
$$\sigma_{,}:I=\frac{1}{r_{,}^{2}}:\frac{1}{r^{2}}$$
$$\therefore\ \sigma_{,}=I\frac{r^{2}}{r_{,}^{2}}.$$

Similarly
$$\sigma_{,,}=I\frac{r^{2}}{r_{,,}^{2}}.$$

The resultant repulsion of the magnetism of the inner surface on the unit pole at the center is $\pi\sigma_{,}$ and the attraction of the outer surface is $\pi\sigma_{,,}$. Hence,

Resultant force of magnetic shell$=\pi(\sigma_{,}-\sigma_{,,})$

$$=\pi \mathrm{I}r^{2}\left(\frac{1}{r_{,}^{2}}-\frac{1}{r_{,,}^{2}}\right)=\pi r^{2}\frac{(r_{,,}^{2}-r_{,}^{2})}{r_{,}^{2}r_{,,}^{2}}$$
$$=\pi \mathrm{I}\frac{r^{2}}{r^{4}}(r_{,,}+r_{,})(r_{,,}-r_{,})$$
$$=\frac{\pi \mathrm{I}}{r^{2}}2rt=\frac{2\pi i}{r}$$

which is the result sought.

We see too that in the equation
$$I=it$$
I is the mean of the two surface densities which must necessarily differ in value unless the shell is a plane.