ART. XXXVIII.—*On Electrical Oscillations of Low Frequency and their Resonance;* by M. I. PUPIN, Ph.D., Columbia College.

Part I. *On the Production of Simple Harmonic Currents of Constant Frequency by Electrical Resonance.*

THE sensitiveness of the telephone for exceedingly small alternating currents is well known. It is probably as great as that of the most delicate Thomson galvanometer for direct currents. Just as this instrument, so the telephone is especially suited to zero-methods. But the telephone does not enjoy that popularity in the precision room which its direct current rival, the Thomson galvanometer, enjoys, although the field of physical research in which alternating currents must necessarily be employed is very extensive indeed. The fault lies with our alternating currents and not with the telephone. The alternating currents which the ordinary induction coil as employed in physical laboratories produces is far from being a simple harmonic current. The consequence is that in very many cases the zero method, for which the telephone is especially suited, has to be abandoned, and the minimum method substituted for it, which, of course, is a poor substitution.

Being engaged in a research in which I had to employ alternating currents I tried, for reasons just given, to devise some method of producing simple harmonic currents of constant frequency, the frequency to be easily and very accurately determinable. The following is my solution of this interesting problem:

A. *On the Production of Alternating Currents of Constant Easily and Accurately Determinable Frequency.*

My earliest solution of this problem consisted in producing an alternating current in the secondary of a very small transformer by making and breaking very rapidly and at a constant rate the primary. The interruptor of the primary current consisted of the following arrangement:

A stiff brass wire was stretched between the pole-pieces of a permanent horse-shoe magnet. The wire was supported, just as in a monochord, on two hard rubber bridges, and by varying the distance between the bridges and the tension of the wire it could be made to vibrate any note between about 60 and 1,000 complete vibrations per second. The middle part of the wire was between the pole pieces of the permanent magnet and carried just a short distance outside of the pole-pieces, a short, thin amalgamated copper wire which dipped into a mercury cup once during each vibration. At every dip it closed the circuit of a gravity cell and the action of repulsion between the current now flowing through the stretched brass wire and the poles of the permanent magnet kept up the vibration of the wire when once started. In fact, when well adjusted the wire would start to vibrate of itself, making the primary current at every downward stroke and breaking it at every upward excursion. The current in the secondary was an alternating current, of course, of exactly the same frequency as the vibration of the wire. The frequency of the vibrating wire could be varied by varying the tension gradually until the vibration of the wire was in exact unison with a standard tuning fork. Varying the tension (in a manner which will be described below) of the brass wire did not interfere with its vibration, so that the tuning could be made very accurately by watching the beats. In this form, this what I call *electrodynamic interruptor*, was shown to Professors Abbe, Barker, Mendenhall, Michelson, and Rowland, during the autumn meeting of the National Academy of Sciences in New York, in 1891, and was very favorably commented upon by these scientists.

In the meantime experience suggested the form given in fig. 1, as best suited to the purpose for which the interruptor

Fig. 1.

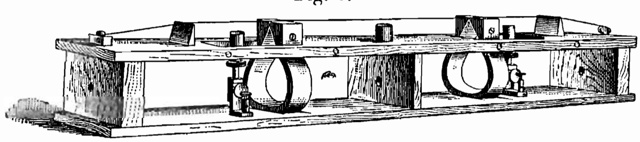
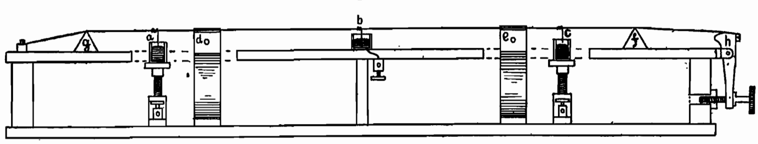


Fig. 2.



was first designed. The diagram of fig. 2 explains the construction of the apparatus more clearly. A stout aluminium, or phosphor-bronze wire, *the vibrator*, is stretched between the polepieces *d*, and *e*, of two permanent Weston magnets, such as this distinguished electrician uses in his voltmeters.

Fig. 3

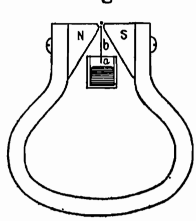


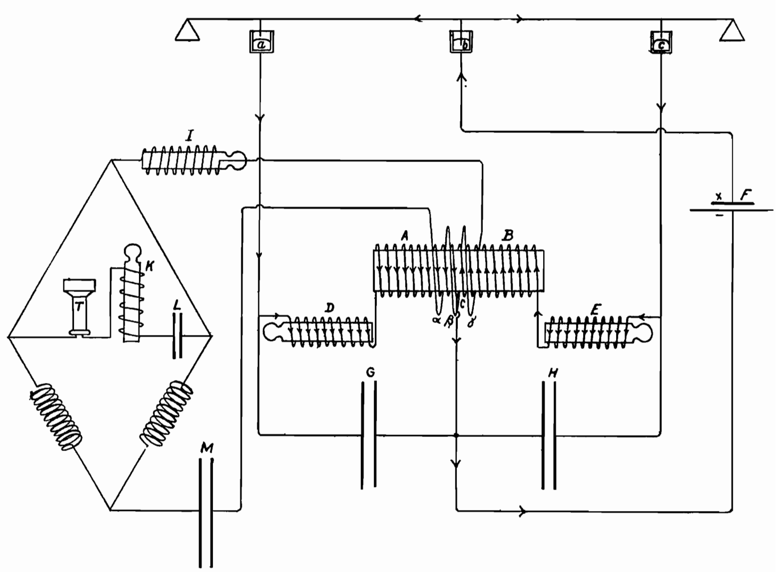
Fig. 3 gives the front view of one of the magnets. The cross section of the vibrator is seen there between the polepieces N S as a black dot. The short line, *a*, *b*, extending from the vibrator to the mercury cup below is the dipper, a short, thin amalgamated copper wire, which is soldered to the vibrator. The vibrator rests on two hard rubber bridges, *f, g.* One of its ends is rigidly attached to the wooden frame of the apparatus, the other end is attached to a lever *h* which, worked by a micrometer screw, varies the tension of the vibrator. There are three mercury cups, *a*, *b*, *c*, and three dippers (which unfortunately do not appear in fig. 1). The middle cup, *c*, is fixed in position, and the middle dipper, being at the nodal point of the vibrator, makes a permanent contact there. The other two dippers make contact with mercury cups which can be raised or lowered by means of a nut and screw as represented in fig. 1, and indicated in diagram 2. The construction of the adjustable mercury cups and the stretching lever were copied from Dr. Max Wien's magnetic interruptor (Wiedem. Ann. 1891 and 1892). The middle cup (see fig. 4), is connected to one pole F, of the gravity or storage cell, the other two cups are connected one to one end, and the other to the other end of the primary of the small coil A, B. From the middle point, C, of the primary a wire leads to the other pole of the cell. Auxiliary small coils, E and D, and condensers, H and G, are inserted in the circuits as indicated. Their functions will be explained further below.

The vibrator vibrates with a node at the middle dipper as soon as the tension has reached a certain, by no means high, limit. A permanent contact is therefore maintained at this point, and the contact is made at one of the other cups just at about the same moment as it is broken at the other cup. Leaving the condensers out of consideration for the present, it is evident that this form of the current make-and-brake produces the same effect upon the iron core of the coil as an alternating current would. The advantage of this needs no comment; for although the iron core consists of the finest iron wire that can be obtained in the market, yet it must be remembered that the vibrator is expected to work sometimes at the rate of 512, or more, complete periods per second. Another immediate advantage which this interruptor offers is a considerable diminution of sparking. The addition of condensers, besides performing other functions which will be discussed presently, reduces the break sparks almost to invisibility, even when currents as large as half of an ampere are used. Each half of the primary coil consists of 532 turns of No. 22 silk-covered wire wound over an iron core of 30^{cm} in length, $4^{\square cm}$ in cross section, and consisting of very fine, soft iron wire.

The vibrator when at work gives a pure, but not objectionably loud, musical note in which the overtones are scarcely perceptible. The frequency ordinarily employed in my work is 256 complete periods per second, and it is obtained by bringing the vibrator in unison with a König standard tuning fork. The tuning is done in a few seconds, without any difficulty, by simply stretching the vibrator gradually by means of the lever and micrometer screw and watching for the beats. The stretching does not interfere in the *slightest* with the vibrations of the vibrator.

The secondary current is, of course, an alternating current having the same frequency as the vibrator. But it is by no means a simple harmonic current. On the contrary, it is a very complex harmonic, its complexity depending on the fundamental frequency, on the ohmic resistances, and especially on the self-induction and electrostatic capacity of the primary and secondary circuits. A telephone placed in shunt with a part of the secondary circuit shows that without the condensers sparking is rather strong, producing that peculiar rattling noise which is full of those exceedingly high notes for which the telephone is especially sensitive. These high notes are due, as is well known, to rapid electrical oscillations which accompany the sparks. If the condensers are put in as indicated in fig. 4,

Fig. 4.

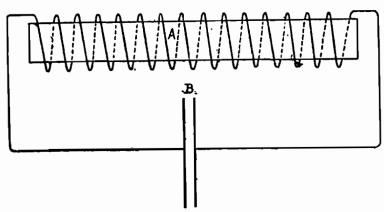


then the telephone shows that it is simply a question of capacity whether this or that overtone is particularly prominent. These overtones mean, of course, that in addition to the alternating current of the fundamental frequency there are also in the secondary circuit higher harmonic currents. In fact, the telephone shows plainly that sometimes these upper harmonics are apparently much stronger than the fundamental current. The method of reducing the complex harmonic current obtained by the means just described forms the next part of this paper.

B. *On the method of weeding out harmonics by electrical resonance.*

If a coil, A, (fig. 5) is connected with a condenser B

Fig. 5



and an impulse starts an electrical disturbance in this system, then electrical oscillations will result from this disturbance. Electrical equilibrium is restored again after the electrokinetic energy produced by the impulse is partly radiated off and partly transformed into heat by the ohmic resistance of the circuit. Not to mention losses due to magnetic and dielectric hysteresis and to convection currents consisting of dust particles charged by contact with the systems. In Hertzian oscillations and in Tesla frequencies the period depends on the self-induction and the capacity of the system only, as is well known. But even in systems of large self-induction and large capacity, where *à priori* we can expect a long period of oscillation, this period can be easily shown to be independent of the ohmic resistance of the system in the majority of cases. An analytical discussion of this matter, as well as of other matters relating to resonance of slow oscillations is reserved for a future paper. Suffice it for the present to refer to these things, only in so far as they bear upon the subject of this paper.

The period of the system represented in fig. 5 is given (provided certain well-known conditions are fulfilled) by

$$T=\frac{2\pi}{10^3}\sqrt{LC}$$

where T is the period in seconds, L the coefficient of self-induction in Henrys, and C the capacity in microfarads. I shall refer to this period as the "*natural period*" of the system. By varying the capacity or the self-induction of the circuit we vary its natural period. I call this variation the *tuning of the electrical circuit.*

Let a complex harmonic, alternating electromotive force E act upon this cirucit.

By Fourier's theorem E can be represented by

$$E = a_1 \sin pt + a_2 \sin 2pt + \ . \ . \ + a_n \sin npt + \ . \ . \ . \ \text{where } p = \frac{2\pi}{T}$$

T being the fundamental period. It is well to observe here that in complex harmonic e. m. forces as produced by ordinary methods the amplitude a_1, of the fundamental harmonic is largest and the amplitudes of higher harmonics diminish with the period of these harmonics.

The current produced in the circuit by the action of this complex e. m. f. will, of course, be a complex harmonic consisting of the same number of single harmonics as the e. m. f. and of the same periodicity; but the ratio of the amplitudes will be different now. The various simple harmonic components have also different phases. In general every one of these harmonics is a forced oscillation of the circuit, but by tuning the circuit we can bring it (within certain practical limits) in *resonance* with any one of the harmonics.

In my work I generally bring the circuits in resonance with the fundamental harmonic. A resonant circuit behaves toward a complex harmonic e. m. f. just the same as an acoustical resonator toward a source of complex sound. It brings out prominently that harmonic with which it is in resonance. To express this numerically, say that the ratio of the amplitude of the fundamental harmonic e. m. force to that of the next higher harmonic (supposing it even to be no higher than an octave) is 2 to 1. Then the circuit can be easily brought into resonance with the fundamental harmonic, in such a way as to increase the ratio of the amplitudes of the corresponding simple harmonic currents to 60 : 1. Theoretically (and to a great extent practically also) that ratio can be made anything we please by increasing continually the coefficient of self-induction and diminishing the capacity without destroying the resonance. In other words, we can by proper single tuning weed out the upper harmonics as much as we please. But, as will be indicated later on, it is not always advisable to avail ourselves too much of the means of weeding out the upper harmonics by using very large self-induction. The best method of tuning depends on the nature of the problem before us. I propose to discuss two cases, after stating briefly the experimental method which I consider as the simplest in detecting resonance. Consider the circuit represented in fig. 5. Put a telephone in shunt with some part of the circuit between the coil and the condenser; insert a small auxiliary coil with movable iron core in series with the large coil. Say the fundamental frequency is 256 per second, make the condenser capacity larger and larger until the deepest note in the telephone (in our case 256), comes out strongest. It is easily recognized, for the difference between the sound of the telephone with the upper harmonics strongly represented and the sound without them, is just about the same as between the sound of a clarinet and that of a drum when playing the same note. Having done that, I then move the iron core of the auxiliary condenser until the telephone sounds loudest. The circuit is then in resonance with the fundamental harmonic.

An interesting phenomenon is observed during the first part of the tuning process. While plugging the condenser, so as to bring the capacity nearer and nearer to the point of resonance, a certain point is reached, when taking out a condenser plug is followed by a bright, snapping, spark. The spark is a sign that the point of resonance is very near, for resonance produces a difference of potential between the condenser plates which is many times higher than the amplitude of the impressed electromotive force. I proceed to consider this phenomenon a little more fully.

Case I.—Method of Tuning for the purpose of producing a high rise of potential at the Condenser plates.

When resonance is established the ratio of the amplitude E_o of the impressed e. m. f. to the amplitude $E_{,}$ of the difference of potential at the condenser plates is given by

$$\frac{E_{,}}{E_o}=\frac{10^6}{\frac{2\pi}{T}CR}.$$

Where C is the capacity of the condenser in micro-farads, R the resistance in ohms and T the period in seconds.

On the other hand,

$$T=\frac{2\pi}{10^3}\sqrt{LC}$$

Hence

$$\frac{E_{,}}{E_o}=\frac{2\pi\,L}{T\,R}.=\frac{\text{Inductance}}{\text{Resistance}}$$

If the priod $T=\frac{1}{400}$ and $E_o=5$ volts, then

$$E_{,}=\frac{2\pi\times400\times L}{R}\times5.$$

Hence to get the rise in potential as large as possible it is necessary to make the resistance as small, and the coefficient of self-induction as large as mechanical (and financial) considerations will permit. It is not difficult at all to construct a coil whose L=5 and R=5, in which case $E_{,}$=over 12000 volts.

In a circuit of this kind the amplitude of the fundamental harmonic would be at least 2000 times as large as that of any of the upper harmonics. In other words, we should have a simple harmonic current in the circuit. In such a circuit the condenser has very small capacity and can be replaced by vacuum bulbs partially coated with tinfoil on the outside and electrical discharges could be produced in them by this enormous rise in potential. Hence the interest attached to this method of tuning. I expect to take up this interesting subject in another communication as soon as time will permit.

Case II.—Method of Tuning for the purpose of supplying a Wheatstone Bridge with a simple Harmonic Current of constant frequency.

In this case the method of tuning is governed somewhat by the well known conditions under which the flow in a Wheatstone bridge system will have the highest sensitiveness. These conditions exclude the possibility of using self-inductions which are very much larger than those in the principal branches of the bridge. *Two kinds of apparatus can be employed. The first* and in a great many respects the most convenient kind of apparatus is the interrupter described above. In this case the most difficult, but at the same time the most interesting part of the tuning consists in establishing resonance between the circuits ADGC, BCHE (fig. 4) and the vibrator. It is done in the following way:

The vibrator is first tuned up to any frequency we wish, say, 256, in the manner described above, all the iron having been previously removed from the coils and a high resistance inserted between F and C so as to reduce sparking at the dippers. The iron cores are then gradually pushed in and the capacity of the condensers G and H varied until the sparking is reduced to a minimum. This process is continued until the iron core of the principal coil AB is entirely in the coil. The final touches to this part of the tuning are given by means of shifting the iron cores of the small auxiliary coils D and E. By watching the sparks in the cups *a* and *c* the point of maximum resonance can be determined with great accuracy. For at this point the sparks are scarcely visible. If there is any defect in this adjustment it shows up immediately when the current is increased, by gradually diminishing and finally removing the resistance which, as a matter of precaution, was inserted in the circuit between F and C. The voltage of the generator F does not (within reasonable limit) seem to cause a variation in the size of the sparks if the adjustment is properly done. I have used as much as four storage cells in series as my exciter and have no doubt that with a properly constructed vibrator as much as 100 volts could be used without any annoyance arising from the sparks. But the interruptor must be kept in a place entirely free from jars or vibrations. For even with the storage battery just mentioned, when the voltage is only 8 volts, vibrations of the floor or table or singing a note which is nearly in unison with the vibrator will disturb its vibrations sufficiently to cause a dissonance between the vibrator and the circuits. This dissonance manifests itself at once by lively sparking which subsides as soon as the disturbance ceases. It is this very state of extreme sensitiveness of the electrically tuned up system which makes the work with the vibrator exceedingly instructive and interesting.

The secondary coil $\alpha\beta\gamma$ (fig. 4) supplies the alternating current for the bridge. The number of its turns is small in comparison to the number of turns in the primary and the current in it is also small in comparison with the current in the primary, on account of comparatively high resistance in the bridge, so that the variation of the secondary current does not interfere with the established resonance in the primary. We can therefore tune this circuit without disturbing the adjustment of the primary. The tuning is performed by means of the auxiliary coil I, the condenser M, and a telephone placed in shunt with a part of the circuit whose resistance is high enough to give sufficiently intense sound in the telephone.

Finally, the bridge itself containing the telephone T is tuned by means of the auxiliary coil K and condenser L. This part of the tuning does not interfere with any of the previous adjustments on account of the extremely small value of the current which has to pass through the telephone to make the sound in it sufficiently intense.

It is needless to observe that the upper harmonics which the vibrator tends to produce are completely wiped out in the telephone circuit and that, therefore, with this arrangement we can employ the zero method of measurement, using as our detector the ordinary Bell telephone.

The second kind of apparatus that can be employed is a coil in series with a condenser as in fig. 5. A spark space is inserted in the circuit between the coil and the condenser and the extremities of this spark space are connected to the poles of an influence machine, or a high Voltage electromagnetic generator, and an air blast is applied to the spark space when the sparks begin to pass. An alternating current of any frequency may thus be generated in this coil-condenser circuit by a proper adjustment of capacity and self-induction and by gradual transformation and tuning deprived of all its upper harmonics. This method is to be preferred when we wish to obtain a simple harmonic current of definite frequency to drive a synchronous alternating current motor, that is to say, a simple harmonic current, carrying with it a large quantity of power. I hope that I shall be pardoned for observing here that this last method was worked out by me some time ago. But in conversation with Mr. Nikola Tesla this distinguished experimentalist informed me that he obtained a patent about a year ago on the method of generating alternating currents of any frequency by means of disruptive discharges. The discovery that I was anticipated in a pretty invention caused me some disappointment at first, but I consoled myself very soon with the idea that a method patented by so excellent an experimentalist as Mr. Tesla, would enable me to obtain the object for which I invented it, the object being *to construct a synchronous alternating current motor which would spin around with great but perfectly constant angular velocity, the angular velocity to be just as adjustable and just as accurately determinable as the time of vibration of my vibrator, the motor to perform the function of a microphonograph.* I expect to be able to offer a favorable report on this matter very soon.

A communication of the results of some of my experiments with the simple harmonic currents obtained by the method described in this paper will be given as soon as time will permit.

Electrical Engineering Laboratory,
School of Mines, Columbia College, March 8th, 1893.