Art. XVIII.—*On the Use of Planes and Knife-edges in Pendulums for Gravity Measurements*;† by T. C. Mendenhall.

In the theoretical discussion of the pendulum it is usual to assume that it vibrates about an axis which is a straight line. In practice it is generally attempted to realize this condition as nearly as possible and the method almost if not quite universally adopted has been to attach what is technically known as a knife-edge to the pendulum and allow it to rest upon a suitably supported plane horizontal surface. The axis about which rotation takes place is at any moment determined by the contact of the edge with the plane surface. To satisfy theoretical conditions it is evident that this edge must be

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formed by the intersection of two perfectly plane surfaces, a condition which can never be quite realized in practice. It is important, therefore, to inquire how it may be most nearly approached and especially by what disposition of parts, both as to form and arrangement, a given departure from it will produce the minimum effect upon the period of the pendulum and upon the value of the force of gravity obtained by its use.

What is believed to be an important departure from the universal practice in regard to the arrangement of parts has recently been experimentally investigated and with very satisfactory results. It consists in an exchange in place of the knife-edge and plane, the latter being attached to the pendulum and the former to the fixed support. This arrangement offers many advantages to which it is desirable to invite attention. It will be best, however, to refer in the beginning to what will at once suggest itself to many as a serious objection to this plan. When the knife-edge is attached to the pendulum it may be assumed to have a constant relation to its mass, at least so long as the pendulum does not suffer an injury which alters its configuration. The axis about which vibration takes place may therefore be regarded as constant as far as relates to different sets of swings, and it will be practically indifferent as to what part of the supporting plane it rests upon. When the plane is attached to the pendulum this axis of vibration is entirely determined by the position of the plane upon the knife-edge.

This difficulty, which at first sight appears to be formidable, readily disappears in practice. In fact, a little calculation will show that the line of contact between the knife-edge and the plane must vary in position by a relatively large amount in order to effect sensibly the period of the pendulum. Even if the placing of the pendulum for successive swings were accomplished by no more accurate devices than the eye and hand, it would not be difficult to avoid sensible error from this source. The apparatus by means of which the pendulum is lifted from and lowered upon the knife-edge is readily made adjustable so that any desired line of contact can be secured and retained indefinitely.

As an illustration of the constancy of period of a pendulum arranged in this way, as well as showing the ease and accuracy with which the period of vibration is ascertained, the following results are exhibited.

They show the period of the pendulum derived from individual swings extending through about an hour each. Twelve such swings were distributed nearly uniformly through a period of twenty-four hours, in order to eliminate any effect of irregular hourly rate of the chronometer. The figures as shown below include this effect, of course, it being eliminated only from their mean.

The two sets shown were made for the purpose of determining a "pressure coefficient," the first being made in air at a pressure of 25^{mm} and the second at a pressure of 600^{mm}. It will be seen that in no case does the variation from the mean amount to as much as one part in a million, a constancy which leaves little to be desired.

Pres. 25^{mm}. Period	v.	Pres. 600^{mm}. Period	v.
·5006904	− 4	·5007368	+ 4
901	− 1	74	− 2
898	+ 2	71	+ 1
900	0	73	− 1
902	− 2	72	0
899	+ 1	75	− 3
901	− 1	73	− 1
898	+ 2	71	+ 1
896	+ 4	70	+ 2
902	− 2	68	+ 4
899	+ 1	69	+ 3
900	0	75	− 3
Mean ·5006900		Mean ·5007372	

The advantages of the new form of pendulum will be made evident on an examination of its application to the differential method or use of a so-called invariable pendulum as well as to the reversible form for absolute determinations of gravity.

There is an advantage in the matter of construction. It is not easy to insert the knife-edge in the head of the pendulum so that it shall be at right angles to the axis of symmetry of mass. The plane used in its stead may be accurately adjusted by simple optical methods.

The advantage of the plane in the matter of permanency or invariability is so evident as hardly to need remark. The knife-edge is usually the most delicate part of a pendulum, that most liable to be injured and generally incapable of being repaired when once damaged. In an invariable pendulum the knife-edge cannot be reground or replaced by another, without destroying the identity of the whole so that swings at different places or times are no longer comparable with each other. A pendulum carrying a plane instead of a knife-edge is evidently vastly less liable to accidental injury and is entitled in a much greater degree to the name "invariable." The knife-edge being no longer an integral part of the vibrating mass can be reground or replaced at will. In fact in practice it is desirable to have several knife-edges and in an extensive pendulum campaign a "standard edge" will be used upon which swings will only rarely be made in order to detect any deterioration which may take place in those in daily use. Knife-edges of different material may also be used if such a course is found to be desirable.

In the reversible pendulum for absolute measures, in addition to the advantages already described there is also the very important fact that the measurement of the length of the pendulum is likely to be more accurate. Whether the knife-edge is a part of the pendulum or of the fixed support, a certain amount of elastic compression will always take place when the weight of the pendulum is upon it and this is likely to be relatively greater the more perfect the edge. If the knife-edge is a part of the pendulum the length of the latter must be greater when vibrating than when at rest and supported with the edge free for length measurement.

The measurement of the distance between the two knife-edges of a reversible pendulum as ordinarily made is a matter of much difficulty. It is believed that without resorting to extraordinary methods the distance between the two planes of the newly constructed reversible pendulum will be capable of measurement with higher accuracy and if necessary or desirable, recourse may be had to the method of Michelson and Morley for relating the distance between surfaces to the length of light waves.

Perhaps the most important gain thus far from the new arrangement is that it has made it possible to investigate the knife-edge, as to form and material, with an ease and thoroughness hitherto unattainable. As long as the knife-edge forms a part of the pendulum it is impossible to study the effects of variation in its angle, width or material because every such alteration must necessarily alter the vibration-period by changing the mass and form. With the new form this difficulty no longer exists; the vibrating body remains constant in mass and configuration and any change in its period is due to the influence of the knife-edge.

Some of the results already obtained are of sufficient interest to justify their publication in advance of a full and complete investigation now in progress.

In order to investigate the effect of a wearing or slight flattening of the edge, such as may, and in fact does result from long and not too careful use, a steel knife-edge was used, the plane, forming a part of the pendulum, being of agate. A steel-edge was used on account of the greater ease with which it could be manipulated in grinding. The agate knife-edge has been in general use, but experiment showed that both steel and agate being made as perfect as possible, the vibration periods were essentially the same. The angle which the two planes forming the edge made with each other was 110°. The edge was first ground until it was pronounced as perfect as was practicable by the artist, Mr. E. G. Fischer, chief mechanician of the Coast and Geodetic Survey, who has shown rare ingenuity and skill in the solution of the mechanical problems arising during the investigation. The width of the edge was then measured, using a microscope magnifying from 100 to 500 times. This was a difficult operation, the question of illumination being troublesome. It seemed tolerably certain, however, that the width of such an edge was not greater than 1^{μ} ($\cdot 001^{mm}$) Several different edges ground in this way were measured with essentially the same result. After the vibration period on one of these had been ascertained it was given one or two light touches upon the stone, producing an edge which was found on measurement to be approximately 2^{μ} in width. After the pendulum had been swung on it it was still further flattened.

Theory shows that a pendulum will vibrate more rapidly upon a slightly flattened or rounded edge than upon one which is perfect. The table below shows the results of experiment conducted as above described. The pendulum used was one of the short, approximately half-second, pendulums of the form referred to a year ago.* The numbers showing the width of the knife-edge must be regarded as approximations only but they are probably relatively not far from correct.

They indicate very clearly and positively the important part played by the knife-edge and the importance of having it perfectly ground. There is good reason to believe that this matter has not hitherto received that attention in pendulum researches which it is here shown to demand. While the effect of a given amount of flattening on the vibration-period would be less with a long seconds pendulum than with one of shorter period, the tendency towards flattening, arising out of greater weight and greater difficulty of handling would be very much increased. There is no doubt that in some important gravity operations knife-edges have been used which, in the light of these results must be considered intolerably poor. It will be observed that going from what may be accepted as a practically perfect edge to one only one two-hundredth of a millimeter wide, changes the period by one part in forty thousand, an amount entirely outside of a reasonable limit of accuracy for work of this kind.

The question of the best angle for the knife-edge is also important and interesting. If the material of which the edge is composed were physically perfect and if the faces were perfect planes all angles, within certain wide limits, would be

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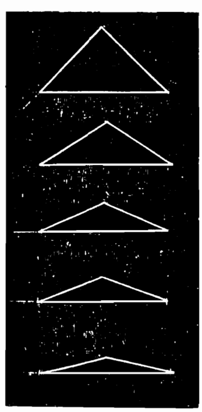
equally good. Imperfection of substance, as to strength, continuity, etc., is the principal difficulty and this is met by making the angle of the faces as great as possible. But the errors arising from the impossibility of grinding the faces to true planes are reduced to a minimum by making the angle as small as possible and hence there is indicated an angle neither very large nor very small which is better than others. In other words, if the angle is too small the edge will be entirely crushed and thus broadened and made imperfect. If the faces are not perfect planes it is clear that the larger the angle the wider will be the limits of the horizontal projection of the line of their intersection and this will be equivalent to widening the edge upon which the pendulum swings.

It is interesting that this seems to be verified in practice, as is shown in the results given below. Edges ground to five different angles, varying from 90° to 160° were used. The results show that while there is no *great* difference in the results with the range of angles examined, it is tolerably certain that the angle should not (with this material—steel) be less than 110° nor more than 140°. An angle of 130° will probably be found the most satisfactory, combining the necessary sharpness (when properly ground) with strength to resist accidental injury or excessive elastic compression.

Steel Knife-Edge.

Width.	Period.	Gain.
μ		
1	·5008880	0
2	8839	41
5	8756	124
6·5	8503	377
10	7626	1254



Angle.	Period.
90°	5008875
110°	8880
130°	8874
150°	8884
160°	8852

Steel knife-edges have been very generally used in pendulum work but they are inferior to those made of agate. The brittleness of the latter is rather an advantage than otherwise. When an agate knife-edge receives a blow sufficient to injure it, a piece is chipped out, leaving the remaining portion of the edge clean and perfect as before and the only effect on the vibratory period is that due to the removal of the matter lost. Under the same circumstances an edge of steel will be flattened or distorted and while there may be no loss of matter the edge may be made so imperfect that the period will be decidedly affected.

The grinding of an agate edge demands much more labor and skill than is required in the preparation of one of steel but it is well worth the additional cost.

It has been found that a knife-edge, even if its mass is considerable, is very susceptible to such distortion of figure as will render it imperfect. To avoid this it has been found desirable to insert the edge in a heavy tablet of brass and grind it *in situ.* The tablet is provided with three feet with hemispherical ends, resting respectively in a conical hole, a V groove and on a plane. In this way the edge is subjected to no strain after it is ground.