ART. XI.—"*Potential*" *a Bernoullian Term;* by GEO. F. BECKER.

POTENTIAL as the name of a function was undoubtedly introduced by Gauss in 1840. He wrote: "Zur bequemern Handhabung der dazu dienenden Untersuchungen werden wir uns erlauben dieses V mit einer besondern Benennung zu belegen, und die Grösse das *Potential* der Massen, worauf sie sich bezieht, nennen."* But Gauss was certainly not the first to employ the corresponding adjective in a similar sense. George Green employed the term *potential function* in 1828 in his famous paper on electricity and magnetism. He introduced the phrase by remarking that certain memoirs of Poisson's "are in fact founded upon the consideration of what have, in this essay, been termed potential functions."† In section 1 of the paper he says: "As this function . . . will occur very frequently in what follows, we have ventured to call it the potential function belonging to the system." Nowhere in this paper can I find the word potential used as a noun, while "potential function" is met with on almost every page.

Most physicists give Gauss full credit for independent investigation of the potential, but Todhunter refers to the matter in a manner which would seem to convey an innuendo. "We may observe," he says, "that the name *Potential* was first used by the late George Green. . . . Gauss used the word in his memoir . . . published in 1840. As Gauss does not refer to any previous authority we are, I presume, to infer that he had independently selected the name."‡ As was pointed out above, this is not a correct statement, since Green did not use the *name* potential but only the adjective; and the effect of the words "I presume" is to suggest a doubt whether Gauss might not have been acquainted with Green's nomenclature, and consequently also with the theory portions of which he is credited with rediscovering. Gauss however never displayed any tendency to plagiarism.

It is not very generally known, though it has been mentioned in modern literature, that Daniel Bernoulli and Euler employed the term *vis potentialis*. A single passage from Euler will illustrate its use: "Quamobrem cum vir celeberri-

* Allgemeine Lehrsätze in Bez. auf . . . Anziehungs- u. Abstossungs-Kräfte. Collected works, vol. v, 1867, p. 200.

† Reprinted in Crelle's Journal, vol. xxxix, p. 77, 1850, and in his collected papers.

‡ Math. hist. theories of attraction and the figure of the earth, vol. ii, 1873, p. 26.

mus . . . Daniel Bernoulli mihi indicasset se universam vim, quae in lamina elastica incurvata insit, una quadam formula quam *vim potentialem* appellat complecti posse, hancque expressionem in curva elastica minimam esse oportere, etc." In stating his problem he says again "atque, secundum Bernoullium, exprimetur *vis potentialis* in laminae portione AM contenta hac formula $\int \frac{ds}{RR}$," s being the arc of the spring and R the radius of curvature.* It is evident that in these passages *vis* is used in the same sense as in *vis viva* and that it is to be translated energy, so that Bernoulli's proposition was that the elastic curve must be such as to make something which he regarded as the potential energy of a bent spring a minimum.

Todhunter in his History of Elasticity, 1886, quotes passages from D. Bernoulli's letters in 1742–3, in which the vis potentialis is mentioned; but since the historian makes no allusion to his earlier remarks on the origin of the name Potential, it must be inferred that no relation between the two terms suggested itself to him. Had Todhunter lived to edit his history, perhaps this omission would have been supplied.

It is well known that Green's great memoir was not published in the ordinary manner, but by subscription in Nottingham, and that it attracted no attention for many years either in Great Britain or on the Continent. Gauss was thus very naturally ignorant of it. On the other hand it is substantially impossible to suppose Gauss ignorant of the famous memoir of Euler quoted above. The methods there developed of finding curves have been superseded by those of the calculus of variations; but the appendix, *Additamentum de curvis elasticis*, from which the passage quoted above is taken, contains the classification of elastic curves into nine species which, so far as I am aware, has received no addition or improvement.

There can thus be substantially no doubt that Daniel Bernoulli's vis potentialis suggested to Gauss the name potential for a somewhat similar but more general function. Either he considered the change of form of the expression and the increased generality of its significance as sufficient to make any reference to Bernoulli needless; or, as seems more probable, he assumed that his readers were as well read as himself and that an allusion was superfluous.†

* De Methodis inveniendi lineas curvas maximi minimive proprietate gaudentes. 1744, pp. 246, 247.

† The potential has received a variety of names in modern times. "Force function," which seems to be used by European mathematicians at least as frequently as potential, was proposed by W. R. Hamilton in his famous memoir on varying action, Phil. Trans., 1834, p. 249. In the same year B. de Saint-Venant called it "latent dynamic capital" (Leçons de Navier, 1861, p. 786) and Ampère suggested "implicit vis viva." Ann. chim. phys., vol. lviii, 1835, p. 438.

It is natural to inquire what Euler and Bernoulli meant in terms of modern nomenclature by vis potentialis. Now Saint-Venant has pointed out* that if M is Young's modulus and J the moment of inertia of the cross section of the spring, the work done in flexure for a length s is

$$\frac{MJ}{2}\int_0^s \frac{ds}{R^2}$$

so that Bernoulli's function is simply proportional to the potential energy as that expression is now understood and to the *Potential*, a fact to which the great French elastician calls attention.

There seems to be a general impression that the natural philosophers of the last century, when they used the quantities now known as kinetic, potential and total energy at all, regarded them from a purely algebraical or geometrical point of view, failing to perceive their great physical significance. In this respect these physicists seem to have been underrated: as some passages from the first John Bernoulli, Euler's teacher and D. Bernoulli's father, will show. In a paper on the true conception of living forces† he generalizes the idea of vis viva and defines it as equivalent to capacity for doing work, or *facultas agendi*, which is simply a Latin equivalent of the Greek energy. In Section I of this paper he says (translated):

"Vis viva does not consist in the actual exertion, but in the capacity for doing work: for it subsists even when it does no work nor has any object whereon it could act; so for example a strained spring, or again a body in motion, has in itself the capacity of doing work, so that if nothing external to itself come in its way upon which it may exert itself, and so long as there is no object present with which it can come in contact, it infallibly retains it all undiminished by time, and does not do the work which it would be capable of doing if it had the opportunity."

This seems a clear and even a vivid statement of the law: "When a system is subjected to no external forces, its energy remains constant."

In Section III he takes a further step. "Vis viva (which would be more aptly named facultas agendi, gallice le pouvoir)‡

* Leçons de Navier, edition of 1864, p. cxij.

† De vera notione virium vivarum, Acta Eruditorum, Leipzig, 1735, p. 210.

‡ The term power is now rarely used for energy, but it is scarcely a generation since this meaning was common enough. Saint-Venant (op. cit., p. 785) in 1864 defined the potential of one or more forces as "leur pouvoir moteur total." B. Peirce in his great work on analytical mechanics 1865 always uses "power" instead of "energy."

is something real and substantial, which has an independent existence and, whatever it consists of, depends upon nothing else. Whence we conclude, that any given vis viva is of determinate quantity of which none can disappear except it reappear in the effect produced. Hence it follows at once, that vis viva is always preserved, and so perfectly that what inhered in one or many bodies before action is now, after action, necessarily found in another or in several others excepting what remained in the first system. And this we call the *conservationem* virium vivarum."

Compare this with the modern statement: In any system the variation of energy is equal to the external work done by the system less the work done by external forces upon the system.

John Bernoulli was under no misapprehension as to the importance of the principles he had stated. He says in substance: Whether bodies are regarded as communicating motion to one another or whether one considers the various modifications of the motion of one and the same body depending on its own force (where nothing can vanish without an equivalent effect), "pro fundamento et principio universali poni debet conservatio virium vivarum, hoc est illius *facultatis agendi*."

That such men as Daniel Bernoulli and Euler should have been deaf to teachings like these is impossible and they must therefore have had an idea of the vis potentialis differing but little from that which the words convey to modern ears.

It would be interesting to know whether they really omitted to show in any portion of their writings why they felt themselves at liberty to base investigations on the statement that the potential energy of a strained solid is a minimum. Indeed it appears to me that the few notes given above are sufficient to indicate an opportunity for a physicist to write a most interesting essay on the evolution of the potential up to the date of Lagrange's memoir on the movement of a number of mutually attracting bodies (1777) in which the potential seems to have been used for the first time in an entirely general form.

Washington, December, 1892.