ART. IV.—*Preliminary Account of the Iced Bar Base Apparatus of the U. S. Coast and Geodetic Survey*;* by R. S. WOODWARD.

Historical Note.—The use of ice in thermometry to furnish a standard temperature naturally suggests the availability of ice to fix the temperature of a standard of length when used in laboratory comparisons or in measuring base lines It does not appear, however, that ice has been generally used even in laboratory work with standards of length,† and I am not aware that any attempt has been made hitherto to measure a base with a bar whose temperature is controlled by means of melting ice. The feasibility of using such an apparatus in base measurement has, nevertheless, been suggested and maintained by several persons. One of the first, if not the first, to outline a scheme for such an apparatus is, I believe, Mr. E. S. Wheeler, a former colleague on the U. S. Lake Survey. Mr. Wheeler's plan is advocated by Professor T. W. Wright in his treatise on the Adjustment of Observations.‡ The late Captain C. O. Boutelle of the U. S. Coast and Geodetic Survey also advocated the use of such apparatus.

Soon after joining the U. S. Coast and Geodetic Survey in July, 1890, I was requested by Dr. Mendenhall, Superintendent, to devise means of testing in the most thorough way

* Communicated by permission of the Superintendent of the Survey. The substance of this paper was presented before Section A of the American Association for Advancement of Science, at the Rochester meeting, August, 1892.

† From published accounts it would appear that the most extensive series of laboratory comparisons of standards, wherein ice was used, are those of the U. S. Lake Survey, conducted under the superintendence of General C B Comstock, Corps Engineers, U. S. A. In these comparisons ice was successfully used during several years. See Professional Papers Corps Engineers, U. S. A., No. 24.

‡ D. Van Nostrand, New York, 1884. See also this Journal, III, vol. xxviii, p. 479.

practicable the efficiency of the various forms of base apparatus used by the Survey and especially the efficiency of long steel tapes or wires. Accordingly, considerable study was given to this subject during the autumn of 1890 and the winter of 1890–1, and the plans and specifications for the iced bar apparatus considered in this paper were matured and approved early in the spring of 1891. It was constructed in Washington, partly by the machinists E. N. Gray and Co., and D. Ballauf, and partly by the Instrument Division of the Survey.

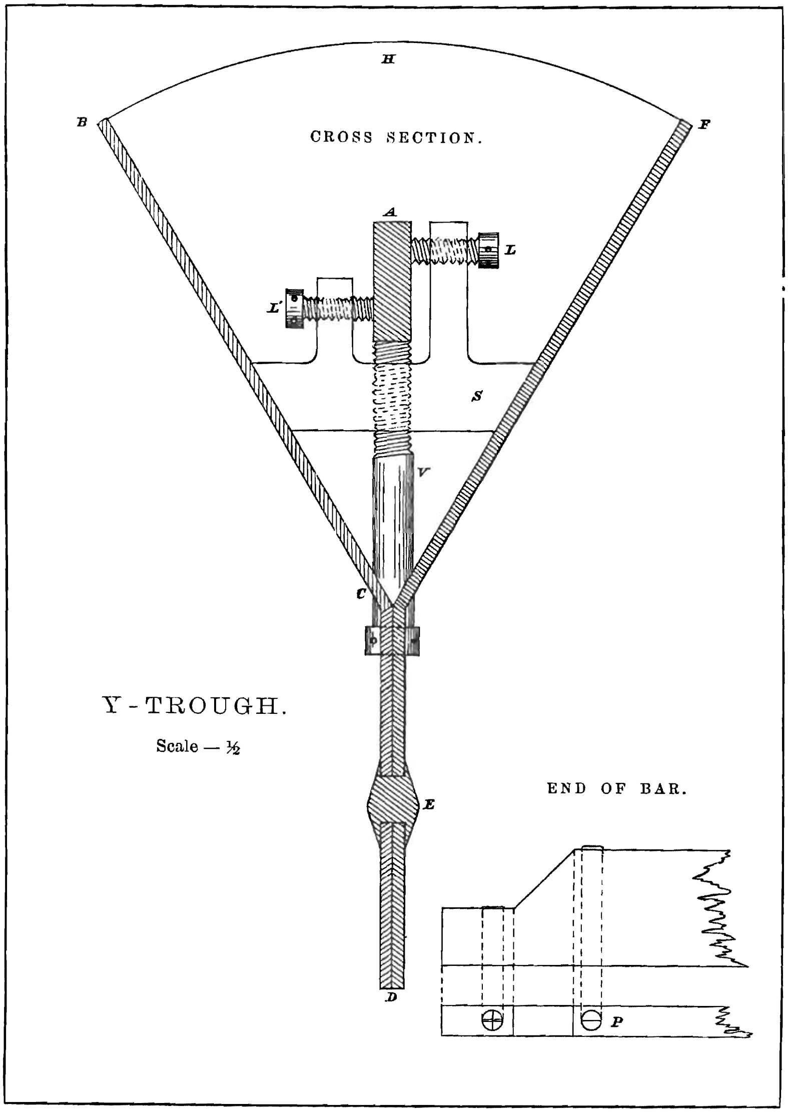
Before proceeding to a description of the apparatus I desire to acknowledge my indebtedness to colleagues of the Survey for valuable suggestions and criticism. I am specially indebted to Mr. John S. Siebert, who verified all of the preliminary calculations relative to the stability and efficiency of the apparatus, and who elaborated many of the designs and made most of the working drawings for its construction I am particularly indebted also to Mr. E. G. Fischer, chief mechanician of the Survey, whose knowledge of and skill in mechanical appliances were frequently appealed to. Finally it affords me pleasure to state that my friend Mr. E. S. Wheeler, who has had extensive experience with base apparatus, happened to visit Washington about the time the plans for this apparatus were completed and gave me the benefit of his advice and criticism.

Description of Apparatus.

The measuring bar.—The measuring bar of this apparatus is a rectangular bar of tire steel. It was rolled in the steel works at Lancaster, Pa. It is $5 \cdot 02^{m}$ long, 8^{mm} thick and 32^{mm} deep. A cross-section is shown at A in the accompanying drawing.

The upper half of the bar is cut away for about 2^{cm} at either end to receive the graduation plugs of platinum-iridium, which are inserted so that their upper surfaces lie in the neutral surface of the bar. Three lines are ruled on each of these plugs, two in the direction of and one transverse to the length of the bar. These lines were ruled by Mr Louis A. Fischer, Adjuster in the Weights and Measures Office. The longitudinal lines, which serve to limit the parts of the transverse lines used, are $0 \cdot 2^{mm}$ apart.

To secure alignment of the bar eleven German silver plugs of 5^{mm} diameter are inserted at intervals of 495^{mm} along the bar so that they project about 1^{mm} above its top surface. The upper surfaces of these plugs are all the same distance, within a few hundredths of a millimeter, from the neutral surface of the bar On the top of each plug is ruled a fine line in the direction of the bar as shown at P in the drawing. The length of the bar as regards alignment is defined to be the distance between the transverse graduation marks when the upper surfaces of the alignment plugs are all in one plane and



when the lines on these plugs are in one straight line. The means of securing these two adjustments are described below.*

The Y-Trough.—The most important and distinctive part of this apparatus is the trough which supports the bar, keeps it aligned, and carries the ice load essential to control the bar's temperature. This trough is called the Y-trough by reason of the resemblance of its cross-section to the letter Y. The drawing shows a cross-section of this trough. It is made of two steel plates $5{\cdot}14^{m}$ long $25{\cdot}5^{cm}$ wide and 3^{mm} thick. They are bent to the angle B C D of the figure and are riveted together as shown at E, thus making the angle of the trough B C F = 60°. The bar, shown in cross-section at A, is supported at every half meter of its length by saddles, one of which is shown in the figure. These saddles are rigidly attached to the sides of the trough by screws at S, S. Each saddle carries one vertical and two lateral adjusting screws as shown at V, L, L′. These screws serve to fix the alignment of the bar. The lateral adjusting screws of the saddles at the ends of the bar are of the same height, which is equal to that of the lower screw L′ of the diagram. The lateral adjusting screws of the intermediate saddles on either side of the bar are alternately high and low. The object of this disposition is two-fold, to wit: 1st, to prevent pinching the bar, which might more readily occur if the lateral screws were all opposite to one another; 2d, to afford means of rotating the bar slightly about its longitudinal axis, so that for a fixed and nearly vertical position of the trough the graduated surfaces of the bar may be made horizontal. The vertical adjusting screws of the saddles project, as shown in the diagram, below the vertex of the trough, and their capstan heads are accessible through slots cut in the web of the trough. These slots serve also as drainage-ways for the melted ice. To prevent circulation of air through them they are stuffed with cotton batting, through which the water percolates freely. The ends of the trough are closed with wooden V-shaped blocks.

The trough is very rigid in all directions and especially so with respect to vertical stresses. It weighs 82 kilograms exclusive of the bar and ice load. The whole trough is covered by a closely fitting jacket of heavy white cotton felt, which protects the trough and ice load alike from direct radiation.

* The form of bar described is evidently not the best form. Theory and experience indicate that a bar having a Y-shaped cross section with metric subdivisions on its neutral surface would best meet the requirements However, the question which presented itself in planning the apparatus was not what is the best form of bar, but what is the most economical form possessing the requisite properties.

For measuring grade angles a sector reading by two opposite verniers to $10''$ is attached to one side of the trough near its middle point. Thus arranged this sector has great stability.

The ice load and ice crusher.—When the apparatus is in use the Y-trough is completely filled with pulverized ice, the upper surface of which is rounded to about the height shown by the curve B H F in the diagram. The amount of ice required for this purpose is about 40 kilograms, or 8 kilograms per meter of the bar's length. The ice, by reason of its weight and the sloping sides of the trough, is kept in close contact with the bar. This is especially the case when the apparatus is in use, for it is then trundled along on its cars with sufficient jarring to overcome any tendency of the ice to pack. For covering the ends of the bar a small quantity of ice is cut with a jack plane. Ice thus cut, like wet snow, packs well and permits making a small conical hole through it to the graduation plugs.

A very essential auxiliary to the use of the apparatus is an ice crusher to pulverize the ice. The machine used is a modification of the Creasey ice breaker manufactured at Philadelphia, Pa. It is a small light hand machine which, as modified, does its work very satisfactorily. With it 40 kilograms of ice may be pulverized in ten minutes or less. The particles of crushed ice vary in size from the smallest visible up to the bulk of a cubic centimeter; and this gradation in size appears to be advantageous as compared with uniformly finer particles like those of snow, since there is less liability of regelation and packing.

The cars and portable track.—The Y-trough is mounted on two cars, the saddles or bolsters of which are attached to the trough 40^{cm} from either end. Each saddle is attached rigidly to the trough above and to a jack screw below. The jack screw is attached to a slide rest which is connected rigidly with the base of the car. The slide rests are provided with screws to give slow motions in the direction of the trough's length and transverse to its length.

The jack screw cylinders have right and left handed threads at their respective ends and are turned by a short capstan bar. They give thus the rapid vertical motion to the trough essential in bringing the bar quickly to focus under the microscopes which define its position.

The cars have each three wheels and run on a portable track whose width is 30^{cm}. Three sections of this track, each 5^{m} long, are provided; and each section is carried forward as the cars are rolled along during the measurement of a line. It thus appears that instead of lifting up and carrying forward the measuring bar as with most forms of apparatus, this rather delicate and difficult operation is supplanted by that of moving the portable tracks.

The micrometer microscopes.—To define the successive positions of the bar in measuring a line, micrometer-microscopes are used. Through the courtesy of General Casey, Chief of Engineers, U. S. A., the Survey was enabled to borrow the four microscopes and the cut-off cylinder of the Repsold base apparatus* used on the U. S. Lake Survey. These are especially well adapted for use with any line measure apparatus. As designed by the Repsolds and as used on the Lake Survey the microscopes were mounted on iron tripods. These latter having been destroyed by fire while stored at the Engineer depot at Willett's Point, N. Y., it was essential to replace them by some equivalent device. In view of the economic and other features of the special work contemplated with the iced bar apparatus it was decided to mount the microscopes on wooden posts set firmly in the ground. To connect the microscope with the post a cast-iron post cap is provided. It fits like a box cover on the end of the post and is clamped rigidly to it by means of a screw.

The microscopes are provided with levels and leveling screws so that their axes may be made vertical. They are mounted on slide rests which give a motion of 2^{cm} in the direction of the line measured or transverse to it. To secure additional displacement in the direction of the line a small rotary motion is provided for in the connection of the microscope with the post cap. The micrometer heads of the microscopes are divided to read microns directly, one revolution of the screws corresponding to $0{\cdot}1^{mm}$. When used in the field the microscopes are shaded from the sun by large umbrellas.

With this method of mounting the microscopes it is advantageous if not essential to set the microscope posts and those supporting the portable track before beginning measurement.

End marks and method of reference thereto.—The method of marking the end of a line is essentially that of the Repsolds and fully described in the Lake Survey Report referred to above. It consists in the use of a metallic bolt terminating in a spherical head, the bolt being embedded in a stone or other stable mass set in the ground. The center of the bolt head is the fiducial point. To refer to this point a cylinder called a cut-off cylinder is used. It terminates at one end with a conical hole which fits over the spherical head. The other end is provided with a transverse level and graduated scale. The scale is

* Fully described in General Comstock's report referred to on page 33.

brought by a rack and pinion motion to focus under the microscope whose position relative to the fiducial point is sought. The scale and level, which are parallel to each other, are placed parallel to the line measured. With the cylinder thus disposed readings of the micrometer on the scale and of the position of the level bubble are made. The cylinder is then turned 180° in azimuth and the scale and level readings are again observed. From these observations and the height of the scale above the bolt head, the horizontal distance (in the direction of the line) between the micrometer zero and the fiducial point may be accurately determined.

Adjustments of apparatus.—The most important adjustment of the apparatus is the alignment of the bar in the Y-trough. This adjustment is made when the ice load is in the trough and after the latter has had time to assume a stable shape. This time does not exceed 15 minutes.

As already stated, the alignment of the bar requires that the upper surfaces of the alignment plugs be in one plane and that the lines on these plugs be in the same straight line. The former requisite is secured by a striding level whose feet are 99^{cm} apart, so that they reach from any plug to the second adjacent plug. Beginning at one end of the bar the plugs are numbered 1, 2, 3,—11. By placing the level feet in succession on plugs 1 and 3, 3 and 5, etc., plugs 1, 3, 5,—11 are brought into the same plane by means of the corresponding vertical adjusting screws, the screws under plugs 2, 4,—10 being loosened if need be to secure this end. Having thus adjusted plugs 1, 3, 5,—11, the level is placed on plugs 2 and 4, 4 and 6, etc., and the vertical screws are brought up to contact with the bar but not raised enough to disturb the previous adjustment of 1, 3, 5—11, which are the principal defining plugs in this adjustment.

To place the lines on the plugs in the same straight line a sharp pointed plumb bob suspended from a fine brass wire stretched over the trough was originally used. This device with the aid of the lateral adjusting screws of the saddles permits placing the lines in proper position within $0{\cdot}1^{mm}$ when the trough is fully loaded with ice. Experience with the apparatus, however, showed that the simpler method of stretching the wire, or better still, a fine thread, close over the plugs when the trough is about four-fifths loaded secures equally good results.

It was feared before using the apparatus that the daily temperature range might produce an appreciable effect on the length of the bar through change in curvature of the trough. Hence the accurate method of measuring such change by the striding level was provided. But experience shows that the change in shape of the trough gives rise to quite insignificant changes in length of the bar. Indeed, the alignment of the bar may be maintained so perfectly that the correction for its curvature will not exceed a few tenths of a micron.

The grade sector of the apparatus is adjusted to zero when the graduated surfaces of the bar are in the same horizontal plane. To secure the latter condition an engineer's level is used; and with appropriate care the difference in height of the ends of the bar can be made zero with a probable error not exceeding $\pm 0{\cdot}1^{mm}$.

The microscopes are provided with fixed levels, which, when once adjusted, enable the operator to make the axes of the microscopes vertical. They are also provided with clamp screws so that they may be rigidly held in proper position.

When posts are used to support the microscopes, as has been the case with this apparatus thus far, they must be set in their proper positions within a centimeter or two. It is easy and convenient, however, to adjust their sides facing the line to be measured with much greater precision. When firmly set, a line parallel to the base may be deliberately ranged out with a theodolite of high magnifying power, and this line may be defined by suitable marks on each post. Then by simply noting the distance of the axis of the bar during measurement from this reference line an accurate correction for deviation of the bar from parallelism with the base may be obtained. This adjustment of the posts though not essential to the use of the apparatus has been followed.

Another convenient adjustment which the use of posts permits is that of making the tops of several or many posts conform to one grade. By this means, since the four microscopes used are closely alike, the grade angles for several or many bar lengths are nearly the same—a condition favorable to precision in determining grade corrections. As an additional precaution in the use of this apparatus the relative heights of the alternate post tops have been determined with an engineer's level.

Method of measurement.—To conduct the measurement of a line with this apparatus eight men are required, to wit: three observers; one recorder; one man to move the microscopes; and three men to move the car tracks, the microscope shades, and the ice and ice crusher.

The operation of measurement proceeds as follows: The position of the microscope relatively to the fiducial point at the end of the line having been observed as explained above, the rear end of the bar is brought to focus under that microscope by the rear end observer. By means of a lever which grips into the track and hinges on the car, the latter observer holds the bar near to bisection under the microscope while the front end observer brings his microscope into position over the front end of the bar; to do which he can make use of the lateral motion of the trough, of the microscope, or both. When the bar is adjusted at both ends the rear observer brings the rear end graduation accurately to bisection between the micrometer wires by use of his lever without turning the micrometer screw. Simultaneously he gives the signal "read" to the front observer, who brings his micrometer wires to bisect the front end graduation mark by moving the microscope, the micrometer wires, or both. The observers then read their micrometers and the recorder notes them down in his book, after which the rear observer turns his micrometer screw a half revolution or less backwards. The observers then exchange positions. The rear observer carrying with him his lever applies it to the front car and brings the front end graduation to bisection without disturbing the micrometer threads from their previous position; while at the signal "read" the front observer bisects the rear end graduation by moving the threads with the micrometer screw. They then announce the readings as before and the recorder jots them down, notifying the observers at the time if the screw revolutions differ from their previous values. This process eliminates the relative personal equation of the observers, and checks any blunders of whole revolutions in reading the microscopes, each of them being read four times, and the four readings being the same within a few microns. The probable error of a bisection is less than $\pm 1^{\mu}$.

While the bar is in position under the microscopes, the third observer measures the distance of the front end (and the rear end at starting) of the axis of the bar from the reference line, and adjusts the sector level bubble to center, taking care at the same time to keep away from the microscope posts when the bar is observed. The grade sector reading is then made and recorded, and the bar is rolled rapidly forward to a new position.

As soon as the rear end of the bar is brought safely to position under a microscope the one previously at the rear end is taken up and carried forward by the microscope porter who clamps and adjusts it on a new post. Likewise, as soon as a section of track is passed over it is carried forward to a new position.

The observers stand on platforms which rest at their ends on the ground at a distance of about one meter on either side of a microscope post.

At intervals of 20 to 40 minutes fresh ice is supplied, the trough being run to the rear of or ahead of the two microscopes which were last used. The trough is completely uncovered in this operation and the ice stirred up and supplemented by the amount requisite to replace the waste. This amount is usually 3 to 5 kilograms.

The speed of measurement has varied somewhat with circumstances. It has usually been about 100^m per hour. 750^m were measured in 7 hours on two different dates; and a kilometer would not be an excessive day's work.

The Efficiency of the Apparatus.

Plan of operations with apparatus in 1891.—The plan submitted by me to the Superintendent of the Survey for the use of this apparatus on the Holton Base, of the transcontinental triangulation in Indiana, contained the following recommendations which were approved and carried out during the summer of 1891: (*a*) To construct a 100-meter comparator near the Holton Base; to standardize this comparator by repeated measurements with the iced bar; and to use this comparator in turn to standardize and study the behavior of 100^m tapes or those of less length, or any other form of base apparatus. (*b*) To use the iced bar in addition to make several measures of a kilometer at least of the base line; so that the efficiency of the different forms of apparatus used in measuring the whole base could be tested on the actual ground over which they were applied.

The plan also contemplated making a determination of the length of the steel bar of the apparatus in terms of one of the International Prototype Meters. This was done, but owing to the small amount of time available, before going to the field it was impossible to reach anything better than a tentative value.

Before giving the results of the measures made with the iced bar it is proper to give a brief description of the long comparator and of the kilometer whereon the apparatus was used.

The 100-meter comparator.—The 100-meter comparator of the Holton Base was a line 100^m long fitted for measurement with the iced bar apparatus. Twenty-one beech wood microscope posts $1{\cdot}8^m$ long and $15^{cm}\times 15^{cm}$ in cross section were set firmly in the ground 5^m apart on a level plat near the north end of the Base. Alongside of the posts a stationary railway track was laid, the support posts of which were half way between the microscope posts. The ends of the line were marked by brass, spherical headed bolts cemented into the upper ends of stone posts, which latter were well set in beds of concrete. The comparator was covered by a shed 110^m long by 3^m wide. Its length extended nearly east and west. It was covered at the ends and on the south side as well as overhead, but the north side was left open in order to permit free access of daylight and air.

This comparator was built by Assistant A. T. Mosman after plans drawn up by Mr. Siebert. It answered its purpose very satisfactorily. An efficient auxiliary applied by Assistant Mosman was a sawdust covering to the ground along the comparator. This covering absorbed dust and moisture, and prevented the transmission of disturbances through the ground to the microscope posts. The stability of these posts may be inferred from the measures of the comparator interval given below.

The standard kilometer.—A nearly level portion one kilometer in length of the Holton Base was selected by Assistant Mosman for measurement with the iced bar apparatus. The base line, whose entire length is 5·5 kilometers, runs in a nearly north and south direction across the Crawfish Flats of southern Indiana. The portion selected for the iced bar measures passes for 600^{m} of its length through a dense forest growth, leaving about 200^{m} at either end in open fields. The whole kilometer is on low ground and the part within the forest is, in a wet season, subject to partial inundation. The soil along the kilometer is a stiff clay which is very firm when dry but which assumes a jelly-like mobility and elasticity when saturated with water.

The way through the forest was cleared and the end stones of the kilometer were set under the direction of Assistant Mosman during May and June, 1891. During the latter half of the following August and early September the microscope and track posts were set along the line. Owing to frequent and heavy rains this was a tedious operation. Many of the posts were set in the water which filled the post holes as fast as they were dug. It is impossible, therefore, to present any statistics as to the speed with which this work can be done under usually favorable circumstances. It may be remarked, however, that it is a work which requires but little skilled labor. In addition, it should be said that the microscope posts were set with considerable precision. Accurate spacing of the posts to 5^{m} apart was secured by means of 100^{m} and shorter steel tapes; while the posts were aligned by means of a theodolite. The probable error in position of a post face with respect to the kilometer line does not, I think, exceed $\pm 3^{mm}$; while the probable error of the reference line fixed on the posts as explained above does not exceed $\pm 1^{mm}$.

The bolts marking the termini of the kilometer were cemented in the end stones by Assistant Mosman early in August, after their proper relative positions had been determined by Assistant O. H. Tittmann with the Survey secondary apparatus.* Intermediate stones dividing the kilometer into four nearly equal sections were set on September 7, 1891. Each of them consisted of a half cubic meter of concrete set in the ground so that its upper surface was about even with the ground surface. On the top of each stone was cemented one of the Repsold cut-off plates which are provided with spherical headed bolts for use with the cut-off cylinder previously described.

When these intermediate stones were set, the ground along the line was so wet that it was a matter of difficulty to keep the water out of the excavations while the concrete was being rammed into place. These stones did not become dry and hard until deep trenches were dug about them on September 18, 1891. For this reason it is probable that these stones were much less stable during the first two measures of the kilometer (Sept. 10–15) than during the last two measures (Sept. 26–30).

Results of measures of 100^m comparator interval.—The earliest experience with the iced bar apparatus showed that the personal equation of the observers may cause appreciable constant error, and hence the method of interchange of the observers already explained was adopted. The first five measures of the comparator interval, however, were made without interchange of the observers. In place of such interchange direct observations for relative personal equation were made on the bar.

The greater part of the measures of the comparator interval were made by starting at the west end and moving the bar toward the east. Some of the later measures were made in the opposite direction and they disclose, apparently, a systematic error depending on the direction of measure. The same kind of systematic error is indicated also by the measures of the kilometer referred to below.

In order to explain the data of the comparator measures clearly and fully let

D = distance between spheres of comparator
= $100^m + 39{\cdot}5^{mm}$ approximately,
B_{17} = length of 5^m steel bar No. 17 in ice,
x = the relative personal equation of the two observers on the bar,
Q = quantity measured on cut-off scale,
v = the most probable correction to Q.†

Then, for one position of the observers a measure of the comparator interval gives an observation equation of the form

* This is an end measure apparatus consisting of two steel rods encased in wood, with mercurial thermometers to give their temperatures.

† The quantity Q involves the error of measuring the line as well as the errors in position of the first and last microscope with respect to the fiducial points.

$$D + 20x - 20B_{17} - Q = v,$$

while for the reverse position the equation is

$$D - 20x - 20B_{17} - Q = v.$$

The following table gives the data furnished by the first group of measures of the comparator. These data include the observations for personal equation, which give rise to observation equations involving the quantity x only. Although the latter equations should have somewhat greater weight than the others they are all treated as of equal weight in this purely preliminary statement.

Data for length of 100m comparator.

Date 1891.	Direction of Measure.	Observation Equations.
		mm mm
July 30	W to E	$D+20x-20B_{17}-39{\cdot}532=+0{\cdot}052$
30	W to E	$D+20x-20B_{17}-39{\cdot}574=+\ {\cdot}010$
30	W to E	$D+20x-20B_{17}-39{\cdot}660=-\ {\cdot}076$
30	....	$+20x\ \ldots\ -\ 0{\cdot}049=+\ {\cdot}013$
31	W to E	$D+20x-20B_{17}-39{\cdot}548=+\ {\cdot}036$
31	....	$+10x\ \ldots\ -\ 0{\cdot}040=-\ {\cdot}008$
Aug. 3	W to E	$D+20x-20B_{17}-39{\cdot}643=-\ {\cdot}059$
3	....	$+10x\ \ldots\ -\ 0{\cdot}061=-\ {\cdot}029$
4	W to E	$D+20x-20B_{17}-39{\cdot}624=-\ {\cdot}040$
4	W to E	$D-20x-20B_{17}-39{\cdot}476=-\ {\cdot}016$
4	....	$+20x\ \ldots\ -\ 0{\cdot}022=+\ {\cdot}040$
7	W to E	$D+20x-20B_{17}-39{\cdot}528=+\ {\cdot}056$
7	W to E	$D-20x-20B_{17}-39{\cdot}424=+\ {\cdot}036$

The resulting normal equations are:

$$\begin{aligned} 9D + 100x &= 9(20B_{17}+39^{mm}) + 5{\cdot}009^{mm}, \\ 100 + 4600 &= 100(20B_{17}+39^{mm}) + 66{\cdot}610\,; \end{aligned}$$

whence

$$\begin{aligned} D &= 20B_{17}+39{\cdot}522^{mm} \pm 0{\cdot}011^{mm}, \\ x &= +3^{\mu}{\cdot}1 \pm 0^{\mu}{\cdot}5. \end{aligned}$$

The probable error of an observed quantity of weight 1, or of a single measure of the comparator without interchange of observers to eliminate personal equation, is $\pm 30^{\mu}$.

Later in the season (Sept. 24 to Oct. 6) a second set of measures of the comparator interval was made. In each of these measures the observers interchanged positions, and on two dates the interval was measured in the direction east to west as well as in the direction west to east. The results of these measures are given in the following table which is arranged in the same form as the preceding table.

Data for length of 100^m comparator.

Date 1891.	Direction of measure.	Observation equations.
Sept. 24	W to E	$D+20x-20B_{17}-39^{mm}\cdot272=+0^{mm}\cdot134$
24	W to E	$D-20x-20B_{17}-39\cdot191=+\ \cdot149$
Oct. 2	W to E	$D+20x-20B_{17}-39\cdot394=+\ \cdot012$
2	W to E	$D-20x-20B_{17}-39\cdot320=+\ \cdot020$
2	E to W	$D+20x-20B_{17}-39\cdot432=-\ \cdot026$
2	E to W	$D-20x-20B_{17}-39\cdot369=-\ \cdot029$
6	W to E	$D+20x-20B_{17}-39\cdot422=-\ \cdot016$
6	W to E	$D-20x-20B_{17}-39\cdot370=-\ \cdot030$
6	E to W	$D+20x-20B_{17}-39\cdot512=-\ \cdot106$
6	E to W	$D-20x-20B_{17}-39\cdot449=-\ \cdot109$

The normal equations from this group are:

$$10D+\quad 0x = 10(20B_{17}+39^{mm})+3^{mm}\cdot731,$$
$$0\quad +4000 = \qquad\qquad +6\ 660;$$

whence

$$D = 20B_{17}+39^{mm}\ 373\pm0^{mm}\cdot019,$$
$$x = \quad 1^{\mu}\cdot7\pm1^{\mu}\cdot0.$$

The probable error of an observed quantity of weight 1, or the probable error of one measure without interchange of observers, is $\pm61^{\mu}$.

The values of D resulting as above from the two groups of determinations differ by $0\cdot149^{mm}$, a quantity which is about ten times the average of the probable errors of the separate values. It seems most probable, in view of our experience on the standard kilometer, that this difference is due to a movement of the end marking stones of the comparator. The residuals of the second group of measures indicate a progressive change of this sort.

The data of Oct. 2 and 6 indicate that measures made in the direction west to east give smaller lengths than measures in the opposite direction. Thus the lengths for Oct. 2 are:—

Direction	W to E	$20B_{17}+39^{mm}\cdot357$,
"	E to W	$20B_{17}+39\cdot400$,

Their difference is 43^{μ} and the corresponding difference for Oct. 7 is 84^{μ}. They show an average difference of 64^{μ}, which is equivalent to an error of $1^{\mu}\cdot6$ per bar length relatively to the mean of a forward and backward measure of a line.

Results of measures of standard kilometer.—In the measures of the kilometer the observers always interchanged positions in reading on the bar. The results may then be regarded as free from personal equation except so far as such equation may differ for the two ends of the bar. Each section of the kilometer was measured in both directions. The bar was kept in the same position, relatively to the Y-trough, it had when used on the comparator. A measure from west to east on the comparator corresponds then to a measure from north to south on the kilometer.

Two series of measures of the kilometer were made; one immediately after the intermediate section stones were set and while the ground was very unstable, and one after the ground was dry and very stable. In addition to errors arising from unstable ground the first measures were subject to some errors arising from delay and temporary stops, which were avoided in the second measures. Accordingly, the first series of measures must be regarded as much less precise than the second.

The following table gives the results of these two series of measures. The numerical quantities given are the excesses measured with the cut-off scale over a round number of bar lengths, the number of bar lengths being fifty for each section and two hundred for the whole kilometer. The first measures were made on Sept. 10 to 15 and the second on Sept. 26 to 30.

Results of measures of kilometer.

No. of meas-ures.	Direction of measure.	Section 0^m to 250^m.	Section 250^m to 500^m.	Section 500^m to 750^m.	Section 750^m to 1000^m.	$0^m - 1000^m$.
		1st series, Sept. 10 to 15, 1891.				
		mm	mm	mm	mm	mm
1	N to S	+6·35	−5·43	+19·08	−20·56	−0·56
2	S to N	+6·52	−4·55	+19·37	−19·28	+2·06
1−2		−0·17	−0·88	− 0·29	− 1·28	−2·62
		2d series, Sept. 26 to 30, 1891.				
3	N to S	+6·97	−5·50	+19·67	−21·28	−0·14
4	S to N	+7·07	−5·60	+19·76	−21·19	+0·04
3−4		−0·10	+0·10	− 0·09	− 0·09	−0·18

Without attempting to discuss these results here, it may be pointed out that they indicate systematic differences depending on the direction of measurement. These differences agree in sign with those developed in the forward and backward measures of the 100^m comparator as explained above. It may be remarked also that the largest differences in the first series are found in the measures of the second and fourth sections where the ground was least stable.

From the differences (1–2) and (3–4) of the table it appears that the probable error of one measure of a kilometer is $\pm 0{\cdot}76^{mm}$ from the first series and $\pm 0{\cdot}09^{mm}$ from the second series. The second of these probable errors, it may be remarked, agrees well with the corresponding value deduced from the measures on the 100^{m} comparator, to wit, using the average value for the probable error of one measure of the comparator interval:

$$\pm \tfrac{1}{2}\frac{(30^{\mu}+61^{\mu})}{\sqrt{2}}\sqrt{10} = \pm\, 0{\cdot}10^{mm}$$

It will be of interest in this connection to compare these probable errors of measurement with those obtained with other micrometer microscope apparatus. The apparatus most nearly comparable in this respect with the iced bar apparatus are the Repsold bimetallic (zinc and steel) of the U. S. Lake Survey and the Brunner bimetallic (copper and platinum) used recently by French geodesists. The best work with the Repsold apparatus, on the Olney base 1879, shows a probable error of $\pm 0{\cdot}40^{mm}$ for one measure of a kilometer.* The work of the French on the Paris and Perpignan bases, 1890, 1891, shows $\pm 0{\cdot}67^{mm}$ for one measure of a kilometer.† These larger values are due probably to imperfect temperature indications of the bimetallic apparatus rather than to any material differences of manipulation.

An idea of the stability of the end and section marks of the kilometer may be gained from the above table by computing the differences between the mean lengths of the several sections resulting from the first pair and second pair of measures. Thus we have:

Mean of first pair *minus* mean of second pair of measures.

Section	0^{m}	to	250^{m}	$-0{\cdot}58^{mm}$
"	250	"	500	$+0{\cdot}56$
"	500	"	750	$-0{\cdot}50$
"	750	"	1000	$+1{\cdot}32$

These figures indicate considerable movements of the marking stones; and it seems not improbable that movements of such magnitude did actually occur, since the stones rose to the surface of the ground and the moisture in the ground varied from the extreme of saturation to the extreme of dryness during the interval which elapsed between the two sets of measures.

* Professional Papers Corps Engineers U. S. A., No. 24, p. 303.

† Comptes-Rendus des séances de L'Association Géodésique International du 8 au 17 Octobre, 1891, p. 182.

Length of measuring bar.—The most important and difficult operation attending the use of the iced bar apparatus is that of deriving the bar's length in terms of one of the International Prototype Meters. As already stated, a preliminary determination of this length was made in July, 1891, just previous to shipping the apparatus to Holton Base, Indiana. The method adopted in this and in most subsequent determinations may be briefly described as follows: Six micrometer microscopes were mounted in a straight line at intervals of one meter. The 5^m distance between the two extreme microscopes was measured with Prototype Meter, No. 21, and this distance transferred from the extreme microscopes to the 5^m steel bar No. 17. All comparisons of this kind were made with both the meter and 5^m bars in melting ice. The 5^m bar was mounted in its Y-trough and the meter bar in a wooden box. 40 to 45 kilograms of ice were used for the 5^m bar and 10 to 12 kilograms for the meter.

Assuming constancy of temperature of the bars under comparison the precision of the method just outlined evidently depends on the stability of the microscopes used. The first series of comparisons of July, 1891, and several series of February to May, 1892, were made on the office comparator. This comparator was not designed to meet the special requirements of the case and it did not meet them satisfactorily. The microscopes were too unstable. Their instability was due primarily to the manner of mounting them. They were each suspended by a cast iron bracket from a wrought iron I-beam about 6^m long which is supported at its ends on brick piers. The focal planes of the microscopes fell about $0{\cdot}5^m$ below the beam. The beam was wrapped with cotton batting and covered with a wooden casing to prevent rapid temperature changes. Resting as the beam does by friction on the piers, it is in general in a state of longitudinal stress, which is frequently relieved by vibrations communicated to the piers by vehicles passing in the adjacent street. Changes in this stress produce changes in the curvature of the beam and entail exaggerated motions in the microscopes. The temperature of the comparing room (which is underground) changes very slowly from day to day when not occupied long by observers or when lighted for short periods. The comparisons in question, however, required occupying the room for some hours per day, and the heat from the observers and the electric lights caused notable changes in the temperature of the beam as well as of the air in the room. These changes no doubt caused more or less irregular displacements of the microscopes.

The observations were so arranged as to eliminate the effects of uniform motions of the microscopes. Thus in the first series of determinations the following program was adhered to:

1. Measure of distance between end microscopes with meter,
2. " " " " " with 5^m bar.,
3. " " " " " " meter;

the second measure with the meter being made in a direction opposite to that of the first. In later measures on this comparator the above program was supplemented by an initial and final measure with the 5^m bar.

The observations were made by two persons observing simultaneously at the respective ends of either bar. The observers also exchanged positions in all cases to eliminate personal equation.

The determinations made in the manner just described showed large ranges, amounting to 1/200000th part at the maximum; and with the hope of overcoming the effects of the irregular motions of the microscopes a different method was tried with the same comparator. The essential feature of this method consists in the use of an intermediary 5^m steel bar subdivided into meter spaces. This bar, known as No. 18, is similar in form to No. 17, except that it is cut down to the mid depth at four intermediate places as well as at its ends. The lines subdividing the bar into meter spaces are ruled on platinum iridium plugs. This bar was mounted in the steel Y-trough where its flexure could be controlled in the same way as that of bar No. 17. The sub-spaces of No. 18 when packed in melting ice were determined by direct comparisons with Prototype Meter No. 21. Three series of six measures each of the sub-spaces, and hence of the whole length of the bar, were made. In the intervals between the first and second and the second and third series, bars 17 and 18 were compared. For this purpose No. 17 was mounted in an auxiliary wooden trough similar to the Y-trough. Notwithstanding the diminished time-interval during which dependence on the stability of the microscopes was required by this process, the results attained were still too erratic to give confidence. The range in values for the metric sub-spaces of No. 18 rose to 1/200000th part and in the values for the whole length, or sum of these spaces, to 1/300000th part; and these ranges appeared to be directly referable to the large progressive, and frequently large sudden, movements of the microscopes.

Such being the unsatisfactory quality of the results attained on the office comparator, it was determined in June, 1892, to build a new comparator designed more especially to meet the requirements of the iced bar apparatus. This comparator was constructed in a lot adjacent to the Survey Office. Briefly described it consists of six brick piers resting at intervals of a meter on a foundation of six cubic meters of well rammed concrete. The foundation and the piers are set in Portland cement and weigh about twelve tons. The foundation, which rises to near the ground surface, is covered to the depth of a decimeter with sawdust. Each pier carries a micrometer microscope. The latter and their connections with the piers are wrapped heavily with cotton batting. The meter and 5^m bars were moved under the microscopes on portable tracks supported on posts isolated from the piers. The track for the 5^m bar was that used in the field, and the mode of handling this bar was precisely the same in all respects as that followed in measuring a line.

The microscopes as mounted on this comparator proved to be very stable notwithstanding the fact that they were subject to the daily range in air temperature except so far as they were protected by cotton batting wrappings. The effect of the observers' presence near the comparator was undoubtedly less than in the office comparing room, and the use of artificial light was avoided entirely.

The program followed in the use of the new comparator comprised three measures of the distance between the end microscopes with the meter and two measures with the 5^m bar in each set of observations. The measures with the meter were made in opposite directions alternately along the comparator. Twenty sets of observations were made with the 5^m bar in each of its two orientations relatively to the Y-trough and microscopes. The consistency of the results attained leaves little to be desired. The probable error of a single determination of the length of the 5^m bar in terms of the Prototype is $\pm 1^{\mu}\cdot 8$ from one group of measures and $\pm 1^{\mu}\cdot 7$ from the other. The range in the one case is $12^{\mu}\cdot 3$ and in the other $8^{\mu}\cdot 5$.

The following table gives an abstract of the results obtained in the manner described above for the length of the 5^m bar. The values given are subject to corrections for flexure of the 5^m bar, which, however, cannot exceed a few tenths of a micron. The probable errors are those which come from the discrepancies between the individual and mean results of a group of determinations. Each result is derived from an equal number of measures with the meter and 5^m bars in their two different orientations. The first, second, and third results were obtained from the Office comparator, the third one depending on the method wherein the auxiliary bar, No. 18, was used. The fourth result was obtained from the new comparator.

Summary of results for length of B_{17}.

Date.	No. of Measures.	Mean length of bar.
July, 1891	10	$5^{m}-11^{\mu}\cdot0\pm1^{\mu}\cdot4$
Feb. and March, 1892	20	$5\ -15\ \cdot2\pm0\ \cdot6$
April and May, 1892	18	$5\ -11\ \cdot7\pm1\ \cdot6$
July and August, 1892	20	$5\ -16\ \cdot8\pm0\ \cdot3$

Without desiring to discuss these data here, it may be said that so far as is known at present the probable errors are fairly trustworthy indices of the precision of the several results. The range among them is but little in excess of the millionth part of the bar's length, and is no greater than the probable errors would lead one to expect.

It may be stated also that the external air temperature varied for the different groups of comparison from 5° to 40° C. The average air temperatures for the two most important groups, namely, those of February and March, 1892, and July and August, 1892, were about 5° C. and 35° C. respectively. Hence it does not appear that the bars in ice were affected appreciably by the external air temperature. Finally, it should be said that a systematic difference in the length of the bar, according as the one or the other of its ends is to the right in the Y-trough, is indicated by each group of comparisons. This difference appears to be an inequality of relative personal equation of the observers at the two ends of the bar, and may be due to the considerable inequality in widths of the terminal graduation lines.

Concluding remarks.—The question may be asked, does the bar take the temperature of melting ice when fully packed in it? I am unable to give a decisive answer to this question at present, but there appears to be no reason to suppose that it takes a materially different temperature. Repeated observations on mercurial thermometers placed in the ice alongside the bar show that they read zero within the unavoidable errors of a few hundredths of a degree. That the bar assumes a fixed length within very narrow limits is, it would seem, demonstrated by the small range among the measures of the 100^{m} comparator and the kilometer sections referred to above, and especially by the recent work on the new comparator. This latter work appears to justify the conclusion that the mean of four determinations of the bar's length in terms of the Prototype meter, made in the manner described above, cannot have a greater probable error than one micron. It is evident therefore, in view of the unavoidable errors of observation in such work, and in view of the fact that the bar's expansion is about 55^{μ} per degree Centigrade, that there is a very small margin for change in the bar's length.

The time required for the bar to acquire a sensibly stable length is less than ten minutes. The rate of temperature-change is so great in the early stages of freezing that ninety per cent or more of the contraction of the bar occurs within a minute after it is well surrounded by ice. The corresponding time required by the Prototype meter to reach a stable length appears to be less than five minutes, which is less than the time essential to properly pack it in ice.

A query may also arise as to whether the bar, resting as it does with considerable friction on the vertical adjusting screws of the Y-trough, may not change length by reason of longitudinal stress communicated by the trough. In answer to this query it may be said that the experiment of putting the trough alternately under tension and compression in quick succession has been tried on several occasions without disclosing any effect on the bar.

With regard to the precision attainable in the measurement of a line with this apparatus, it would appear that the error of operation may be rendered insignificant in the mean of a few measures of a line, so that the probable error of the mean length may be diminished to that of the bar when expressed as a fraction of its length. It appears practicable to determine that length with a probable error not exceeding 1/5000000th part, and this, therefore, would appear to be an attainable precision in the measurement of a line with the apparatus.

Although the use of this or similar apparatus is not to be recommended for primary bases in general, since it gives a needless precision, it will compare favorably I think on the score of economy with any of the earlier forms of apparatus which have given a precision approaching the millionth part of a measured line. The proper function, however, of the iced bar apparatus appears to be that of an intermediary between the lighter and cheaper forms of base apparatus and the standard meter. By means of this bar it appears practicable to standardize long steel tapes so accurately that they will give all needful precision for bases in general at much less cost than other forms of apparatus.*

Office Coast and Geodetic Survey, Sept. 15, 1892.

* Experience of the author in the use of 100^{m} steel tapes with mercurial thermometers to give temperature, on the Holton Base, 1891, shows that the length of such a tape can be determined with a probable error not exceeding 1/2000000th part, and that the probable error due to errors of operation and temperature in the mean of n measures of a line k kilometers long need not exceed

$$\pm 2^{mm}\sqrt{\frac{k}{n}}.$$