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Art. XXXIV.—*On a Color System;* by O. N. Rood, Professor of Physics in Columbia College.

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It is possible to combine by rotation a circular disc of cardboard painted with vermilion, with a similar disc covered with a bluish-green pigment exactly complementary to the red, and to ascertain whether by mixture a gray is produced when the two colored areas are equal. If such should be the case, in a certain sense we could say that these two colors were equal, in spite of differences of luminosity or amounts of white light mingled with them. If it were found that to make the mixture neutral, more or less red were required, we could assign to the vermilion disc a coefficient of 1, and to the blue-green disc its proper coefficient whatever that might be. Supposing for the sake of simplicity that the coefficient of the blue-green disc was also equal to unity, we then could mark the positions of the two colors at the extremities of a straight line and at its middle point locate white (gray), and we would then have a line containing, so to speak, all possible mixtures of these two colors, and by simple processes could indicate on it the positions, and consequently coefficients, of any red or blue-green surfaces having hues of the same character; for example, a red surface reflecting half the amount of red light of our standard disc, would be located half way between the positions of the standard red and of white, while another red disc might even find its position on the same line outside of our standard. In all these cases the vermilion disc would be the material standard to which a great number of reds and blue-greens, more or less pale or intense, could be quantitatively referred. But it might be possible to go farther, and to be able to say that our standard vermilion disc reflected the same amount of red light that is furnished by white card-board under the same illumination, or that at all events it reflected a known fractional part of it. All the colors just spoken of would then be virtually referred to white card-board, or to some other white reflecting surface as might be determined on, and the standard red disc could at will be reproduced at any time or in any part of the world. To the reproduction of standard red discs I have not devoted much time, as before they can be made valuable a much more difficult problem must be solved.

For if we undertake to go farther than the proceeding just indicated, and build up a whole system of colors we are immediately confronted by the fact that we are quite unable to leave our line; or for example, to express the amount of orange colored light or of green light reflected by other discs in the same terms, or to bring them all into one system. As Helmholtz has remarked,* masses of colored light which when mixed with their complements furnish in pairs the same white, may be said to belong to the same system and may be located at equal distances from the same center. A set of ideal discs which when combined pair-wise all furnished the same white, would in this system find their positions on the circumference of a circle with white at the center. But it is not practicable to experiment directly in this way, as it is nearly impossible to measure directly the amounts of white light mingled with the colored in the case of painted surfaces.

I have employed a different proceeding and built up with it a reproducible system, which, as far as I can ascertain, is the one above indicated. Let us suppose that the positions of our standard red and its complement have been laid down at the extremities of a line with white at its center: we then combine with the red disc a green which is a little too blue to be strictly complementary, and obtain the best neutralization, the gray furnished having the least perceptible color, and we will suppose that this takes place when the areas of the two colored surfaces on the composite disc are equal; we shall then know that the new color also belongs on the circle not far from the complement of vermilion. One step has thus been taken, we have been able to leave our first line of mixtures. We can then with ease and certainty locate the complement of this new hue, which will be a little more orange than the vermilion; or better still, we may combine the new color with one that is

* Handbuch der Physiologischen Optik, p. 287.

still more refrangible than its true complement, and locate a disc which is still more orange, advancing thus more rapidly in the work. In the same way this orange-red disc will enable us to add a still bluer green, until in this way all the colors of the chromatic circle have been provided for and all our colored discs have *received coefficents*, for of course it will rarely happen that the actual positions of the colored discs fall on the circle, mostly they will be well inside of it. The first part of the work then consists in assigning coefficients to a series of painted discs which are arranged so as to have only small progressive differences of color.

Mode of experimenting.—To compare together colors which are not strictly complementary a number of precautions must be taken. In the first place, the pairs of discs employed should always have as nearly as practicable about the same coefficients; consequently the selection of the colored papers offers very considerable difficulty, and in the majority of cases it has been necessary for me to prepare the paper specially for each case as it arose. Of course it is desirable to use papers with as high coefficients as possible, but cases occur where one is compelled to use lower coefficients (blues and cyan blues) and in these it is always necessary, so to speak, to take a fresh start, and to descend to the lower coefficient by a line passing through the center of the circle and not outside of it, that is by reaching the other side of the circle by the aid of a *complementary* disc with a lower coefficient.

It is absolutely necessary that the eyes should be thoroughly rested after each individual observation, and they should be protected from side lights by a hood. Only the central portion of the retina should be used; that is, the colors should be observed through a small aperture in blackened card-board held at arms length. It is an advantage and a saving of time, to protect the eyes with quite dark neutral spectacles while engaged in the manipulations not involving observation of the colors.

As a source of illumination, sunlight falling on a stretched sheet of bleached cotton cloth was employed, and only the hours near noon were utilized. The frame with the white cloth was outside of the window, inclined at a suitable angle, and consequently was behind the observer as well as placed symmetrically to the colored discs. Extraneous light from buildings and blue sky was screened off and the room suitably darkened. If light from a uniformly overcast sky is employed a different set of coefficients will be obtained, and there is reason to believe that they vary somewhat with the nature of the sky on such days.

In some observations it was found advisable, for the sake of using high coefficients, to employ paper which was slightly fugitive, but by properly arranging the process the evil effects of this were obviated.

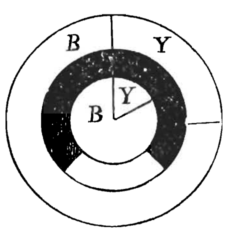
The coefficients of two strictly complementary discs are as their distances from the center or from white, and this distance is of course measured along the two radii that make up the diameter, but in the method used by me for colors not strictly complementary, the cosine of a small angle is necessarily employed and treated as if it were a radius. The angles used however were quite small, from 1°·5 to 3°, and consequently the error in any individual case was insignificant, but it is cumulative and finally becomes comparable with those of observation. After the uncorrected results have been plotted in a Newton's diagram, the angular distances of the colors will be known, very nearly, and the small errors from this geometrical source can be corrected and a fresh diagram constructed.

As above stated, it was not found advisable to employ angles larger than about 3°, since with larger angles the results were more uncertain, while with the angle above indicated the manipulation was hardly more difficult than in the case of truly complementary discs.

Mode of ascertaining the most neutral mixture.

Let us suppose that we wish to obtain the most neutral mixture that can be furnished by yellow and blue discs that are not complementary; the best method of procedure according to my experience is the following: a pair of large blue and yellow discs are combined as shown in figure 1, and on the same axis comes a black and white disc destined to furnish the comparison grays, and again on this, smaller yellow and blue discs cut from the same papers. Of course these three compound discs can be varied independently of each other. We start now with an amount of yellow on the larger disc which is far too small for approximate neutralization, and on the smaller disc with still much less yellow, and call the difference between the sectors the interval. This interval in any given case is always to be kept roughly constant, and the more nearly complementary the discs are, the smaller it may be made. There is however no gain whatever in trying unduly to diminish it, and such attempts tend only to defeat their object, it being far better and safer to have the interval too large than too small. The arrangement in figure 1 indicates

1.



about the first position for discs that have nearly the same coefficient, but are quite removed from being complementary. As soon as the compound disc is set in rotation it will be noticed that the mixture-tint furnished by the larger disc is the least colored of the two, and the amount of yellow in it is to be increased, the smaller disc following with the same interval, and this proceeding is to be pushed forward till the observer is unable to decide whether the larger or the smaller disc is the paler, the least colored. Both will differ sensibly from the pure gray mixture, and the two pale mixtures will differ from each other in hue, still after some practice the observer will learn to decide with promptitude that they differ about equally from gray. This is a much more certain and easy process when in general only a little color is present, than when the hues presented are at all decided, and this is the reason that in building up the system the advances were made by small steps. The first half of an observation being now made and the mean noted, the yellow element is made from the start to preponderate considerably, the interval is preserved, and with a reversed approach the best neutrality obtained. The two readings constitute one complete observation, and if the maximum reliability is demanded, not more than five complete observations in any one case should be made on the same day, it being more profitable to devote the remaining time to other determinations.

As an example of the results to be attained I give three sets of determinations for five discs with rather low coefficients, each number being the mean of only three complete observations made on the same day:

CB9*a*	·585	·575	·585
Y6⅓	·506	·480	·486
CB8	·466	·442	·452
Y6½	·432	·421	·428
B28	·394	·384	·398

When it is remembered that in photometric observations involving light of the same color the error in any one observation may easily be two per cent and more it will be seen that these results are satisfactory so far as uniformity goes.

By the methods above indicated I obtained the coefficients of a series of slightly differing discs ranging in color from orange-yellow through red into purple, and on the other hand from a somewhat yellowish green through cyan-blue to blue. Then, instead of extending the process all around the circle, a Newton's diagram was constructed using the standard vermilion disc, the extreme green attained and the blue. The angular positions of these colors were not arbitrarily assumed, but determined from the mixture equation and the ascertained coefficients. With the aid of this triangle and color equations experimentally obtained, I plotted in the usual way the positions of five of the discs used in the building up process, and calculated their coefficients with the following result:

	Direct coef.	Calculated.
G1	1·119	1·109
O1	1·221	1·231
P12	1·229	1·257
G18	·867	·892
P1	·571	·566
GY5	·806	·795

GY5, a greenish yellow, was quite beyond the limits of the triangle.

This agreement shows that the zig-zag process employed by me is homogeneous with that ordinarily used in Newton's diagram, and from this it follows that it is not necessary to determine by my method coefficients for a greater number of discs than will furnish a properly shaped triangle, and that the remainder of the work can be done by the older method. It may here be remarked that in obtaining the fundamental equation for the construction of the diagram and those for locating the colors, it is necessary to exercise far more suspicious care than is the case in the direct determinations of the coefficients. Although a pure gray is always attainable, theoretically, yet it seldom is obtained quite to the satisfaction of the observer; the best way is to employ on the same axis duplicate sets of discs separated by the comparison gray, to vary the manipulation and multiply the observations. The work having progressed thus far it becomes possible to ascertain the coefficient of any colored surface and to assign it a position in the diagram.

It may be remarked that the angular positions assigned to colors by the method just described agree in their main features with those given by me in a diagram on page 250 of "Modern Chromatics," which is founded on contrast observations, the main differences being in the angular positions assigned in the diagram to violet and blue-violet; these would have to be moved about 15° to the right, i. e., nearer to the other blues to make them coincide with positions furnished by this system.

I give here some of the coefficients obtained: the vermilion with a coefficient of 1 had only a moderate brilliancy, being made of English vermilion applied to card-board as a paste: since then I have found in the shops a red, supposed to be vermilion, with a coefficient as high as 1·174.

Chrome yellow (paste)	·846
" "	·933
"Pale" " "	·991
Emerald green "	1·119
Prussian blue (wash)	·354
" " "	·392
Cobalt blue (paste)	·562
Artificial Ult. blue (paste)	·506
Crimson lake (wash)	1·23

The system which is here proposed can be built up by experimenters and ought to be practically identical in all cases if made by normal eyes, provided the same standard vermilion is used, or the coefficient of it is known, or in general if the starting point is made from any disc of bright color the coefficient of which is known. As before stated it may be possible hereafter to refer the coefficient of the vermilion disc to white card-board, which indeed has been done in the case of my own disc with some approximation to correctness. The system may then be considered a fairly reproducible one, but the question still remains as to its nature. The thought that in it we have one where complementary colors having a determined coefficient of unity furnish the same gray, naturally suggests itself and is rendered probable by the following considerations. The standard red and its true complement when mixed furnish a certain gray, and it is evident that other pairs situated at small angular distances from them, such as 10° on either side, will sensibly do the same. This furnishes two sectors of 20° on either side of white; but the coefficients of the colors situated within them are reproducible with the aid of the triangle, which would hardly be the case unless the colors situated at its angles obeyed the same law, viz: furnished the same white when mixed with their true complements; this again applies to the reversed triangle and the two provide for about two opposite quadrants of the circle, and what holds good of these ought, it would appear, to apply to the remaining quadrants.

The question whether in this system, all the colors located on the circumference furnish by mixture the same white when treated in pairs, is quite difficult to answer by actual experiment, as in the case of most of the discs it is very hard to ascertain the relative amounts of colored and white light reflected, owing to the well known fact that in most cases portions of the colored light mix, and produce what amounts to an indeterminate amount of white light. Some attempts, with the aid of the spectroscope, of colored glasses and of gray discs were made to measure the actual amount of standard white (gray) furnished by the mixture of the *pure* fundamental red and its pure complement; a number was obtained expressed in terms of the luminosity of white card-board, and the same operation was performed for complementary yellow-green and purple as well as for yellow and blue discs. Without laying much stress on the results obtained, it can at least be said that they did not render improbable the idea that these pairs all furnished equal grays when united. To settle this problem experimentally would of course require a considerable amount of elaborate work, if indeed it is at present capable of such solution.

I shall be glad to furnish without cost samples of colored paper with coefficients to those who are particularly interested in these matters.