ART. XVII.—*Calibration of an Electrometer;* by D. W. SHEA.

THE mathematical theory * of the quadrant electrometer leads to the general formula:

$$\Theta = \alpha\ (A-B)\ (C-\tfrac{1}{2}\ (A+B),$$

in which θ represents the moment of the couple which turns the needle, A, B, C, the potentials of the two pairs of quadrants and of the needle respectively, and α is a constant which defines the sensibility of the instrument. The deflection of the needle is proportional to the moment θ. Hence, if α is a constant † as the theory supposes, the deflection of the needle should be proportional to the product

$$(A-B)\ (C-\tfrac{1}{2}\ (A+B),$$

and the curve of calibration obtained in any given method of setting up the electrometer should have a perfectly definite and constant form. But in the various forms of the quadrant electrometer, and in the different methods of setting up the same instrument, the curves of calibration obtained correspond in a very irregular manner with the curves given by theory.‡

Some observations with an electrometer of the Mascart form, which show variations apparently due to change in the sensibility with variation in the temperature,§ are given in the following pages. It is possible that they may be of interest to those who use this form of electrometer.

The results here given were obtained with the electrometer set up in the following manner:

The needle was suspended by a bifilar suspension consisting of a single fiber of cocoon silk, the ends of which were fastened to the drum and the loop to the hook on the upper end of the rod carrying the needle. The two parts of the fiber were separated as widely as the construction of the instrument admits of. The needle was charged by a water-battery. The positive pole of the battery was attached to the electrode connecting with the inner coating of the Leyden jar, and the negative pole and the electrometer case were to ground. The water-battery was made up of four hundred cells of zinc-copper elements, arranged in boxes of eighty cells each. The difference of poten-

* Maxwell, Electricity and Magnetism, vol. i. p. 311.
† Hopkinson, Phil. Mag., V, xix, p. 297.
‡ Mouton, Journal de Physique, vi, p. 13.
Benoit, Journal de Physique, vi, p. 118.
Boltzmann, Pogg. Ann., bd. cli. p. 487.
Hallwachs, Wiedemann's Ann. der Phys. und Chem., xxix, p. 35.
§ Hallwachs, ibid., p. 41.

tial between the positive pole of any one of these boxes of cells and the ground, with the negative pole to ground, was about forty volts.

In making the calibration, a battery of small gravity cells was used, with circuit closed through an external resistance of ten thousand ohms. One and the same end of the set of resistance coils was to ground in all observations. Points on the box of coils such that one, two, three, etc., thousand ohms were included between them and the point to ground were successively connected to one pair of quadrants, while the other pair of quadrants was to ground, and the quadrant connections were alternated in order to get readings on the right and left of the zero, which was placed at the middle point of the scale. The needle was charged by one, two, three, etc., boxes of cells successively, and the number of cells in the gravity battery was decreased, as the charge of the needle was increased, so as to keep the difference of potential between the ends of the set of resistance coils such that the spot of light always remained on the scale.

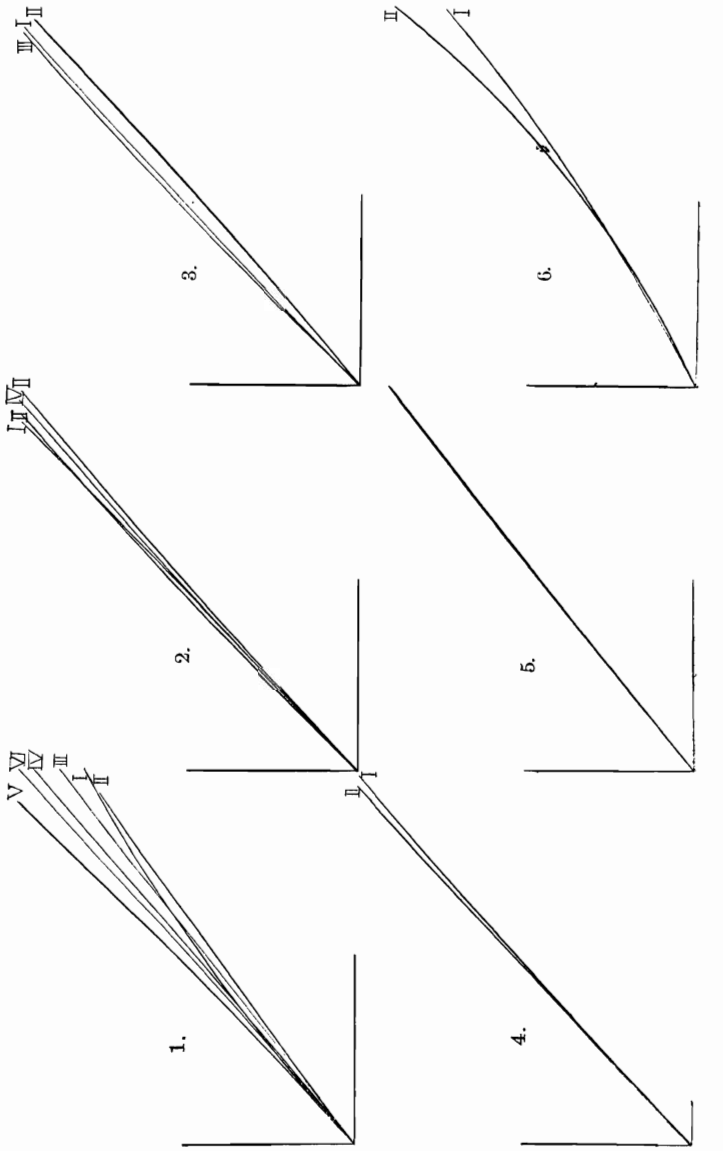
It was found that the form of the curve for a given charge of the needle did not long remain constant, and that even the direction of curvature changed. The change in the form of the curve was greatest when the charge of the needle was smallest, and it was not until the whole water-battery of four hundred cells was employed that a curve was obtained which changed so little as to admit of accurate work. The change in the curves was most rapid when the temperature of the room was changing rapidly between certain limits. Beyond these limits there was little change in the curves. But for any given temperature between the limits the form of the curve was not constant, even though the temperature of the room had been so constant for several hours that all parts of the electrometer could reasonably be supposed to have the temperature of the room.

The changes in the form of the curves for various charges of the needle were followed through the range of temperature attainable, at the time, in the room where the electrometer was set up. The following tables will serve to show these changes.

In all the observations the scale was at a distance of 126^{cm} from the mirror.

The curves shown in the plate were plotted by taking the potentials of the quadrants as abscissas and the deflections of the needle in degrees as ordinates. Fig. 1 shows curves when charge of needle was forty volts; fig. 2, when charge was eighty volts; fig. 3, when charge was one hundred and twenty

volts; fig. 4, when charge was one hundred and sixty volts; fig. 5, when charge was two hundred volts; fig. 6, I, when



charge was twenty volts, and II, when charge was ten volts. The Roman numerals in the figures refer to the tables.

Charge of Needle, 40 *volts.*

I.

TEMPERATURE, 6° C.

Resistance between point to ground and point to pair quadrants in ohms.	Charge of quadrants in volts.	Scale readings in cm. Right.	Scale readings in cm. Left.	Mean of two scale readings.	Deflection of needle in degrees.
1000·	2·5	2·13	2·17	2·15	0° 29′·5
2000·	5·0	4·30	4·20	4·25	0 58
3000·	7·5	6·28	6·20	6·24	1 25
5000·	12·5	10·07	9·85	9·96	2 15·5
7000·	17·5	13·46	13·34	13·40	3 2
8000·	20·0	15·05	15·00	15·02	3 24
9000·	22·5	16·49	16·41	16·45	3 43
10000·	25·0	17·92	17·94	17·93	4 2·5

II.

TEMPERATURE, 8° C.

1000·	2·5	1·93	1·92	1·925	0 26
2000·	5·0	3·81	3·79	3·80	0 52
3000·	7·5	5·64	5·61	5·625	1 17
5000·	12·5	9·17	9·12	9·145	2 4
7000·	17·5	12·85	12·84	12·845	2 54
8000·	20·0	14·60	14·68	14·64	3 19
9000·	22·5	16·29	16·23	16·26	3 40·5
10000·	25·0	17·79	17·78	17·785	4 1

III.

TEMPERATURE, 11° C.

1000·	2·5	2·03	2·04	2·035	0° 28′
2000·	5·0	4·03	4·03	4·03	0 55
3000·	7·5	5·94	6·02	5·98	1 22·5
5000·	12·5	9·95	9·99	9·97	2 16
7000·	17·5	13·70	13·76	13·73	3 6·5
8000·	20·0	15·76	15·84	15·80	3 34·5
9000·	22·5	17·89	17·91	17·90	4 2
10000·	25·0	19·81	19·87	19·84	4 28

IV.

TEMPERATURE, 12° C.

1000·	2·5	1·91	1·89	1·90	0 26
2000·	5·0	3·85	3·87	3·86	0 52·5
3000·	7·5	5·90	5·95	5·92	1 20·5
5000·	12·5	10·08	10·12	10·10	2 17·5
7000·	17·5	14·30	14·33	14·315	3 14·5
8000·	20·0	16·50	16·54	16·52	3 44
9000·	22·5	18·71	18·80	18·75	4 13·5
10000·	25·0	20·79	20·82	20·805	4 47

V.

TEMPERATURE, 14° C.

Resistance between point on coils to the ground and point to quadrants in ohms.	Charge of the quadrants in volts.	Scale readings in cm. Right.	Scale readings in cm. Left.	Mean of the two scale readings.	Deflection of needle in degrees.
1000·	2·5	2·12	2·14	2·13	0° 29′
2000·	5·0	4·29	4·29	4·29	0 58·5
3000·	7·5	6·58	6·58	6·58	1 29·5
5000·	12·5	11·30	11·32	11·31	2 34
7000·	17·5	16·41	16·44	16·425	3 43
8000·	20·0	19·02	19·06	19·04	4 18
9000·	22·5	21·86	21·97	21·915	4 56

VI.

TEMPERATURE, 16° C.

1000·	2·5	1·93	1·89	1·91	0 26
2000·	5·0	3·90	3·94	3·92	0 53
3000·	7·5	6·00	6·03	6·015	1 22
5000·	12·5	10·37	10·37	10·37	2 20·5
7000·	17·5	15·15	15·18	15·17	3 26
8000.	20·0	17·64	17·71	17·68	3 59·5
9000·	22·5	19·85	19·88	19·865	4 28·5
10000·	25·0	22·15	22·24	22·20	5 0·0

Charge of Needle, 80 *volts.*

I.

TEMPERATURE, 6°·5 C.

1000·	1·3	2·35	2·35	2·35	0° 32′
2000·	2·6	4·72	4·69	4·705	1 4
3000·	3·9	7·15	7·13	7·14	1 37
5000·	6·5	12·01	12·04	12·025	2 43·5
7000·	9·1	16·94	16·95	16·945	3 49·5
8000	10·4	19·32	19·30	19·31	4 21·5
9000·	11·7	21·72	21·72	21·72	4 53·5

II.

TEMPERATURE, 9° C.

1000·	1·3	2·26	2·26	2·26	0 31
2000·	2·6	4·57	4·57	4·57	1 2
3000·	3·9	6·83	4·75	6·79	1 32
5000·	6·5	11·34	11·34	11·34	2 34
7000·	9·1	16·13	16·14	16·135	3 38·5
8000·	10·4	18·40	18·35	18·375	4 8·5
9000·	11·7	20·59	20·53	20·56	4 37

III.

TEMPERATURE, 12° C.

1000·	1·3	2·47	2·46	2·465	0° 33′ 5
2000·	2·6	4·90	4·85	4·875	1 6·5
3000·	3·9	7·31	7·26	7·285	1 39·5
5000·	6·5	12·12	11·95	12·035	2 44
7000·	9·1	16·74	16·73	16·735	3 46·5
8000·	10·4	19·08	19·07	19·075	4 18
9000·	11·7	21·43	21·36	21·395	4 49

IV.

TEMPERATURE, 17° C.

Resistance between point on coils to ground and point to quadrants in ohms.	Charge of the quadrants in volts.	Scale readings in cm. Right.	Scale readings in cm. Left.	Mean of the two scale readings.	Deflection of the needle in degrees.
1000·	1·3	2·60	2·59	2·595	0 35·5
2000·	2·6	5·05	4·97	5·01	1 8
3000·	3·9	7·60	7·51	7·555	1 43
5000·	6·5	12·55	12.41	12·48	2 47
7000.	9·1	17·35	17·23	17·29	3 54·5
8000·	10·4	19·50	19·57	19·535	4 24·5
9000·	11·7	21·85	21·78	21·815	4 54·5

Charge of Needle, 120 *volts.*

I.

TEMPERATURE 8° C.

1000·	0·7	2·29	2·30	2·295	0°31′
2000·	1·4	4·62	4·61	4·615	1 3
3000·	2·1	7·04	6·99	7·015	1 35·5
5000·	3·5	11·77	11·82	11·795	2 40·5
7000·	4·9	16·47	16·41	16·44	3 43
8000·	5·6	18·84	18·79	18·815	4 14·5
9000·	6·3	21·22	21·20	21·21	4 46·5

II.

TEMPERATURE 12° C.

1000·	0·7	2·23	2·24	2·235	0°30′·5
2000·	1·4	4·37	4·32	4·345	0 59
3000·	2·1	6·56	6·49	6·525	1 27·5
5000·	3·5	10·99	11·01	11·00	2 27
7000·	4·9	15·52	15·56	15·54	3 31
8000·	5·6	17·99	18·00	17·995	4 3·5
9000·	6·3	20·20	20·20	20·20	4 34

III.

TEMPERATURE 17° C.

1000·	0·7	2·43	2·44	2·435	0°33′
2000·	1·4	4·86	4·87	4·865	1 6
3000·	2·1	7·25	7·26	7·255	1 39
5000·	3·5	12·02	12·01	12·015	2 44
7000·	4·9	16·74	16·76	16·75	3 47
8000·	5·6	19·03	19·12	19·075	4 19·5
9000·	6·3	21·51	21·53	21·52	4 51

Charge of Needle, 160 *volts.*

I.

TEMPERATURE 7° C.

1000·	0·55	2·28	2·28	2·28	0°31′
2000·	1·10	4·53	4·52	4·525	1 1·5
3000·	1·65	6·80	6·79	6·795	1 32·5
5000·	2·75	11·28	11·21	11·245	2 33
7000·	3·85	15·78	15·67	15·725	3 33·5
8000·	4·40	18·00	17·93	17·965	4 4
9000·	4·95	20·30	20·20	20·25	4 34·5

II.

TEMPERATURE 18° C.

Resistance between point on coils to ground and point to quadrants in degrees.	Charge of quadrants in volts.	Scale readings in cm. Right.	Scale readings in cm. Left.	Mean of the two scale readings.	Deflection of the needle in degrees.
1000·	0·55	2·30	2·33	2·315	0°31′·5
2000·	1·10	4·61	4·65	4·63	1 3
3000·	1·65	6·87	6·97	6·92	1 34
5000·	2·75	11·41	11·61	11·51	2 36·5
7000·	3·85	14·98	15·20	15·09	3 25
8000·	4·40	18·21	18·49	18·35	4 8
9000·	4·95	20·50	20·73	20·615	4 37·5

Charge of Needle, 200 *volts.*

TEMPERATURE 17° C.

1000·	0·46	2·00	1·99	1·995	0°27′
2000·	0·92	3·99	3·99	3·99	0 54
3000·	1·38	5·99	5·96	5·975	1 22·5
5000·	2·30	9·98	9·94	9·96	2 15·5
7000·	3·22	14·00	13·96	13·98	3 10
8000·	3·68	16·02	15·96	15·99	3 37
9000·	4·14	18·00	17·96	17·98	4 3·5
10000·	4·60	20·00	19·90	19·95	4 30

It was found that the curve of calibration, when the charge of the needle was 200 volts, was practically constant for a range of temperature from 6° to 25° C.

The curves of calibration for the cases where the needle has charge of 10 and 20 volts were not examined very carefully, but it was observed that the variation was much greater than in the other cases. The following tables will serve to show the form of the curves for these cases:

Charge of Needle, 10 *volts.*

TEMPERATURE 18° C.

Resistance between point to ground and point to quadrants in ohms.	Charge of quadrants in volts.	Scale readings in cm. Right.	Scale readings in cm. Left.	Mean of two scale readings.	Deflection of needle in degrees.
1000·	4·3	1·02	1·04	1·03	0°15′
2000·	9·6	2·24	2·24	2·24	0 30·5
3000·	12·9	3·69	3·71	3·70	0 51
5000·	21·5	7·41	7·39	7·40	1 41
7000·	30·1	11·83	11·81	11·82	2 41
8000·	34·4	14·25	14·27	14·26	3 13·5
9000·	38·7	17·21	17·16	17·185	3 53
10000·	43·0	20·25	20·26	20·255	4 34·5

Charge of Needle, 20 *volts.*

TEMPERATURE 17° C.

Resistance between point to ground and point to the quadrants in ohms.	Charge of the quadrants in volts.	Scale readings in cm.		Mean of two scale readings.	Deflection of needle in degrees.
		Right.	Left.		
1000·	3·2	1·19	1·22	1·205	0°16′·5
2000·	6·4	2·55	2·55	2·55	0 35
3000·	9·6	3·95	3·93	3·94	0 53
5000·	16·0	7·07	7·04	7·055	1 36
7000·	22·4	10·50	10·53	10·515	2 23·5
8000·	25·6	12·44	12·45	12·445	2 46·5
9000·	28·8	14·42	14·41	14·415	3 16
10000·	32·0	16·61	16·63	16·62	3 44·5

In making the calibrations, examinations for leakage were frequently made by charging the quadrants and breaking their connection with the battery circuit. The constancy of the gravity and water batteries was determined by means of a constant cell, devised by Dr. Willson.* The electro-motive force of this cell, which was taken as the standard in these observations, is 1·085 to 1·088 volts. This variation is so small that it is not observable with the electrometer, which is not capable of measuring less than ·01 of a volt, when set up in the manner described, and the needle having a charge of 200 volts.

It will be noticed that the variation in the sensibility decreases as the charge of the needle becomes great relatively to that of the quadrants. The mathematical theory supposes that the charge of the needle is high when compared with that of the quadrants, but it gives no idea of what the order of the charges should be. These observations seem to show that in making use of an electrometer for electrical measurements, we should ascertain by experiment what charge the needle must have, in order that the sensibility may remain constant for the range of charges to be given the quadrants.

Much trouble was experienced at first through the electrometer being set up in a room in which several students were at work upon various experiments in electricity. This trouble seemed to be due to induction effects on the quadrants, which the electrometer case did not very completely shield, for on enclosing the electrometer in a box coated with tin-foil, and put into connection with the ground, the trouble was removed. After removing another difficulty, i. e., leakage, due to use of glass rods in the construction of a commutator, by substituting paraffine for the glass, it was found that the electrical zero suffered a displacement to the side to which the spot of light was deflected.† This displacement increased with

* Über Daniell'sche Normal-Elemente, Inaugural Dissertation von Robert W. Willson, aus Cambridge. U. S. Würzburg, 1886. Pamphlet.

† Thomas Gray, Phil. Mag., V, vol. xxiii, p. 46.

the magnitude of the deflection, and was as great as one centimeter, at the end of two or three minutes, for a deflection of twenty centimeters. The suspending fiber had been in the instrument for some time, inquiry showed that it was drawn from floss silk.* On substituting a fiber drawn from cocoon silk which had been well washed, no displacement of the zero was observed except where the room had been kept at about 0° C. for several hours. Some trouble was experienced from sudden movements of the needle, apparently caused by the working of a dynamo, from which wires extended through the various parts of the building in close proximity to the pipes for gas and water, the pipes being made use of as ground connections for the electrometer and batteries. These deflections of the needle were most marked in the cold, dry weather of winter; from this it seems probable that the pipes did not serve as very good ground connections at the time and under the circumstances.

Jefferson Physical Laboratory, Dec. 5th, 1887.