ART. XXXVI.—*An Instrument for Demonstrating the Laws of Transverse Vibrations of Cords and Wires;* by GEORGE S. MOLER, A.B., B.M.E.

[Contributions from the Physical Laboratory of Cornell University. Communicated by Edward L. Nichols.]

THIS piece of apparatus was designed to meet a want, felt in the laboratory, for an improvement over Melde's method of producing transverse vibrations of cords and wires.

It seemed desirable to produce a circular vibration which would appear of the same amplitude when viewed from all sides and which could be maintained for an indefinite length of time; so that the proper adjustments of length or of tension could be made while the cord was in motion. It also seemed desirable to maintain a constant rate of vibration when cords are used which offer more or less resistance to the motion.

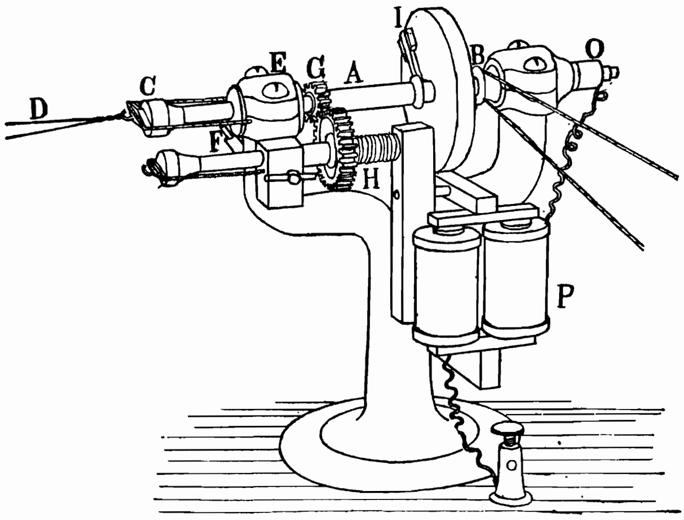
The general arrangement of the apparatus is shown in figure 1. To produce the circular vibrations the cord is attached to a crank of small throw, which is kept at a constant but high rate of speed by means of a regulator. A is the main shaft, driven by a belt running over a pulley at B. Upon the crank-pin at C is a small block having a hook to which the cords D are attached. To prevent the tension of these cords, which may sometimes amount to four or five kilograms, from exerting a pull upon the shaft and thereby greatly increasing the friction, a cord is carried from the box E to the block C through holes in each end of the latter and back to the box E; then to a tightening pin F as shown in figure 2. This device allows the block to move freely and easily under the largest loads.

At G is a pinion which drives another shaft with a similar crank and block. On this shaft is a spring which holds the wheel H in line with the pinion. This wheel is fastened to a slotted sleeve which, when it is not desirable to drive both cranks at the same time, can be moved, by compressing the spring, until it slides past a pin in the shaft. A turn of the sleeve then holds the wheel in a position where it will not engage with the pinion.

From these cranks the cords are carried, as shown in figure 5, to two movable blocks which can be clamped at any desired position upon the base of the instrument. These blocks carry small pulleys over which the cords pass, the ends of each cord being fastened to a scale pan and the proper tension secured by the use of weights.

A cord one meter long is commonly used, but the apparatus works quite as well with shorter or indeed with much longer

1.



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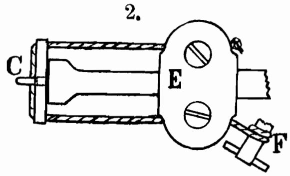

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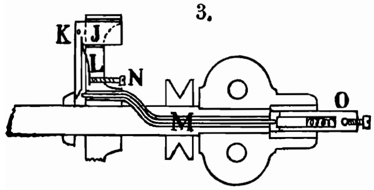

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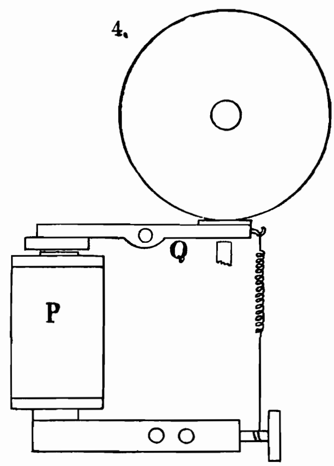

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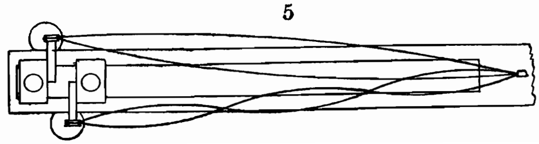

cords. By carrying a cord as far as I conveniently could, a distance of thirty-four feet, and adjusting the tension, I obtained twenty-five well formed segments.

The regulating device is an electrical governor, shown at I figure 1, but more in detail in figure 3. The lever J is pivoted at K and whenever the speed exceeds a certain amount it is pulled outward with sufficient force to bend the spring L and make contact with the insulated wire M. The speed at which this occurs is regulated by altering the tension of the spring by means of the screw N. The contact points are of platinum. The insulated wire runs out through the hollow shaft to the binding post O, thence to the electro-magnet P and the battery. The other pole of the battery is connected with the frame of the machine. The armature of the electro-magnet is attached to a lever Q (fig. 4), and upon the other end of the lever is a leather pad, which is brought into contact with the wheel whenever the circuit is closed. The friction thus produced reduces the speed until the contact breaks when the speed again increases and closes the circuit. The delicacy of this device is such that these variations of speed are imperceptible.

By driving two cords from the same crank a very accurate secondary speed regulation was obtained. For this purpose a cord of considerable mass was made use of, with tension such as to cause it to vibrate in a single segment. The shaft was driven by means of a small turbine motor, capable of any speed up to seven thousand revolutions per minute. When the vibrating segment of the heavy cord was once well formed it was found that small variations in the motive power made themselves felt in swelling or diminishing the amplitude of the segment without producing changes in speed, and that it was possible to make all necessary adjustments of length or tension in the second cord without in any way interfering with the uniform motion of the instrument. If, however, the governing segment were destroyed in any way the electrical governor already described would soon restore the speed to its normal amount. The amplitude of the governing segment served likewise as a delicate indicator of the condition of the motive power, excess of power causing a swelling out of the segment to large diameter, and diminution of power showing itself by decrease in the amplitude of vibration.

By means of this apparatus all the laws demonstrated in Melde's experiment may be verified. The following is the record of a single set of observations made to test the working of the instrument.

The cords used were of braided silk, measuring $\cdot 55^{mm}$ in diameter. Four strands of this size were taken for the controlling cord. This was attached to the crank of the main shaft and with a load of 2·5 kilograms gave a speed of 4986 revolutions, as a mean of four consecutive determinations; the greatest variation from the mean being 10 revolutions, which error lies within the limit of accuracy of the determination. At this speed the controlling cord was formed into a single segment having a diameter of about 8^{cm}.

During the first half of the experiment, in addition to the controlling cord, a single cord was used. It was then replaced by four strands and finally by a single cord driven from the second crank.

Now if N be the number of vibrations per unit of time, L the length of the cord, n the number of segments and V the velocity of transmission of an impulse transmitted to the cord, we have the familiar formula expressing the transverse vibrations of flexible cords;

$$N = \frac{n}{2L} V.$$

If P is the tension of the cord, s its cross section and d its density: we shall have

$$V = \sqrt{\frac{P}{sd}}.$$

Finally if λ is the wave length,

$$\lambda = \frac{2L}{n},$$

$$V = N\lambda,$$

$$\text{and } \sqrt{\frac{P}{sd}} = \frac{n}{N2L}.$$

In the following table the single set of observations already referred to are given, for the purpose of exhibiting the performance of the apparatus under variations of cross section, length, speed and tension. The results agree quite closely, obtained as they were from a single determination of each quantity, and they show the application of the apparatus to the demonstration of the laws embodied in the general formula.

A very interesting form of vibration is obtained by stretching a cord at 45° with the line of the shaft. The crank then gives a longitudinal impulse and a transverse impulse at the same time; the longitudinal impulse being an octave lower than the transverse. The cord thus has a resultant motion in which its vibrations as a whole and in two segments are plainly

TABLE.

Observation.	Cross Section. S.	Length. L.	Rate of Vibration. N.	Number of Segment. n.	Tension in Grams. P.	Square root of Tension. $\sqrt{P}$	Constant $\frac{n}{NL}\sqrt{\frac{P}{sd}}$
No. 1	1 strand.	1	2	1	627·	25·04	12·52
" 2		1	2	2	162·	12·70	12·70
" 3		1	2	3	74·	8·60	12·90
" 4		1	2	4	41·	6·40	12·80
" 5		1	2	5	26·	5·10	12·75
No. 6	1 strand.	1	2	1	627·	25·04	12·52
" 7		$\frac{2}{3}$	2	1	266·	16.30	12·22
" 8		$\frac{1}{2}$	2	1	166·	12·90	12·90
No. 9	4 strands.	1	2	1	2500·	50·00	12·50
" 10		1	2	2	625·	20·00	12·50
" 11		1	2	3	280·	16·70	12·52
" 12		1	2	4	158·	12·60	12·60
No. 13	1 strand.	1	1	1	166·	12·90	12·90
" 14		1	1	2	41·	6·40	12·80
" 15		1	1	3	18:	4·20	12·60
" 16		1	1	4	9·5	3·10	12·40

visible. By interposing a long narrow slit between the cord and a strong light, a single point may be illuminated, which when the cord is in motion, traces in the most beautiful manner, by persistence of vision, the path of that portion of the vibrating segment. By the use of a series of parallel slits any number of such illuminated paths may be made visible at once, bringing out the character of vibration of the entire segment with an indescribably striking effect. The curves followed by the illuminated points are Lissajou's figures and kindred forms. The regulation attained by means of the controlling cord is so excellent that it was found possible to photograph the curves with an ordinary dry plate. During an exposure of three minutes the changes in the form, size and position of the illuminated cross-section of the vibrating segment were not such as to ruin the picture.

The behavior of the vibrating cord as a controlling device is such that it seems possible to use it to advantage as a means of obtaining a uniform motion in chronographs and similar instruments. Its length or tension could be adjusted without stopping the apparatus and any desired rate of speed could be thus obtained without interrupting the work which might be in progress.

June 2, 1888.