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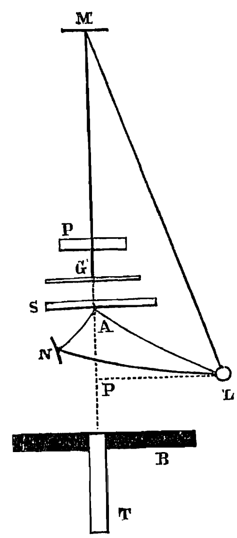
ART. I.—*Photometric Experiments.* PART SECOND.—*On the amount of light transmitted by plates of polished crown glass at a perpendicular incidence;* by OGDEN N. ROOD, Professor of Physics in Columbia College.

IT would seem that in the direct photometric experiments on this matter, the instrumental means employed have been more or less defective, in consequence of which the results gained, although having a certain amount of general correctness, still leave much to be desired. The main sources of error have been, first, the use of a defective mode of making the compensations, not allowing the experimenter to take full advantage of the sensitiveness of the eye, and secondly the employment of *two sources* of light, which always brings with it another set of unavoidable errors. Apart from this, in cases when the amount of reflected or transmitted light has been thus approximately obtained, the indices of refraction of the substances used do not seem to have been determined, so that for *perpendicular incidence*, slender data exist for comparing the results of theory and experiment.

It will be seen in what follows, that, to avoid the variability caused by two sources of light, I have devised a method in which only a single gas-flame is employed, the light being divided in such a way that a certain portion of it always illuminates the posterior side of the screen, while the other portion is reflected from a movable mirror, and pursues its way unobstructed to the screen, or is transmitted to it by the plate under experiment. The compensation is made by moving the mirror, and when once effected of course remains undisturbed by fluctuations of the flame. In addition to this it will also be observed that a more delicate screen, constructed, however, on the general principles indicated in the first part of this paper, has been employed, and these modifications, taken in connection with certain precautions described below, have given the results an accuracy superior to that attained in the experiments on silver mirrors.

Arrangement of the photometer with a single gas flame for experiments on the amount of light transmitted by plates of various substances.

The source of light is a small gas flame at L, about an inch in height, the gas flowing as before from a plain circular opening. A portion of its light, L A, directly illuminates the screen S, while another portion N A is reflected on the screen by a Liebig's mirror, one protected by a coating of copper being selected: this equalizes the light on both sides of the bare "spot" in S. A third portion of the light from L falls on M, a mirror like that just mentioned, and there is reflected perpendicularly through P, the plate under examination, on G the ground glass. It will be observed that the path pursued by the light in this latter case is the distance L M+M G: by the graduation on the instrument M G is actually measured, G P is known; also P L; hence we have the means of calculating the distance M L, which is to be added to M G, giving the distance required. As these calculations involve some labor, I constructed for my instrument a table by which the *measured* distance G M, is readily converted into the *total* distance.



The photometer was the same as that described in the first part of this paper, the following changes having been introduced: the "spot" on the screen was made much smaller, being only about $\frac{1}{30}$ of an inch in horizontal diameter, with a length about three times as great; it was observed by a small telescope T, magnifying six diameters. This reduction in the size of the "spot" rendered it possible to illuminate the ground surrounding it in a manner beyond reproach.

Mode of adjusting the apparatus for experiment.

1st. The center of the flame at L, and the center of the mirror M, must be at the same height above the common base.

2nd. The screen and plates to be examined are to be made perpendicular to the axis of the instrument, by the aid of the small gas flame mentioned in the first part of this paper: the telescope T is also most readily collimated by the use of the same flame, the screen, &c., having been temporarily removed.

3rd. The mirror M evidently must be brought into such a position that it shall send the ray MP perpendicularly to P, or along the axis of the instrument when its distance is such as to effect compensation; for if the reflected ray were sent obliquely through the ground glass plate G, noticeable errors would be introduced. This adjustment, which is important if good results are expected, is best effected by making one or two compensations so as to determine approximately the correct distance of the mirror, which is then rotated on its vertical axis so that the reflected image of the flame is made to fall in the center of the field of the telescope, the screen having been previously removed. By a repetition of this operation finally the mirror is brought into its proper position, when a *single* series of measurements can be made, the minute differences in the individual compensations introducing no appreciable errors. When, however, the compensation point itself has been shifted by the introduction or removal of a plate of glass, the mirror will correspondingly be moved away or toward S, and of course the ray MP will be sent a little to the right or left, and it becomes necessary to devise some simple way in which this difficulty can be obviated, without in each instance removing the screen and altering the focus of the telescope.

The mirror then being actually in adjustment, this, when lost, can be recovered as follows: in the screen B about four inches above the telescope is a circular aperture $\frac{1}{10}$ of an inch in width; the reflection of the flame in the mirror is observed through it with the naked eye, and the image of the flame is seen higher up on the mirror, and is made to coincide with a short black line drawn there previously. The ray MP can always thus be made to coincide with the axis of the instrument, it not even being necessary for the observer to leave his seat or diminish the sensitiveness of the eye by exposure. I am particular in describing this precaution, having in my own case rejected the results of 1300 compensations which were made with a comparative lack of attention to this single point.

Mode of measuring the amount of light transmitted by a plate of glass, &c.

1st. The plate is placed at P and collimated by the use of the small gas flame.

2nd. A compensation is effected by moving the mirror.

3rd. The mirror is adjusted by the use of the aperture in B, and another compensation made, followed by a second adjustment of the mirror, if necessary.

4th. A series of careful compensations are now made, *alternately by the approach and recession of the mirror;* these are registered on the fillet of paper.

5th. The plate is removed without the experimenter changing position or exposing the eye to bright light, and a compensation is made which necessitates of course a new adjustment of the mirror by the aid of the aperture in the screen B. Finally, a series of compensations are effected with the free flame and registered.

At this point it may be well to notice an objection which might be urged to this mode of experimenting, viz: it is evident that the angles at which the light is reflected from the mirror will not be identical in the presence and absence of the plate of glass, and as the amount of light reflected by glass and silver varies with the angle of reflection, this might become a source of error and necessitate a correction. With the plates of colorless glass employed by me, however, the angle in one case was 5° 30′ and in the other 5° 18′, the difference of 12′ being much too small to produce an appreciable effect, as we know from the results of older experiments.

For the purpose of showing what can be effected by this method, I can perhaps do no better than give the results of five sets of experiments; each of them consisted of four sets of eight double, or sixteen single compensations, and during the progress of each trial the conditions remained the same. The experiments were made as follows: eight double compensations were effected with the photometer and registered; then everything being arranged as before, a new set of compensations were made, &c., all of which was repeated four times. The arithmetical mean of the distances of the source of light from the screen was then obtained separately for each of the four experiments, and from these four mean values of the distance, a final mean value was deduced. This last quantity was then squared and compared separately with the squares of each of the first four mean distances, which proceeding tells at once the difference of the four results from the final mean result, in per cents or fractions of a per cent, and is adapted to give an idea of the degree of accuracy to be expected in this kind of observation.

Differences from mean in per cents:

No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
+·012	−·275	−·252	+·573	−·102
+·347	+·126	−·137	−·208	+·057
+·158	−·124	+·385	−·060	+·041
−·512	+·275	+·007	−·308	+·005

It will be observed that in the twenty experiments, only two cases occur in which the difference rises slightly above one half per cent, or above $\frac{1}{200}$ of the whole quantity of light, while in nearly all the other cases the difference falls considerably below this quantity, and the average of their differences is less than 0·2 per cent, or less than $\frac{1}{500}$ of the whole amount.

As the reliability of measurements with a photometer is evidently connected with the power of the eye in distinguishing different degrees of brightness in adjacent surfaces, it is well to review here briefly the results that have been obtained by different observers, as to the sensitiveness of the eye under favorable conditions.

The least difference which Boguer with his method, (two shadows,) was able to distinguish was $\frac{1}{64}$ of the whole; Fechner's friends using the same method were able to reduce this quantity to $\frac{1}{100}$. Arago remarked that where one of the illuminated surfaces was in motion smaller differences could be perceived, and was able under favorable circumstances to distinguish $\frac{1}{131}$, while Masson, who used revolving discs, along with his friends, was able to distinguish between $\frac{1}{50}$ to $\frac{1}{120}$ of the whole quantity of light. Near a window Helmholtz was able to distinguish with certainty a difference of $\frac{1}{133}$, and occasionally as small a quantity as $\frac{1}{167}$, while in the middle of a room he was able to appreciate certainly only $\frac{1}{117}$, seldom and uncertainly $\frac{1}{133}$.*

In all the better of the above mentioned experiments, photometers in any proper sense of the term were not employed, the best of them being made by the use of revolving discs of white paper furnished with narrow black sectors, which are calculated to discover the absolute sensitiveness of the eye under the most favorable conditions. It must not however be imagined that because a good eye can distinguish a difference of $\frac{1}{120}$ on a revolving disc, that the same eye would be able to obtain a similar result with ordinary forms of the photometer, where the conditions are far less favorable, and accordingly it seemed of interest to me to determine by a series of experiments the average sensitiveness of the eye while using the photometric method described in this paper, particularly as the plan of using

* Physiologische Optik, von H. Helmholtz, page 311.

a single flame and mirror offered great facilities for making these observations in a reliable manner. The following mode was employed: the light from the mirror was allowed to fall on the unobstructed screen, and the mirror was *drawn up* till the spot had disappeared, when the observation was registered; the next compensation was made by *pushing* the mirror *away*, and so on alternately, *the compensations by pull and push being registered on separate fillets of paper by two pens.* In the first experiment fourteen compensations were made by the advancing, and an equal number by the retreating mirror, and the mean of these twenty eight quantities was taken as the true distance of the mirror from the screen; then on comparing the mean of either set of compensations with this true distance, it was easy to ascertain how great a difference from the truth had been tolerated by the eye under the given circumstances. In the first experiment this quantity was found to be $\frac{1}{114}$, in a second similar trial $\frac{1}{125}$ of the total amount of light. The "screen" in these experiments was quite *new*, and an inferior result was obtained by using a "screen," which had for six weeks been exposed quite unprotected to the action of the air; here the usual darkening of the edges had begun and progressed so far as to be faintly visible during compensation.

In the experiments just detailed the highest *average* sensitiveness of the eye is only $\frac{1}{125}$, while as before shown in practical use the average difference from the mean is less than $\frac{1}{500}$ of the whole. This higher degree of accuracy results naturally from the mode in which the compensations are made, (alternately by approach and recession,) and the fact that the compensations are thus interwoven, in such a manner as to eliminate errors introduced by the slightly varying sensibility of the eye.

Experiments on the amount of light transmitted by plates of glass at a perpendicular incidence.

A knowledge of the amount of light transmitted and reflected by colorless transparent substances has a certain degree of interest from a technical point of view, enabling opticians to calculate the loss necessarily experienced in various instruments from this source, as well as the intensity where the light is reflected from a single or from two parallel surfaces of glass or other material. Of far more importance, however, is the interest attaching to this subject from a theoretical point of view, and especially in connection with the Undulatory Theory of Light. Formulas for the intensity of the reflected and transmitted beams at all angles have been deduced by eminent supporters of this theory, which in some cases have occasioned considerable discussion. The following very simple formula for the intensity of common light reflected at a perpendicular incidence was first given by Thomas Young; $I = \frac{(n-1)^2}{(n+1)^2}$; the intensity of the incident beam being equal to unity, and n being the index of refraction of the reflecting substance. The same formula was afterwards reached by Poisson in a rigid and learned analysis of the subject, and it was again deduced by Fresnel.* As is well known Fresnel's formulas were subsequently modified by the celebrated Cauchy, but in Cauchy's formula for reflection as soon as the incidence of the light differs considerably from the polarizing angle, the small quantities which depend on e, the coefficient of ellipticity, become so much reduced that they can be omitted from the numerator and denominator, and Cauchy's formula becomes identical with Fresnel's.† It hence appears that the formula above quoted is as well theoretically established, as any which has been deduced under the guidance of the Undulatory Theory, and one would naturally suppose that it had been often tested carefully by experiment. This does not seem to have been the case, and I do not know that it has ever been rigidly tried by a delicate photometric method. On this account I have made a series of observations on plates of colorless glass which are detailed below.

Mode of experimenting.

When a beam of light falls on a plate of glass or other transparent colorless substance, a certain portion will be reflected, another portion transmitted, a third absorbed. If the plate of glass be colorless and thin, the portion absorbed will be smaller than the necessary errors of observation, so that it can safely be neglected. For example, Bunsen found that in using a plate of crown glass 4·7 millimeters thick, that it absorbed only $\frac{4}{10}$ of one per cent of the *chemical* rays that fell on it at a perpendicular incidence.‡ The thickest plate of glass employed by me was 1·67 millimeters from surface to surface, i. e. about one-third of that used by Bunsen, and as we know that the chemical rays are extinguished by glass in a far larger proportion than those which are luminous, it follows that in the plates mentioned below we can safely neglect the internal extinction. This point being settled, the mode of proceeding becomes quite simple; it is only necessary to measure the amount of light transmitted, and the difference between the incident and transmitted light gives the amount of that reflected, and after making a correction for internal reflection, we shall have the means of comparing the results of theory and experiment.

* Pogg. Annalen. Bd. xxii, p. 98.

† Jamin in Pogg. Erg., Bd. iii, p. 256.

‡ Pogg. Annalen, Bd. ci, p. 241.

Mode of determining the indices of refraction.

In my experiments on transparent substances, always where it was possible prisms of the substance were ground and the index of refraction of the sodium line determined as usual with a graduated circle, collimating and observing telescope. In the particular experiments detailed at the termination of this article, the plates of glass used were so thin that it was not practicable to grind from them prisms, and for all such cases I contrived, tested and used two somewhat new modes of procedure, as neither the method of the Duke de Chaulnes (alteration of the focus of a microscope,) nor that of Bernard (displacement of an image viewed obliquely through a plate,) were here found to give reliable results.

1. A minute angular fragment of the glass to be experimented on was placed in a cell, on a glass slide, like those used for mounting microscopic objects, and surrounded with a mixture of "body-sperm-oil" and oil of cassia, the proportions being varied till the refraction of the glass for the sodium line had been exactly compensated by the oil. *Olive oil* became turbid when mixed with the oil of cassia and hence could not be used. The mode of comparing the refractive power of the mixture of the oils and glass was as follows: at the distance of half an inch below the level of a microscope stage was a fine slit, cut in tinfoil which had been pasted on glass; the microscope was focussed on this, a sodium flame being used to illuminate it; the cell with the oil and fragment of glass was then placed on the stage of the microscope, and moved so that the light from the slit passed through the angular fragment, when it would happen that the line of light would be refracted to the right or left hand according as oil or glass predominated in refractive power, which made it instantly evident whether sperm oil or oil of cassia was needed. A number of experiments were made to test this method, which was found to answer well, the index of refraction as determined by prism corresponding with that obtained by the use of the fragment: so in the case of a sample of crown glass a triangular prism gave the index of refraction as 1·526, while by the new method it was found to be 1·529. This method, however, is capable of still more accurate results, as in the above mentioned experiments, the compensation was pushed only far enough to answer my immediate purpose; that is to say, the fragment may be considered to consist of one *large* triangular prism with a moderate angle, and a number of smaller prisms, some of which are sure to have very large angles; these latter become effective when the glass is under oil, total reflection no longer taking place, and they act powerfully on the light coming from the slit, still furnishing faint images having

a considerable deviation even after the main portion of the glass fragment has ceased to perceptibly deflect it.

In no case did I push the compensation far enough to gather in all these outstanding beams.

2. Another method, which in the case of crown glass was found to answer quite well enough for my purpose and to be very convenient, consisted in fusing to a spherical globule a fragment of the glass, placing it in the mixed oils and effecting compensation by observing when the globule ceased to act as a lens, for which purpose a small telescope or the microscope can be employed. Thus, for example, a certain kind of crown glass when ground into a prism gave as index of refraction for the sodium line 1·526, while when tested according to this second method, the results of two experiments were 1·5232 and 1·5235.

I give now the results of careful sets of experiments on the amount of light transmitted by two different samples of crown glass: in each case the results of four independent trials are given, each trial being worked out with the aid of seven double compensations. In the first case the thickness of the glass was ·15 millimeters, the index of refraction 1·5236, and allowing for the effects of internal reflection, it should, according to theory, have transmitted 91·736 per cent of the light falling on it. Experiment gave:

92·227
91·371
91·019
91·143

91·440

The difference ·296 being hardly larger than the necessary error connected with the method of making the determination.

In the second case the index of refraction was 1·5225, the thickness 1·677 millimeters, and by theory it should have transmitted 91·763 per cent of the light falling on it. Below are the actual results obtained:

90·886
90·948
90·892
91·895

91·155

The difference here of ·500 per cent, or $\frac{1}{200}$ of the whole amount, is almost equally satisfactory, and these experiments show, I think, that the reflecting power of glass with the above index of refraction, *conforms in the closest manner to the predictions of theory.* Elaborate experiments were also made with flint glass, quartz and calc-spar, but I suppress the results, as it afterwards turned out that they were contaminated with minute errors of the character described in this article under the head "mode of adjusting, &c." 3rd; the adjustment alluded to having indeed been always made, but not with a sufficient degree of care to exclude the last trace of error.

New York, March 29th, 1870.