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ART. I.—*On Certain Harmonies of the Solar System*; by Professor DANIEL KIRKWOOD, Indiana State University.

I.—THE ROTATIONS OF THE PLANETS.

IN 1849, a very simple formula connecting the rotations of the planets, and harmonizing in a remarkable manner with the elements of the solar system so far as known, was communicated to the American Association for the Advancement of Science. This formula represented the rotation-period of Uranus, which had never been observed, as greater than that of any other planet. Hence a determination of the true rotary velocity would have been regarded as a test of the new harmony. The number of powerful instruments had then been recently increased, and Uranus was approaching a more favorable position for accurate examination. Little doubt was therefore entertained that the claims of this planetary law would soon be decided by telescopic discoveries. Inasmuch, however, as no definite results have yet been obtained, a brief review of the facts may not be destitute of interest.

If the solar system has resulted, as was supposed by Laplace, from the gradual contraction of a rotating nebulous spheroid, what was probably the physical constitution of the abandoned equatorial rings in the first stages of their separate existence? The celebrated author of the nebular hypothesis supposed each of the rings in which the several planets originated to have revolved, during an indefinite period before its dissolution, as one continuous mass. "These zones," he remarks, "ought, according to all probability, to form by their condensation and by the mutual attraction of their particles, several concentric rings of vapor circulating about the sun. The mutual friction of the molecules of each ring ought to accelerate some and retard others, until they all had acquired the same angular motion. Consequently, the real velocities of the molecules which are *farther* from the sun ought to be greatest."[1] This view has also been generally adopted by later advocates of the nebular theory. Instead, however, of each planet's having existed as a single, continuous ring, may not each have consisted of many? Such, according to the investigations of Professors Bond, Peirce, and Maxwell, is the present constitution of Saturn's rings.[2] The conclusion reached by the last named writer is "that the rings must consist of disconnected particles: these may be either solid or liquid, but they must be independent. The entire system of rings must, therefore, consist, either of a series of many concentric rings each moving with its own velocity and having its own system of waves, or else of a confused multitude of revolving particles not arranged in rings and continually coming into collision with each other." Now the physical condition of the *primary* rings was probably somewhat analogous. After the process of separation commenced, we may suppose a continued succession of rings to have been thrown off in close proximity to each other, each revolving round the central mass in accordance with Kepler's third law.[3] The result, then, of a gradual condensation would be a central body surrounded by an indefinite number of concentric rings, or, rather, of disconnected planetary molecules, all moving in the same direction. The mutual attraction of some of these particles, when coming into close proximity to each other, would cause them to unite, and in this way we may suppose the planetary nuclei to have been first established. In the subjoined diagram, let S be the center of the solar mass; ABC and DEF the orbits of two adjacent planetary nuclei, B and E; and H the point of equal attraction between them. Let also p, p' be particles revolving round the center, S, in approximate accordance with the third law of Kepler. Their motion is disturbed by the attraction of E, and, in consequence, they finally coalesce with it. But the orbital velocity of p is *less* than that of E, while, on the other hand, that of p' is *greater*. It is obvious, therefore, that they would not approach the nucleus in lines normal to its surface. The point of contact of the *outer* particle would be *behind* the center, that of the *inner* one in *advance* of it. These particles, then, would act as oblique

[1] Syst. of the World, Harte's Translation, vol. ii, p. 359.

[2] Gould's Astr. Jour., vol. ii, No. 1, May 2d, 1851; vol. ii, No. 3, June 16th, 1851; and vol. iv, No. 14, Sept. 5th, 1855. Also, Maxwell's Essay on the Stability of the Motions of Saturn's Rings, p. 67.

[3] This law, we are aware, would but imperfectly represent the motions of the individual particles of the rings.

forces; their tendency being to produce a rotation in a direction *contrary* to that of the orbital motion. When the planetary mass, however, had gained considerable magnitude, the solar

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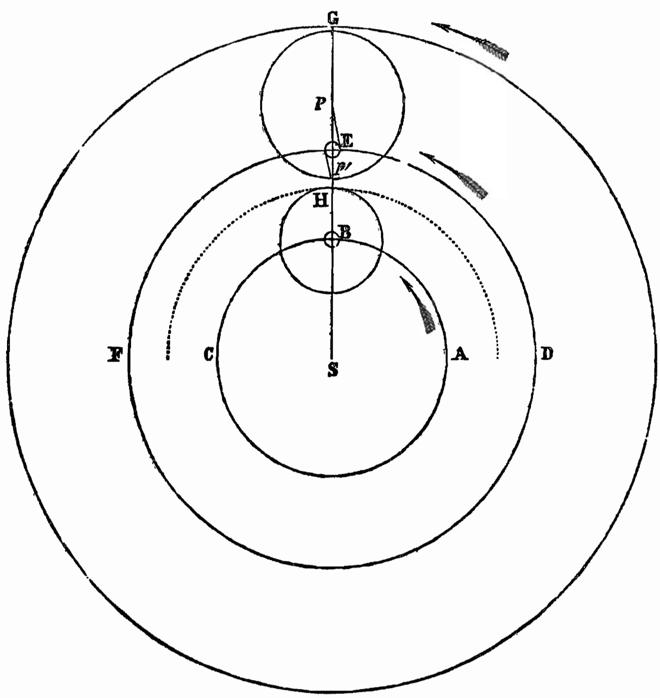


attraction would produce a tidal elevation on the hemisphere toward the sun. The gravitating force of this protuberant matter would maintain the greatest axis, during an indefinite period, in the direction of the central body; thus causing an equality between the angular velocities of rotation and orbital revolution, such as is now found to obtain in the case of the secondary planets.

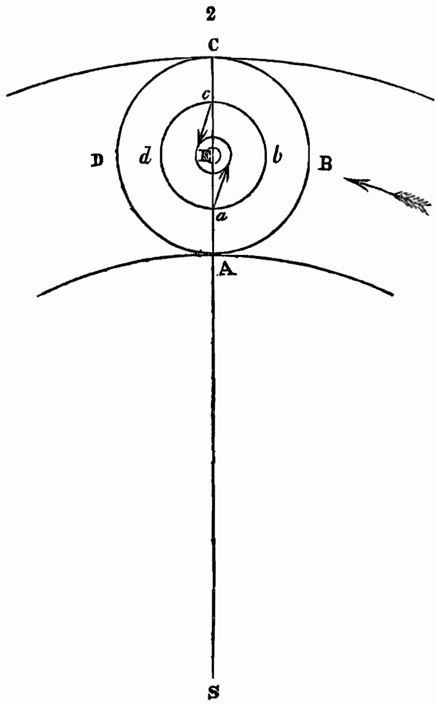
Let us now consider the consequences of further condensation.

Let S be the center of the solar mass;

AC = D = the diameter of a planet's sphere of attraction;

and *ac* = the diameter of the vaporiform planet at the close of the epoch of equality between the angular velocities of the rotary and progressive motions.

It is obvious that the orbital velocity of a particle at *c* must be *greater* than that of the center of the mass, while that of a particle at *a* must be *less*. Any further contraction of the spheroid must tend therefore, to accelerate the rotation *in the direction* *of the orbital motion.* It must also be manifest that the rotation-period at the epoch of solidification will depend upon the ratio of *ac* to SE. Now, it may be impossible to determine in any particular case what was the diameter of a gaseous planet at the commencement of the acceleration of the rotary velocity. We may assume, however, with a high degree of probability, that its ratio to the diameter of the planet's sphere of attraction was the same or nearly the same in each instance; or that



$$\Delta : \Delta' :: D : D'; \qquad (1)$$

where D, D′ are the diameters of the spheres of attraction, and Δ, Δ′, the diameters of the planets immediately before passing the limit of equality between the periods of rotation and revolution.

The law of rotation as originally announced is as follows:—

"Let P be the point of equal attraction between any planet and the one next interior, the two being in conjuction: P′ that between the same and the one next exterior.

Let also D= the sum of the distances of the points P, P′ from the orbit of the planet; which I shall call the diameter of the sphere of the planet's attraction;

D′= the diameter of any other planet's sphere of attraction found in like manner;

n = the number of sidereal rotations performed by the former during one sidereal revolution round the sun;

n' = the number performed by the other; then it will be found that

$$n^2 : n'^2 :: D^3 : D'^3.\text{"*} \qquad (2)$$

Let, now, T, T′= the periodic times of two planets;

t, t'= their periods of rotation;

and d, d'= their mean distances from the sun;

* Proc. Am. Assoc. for the Adv. of Sci., 2d Meeting, held at Cambridge, 1849, p. 207.

then (2) becomes

$$\left(\frac{T}{t}\right)^2 : \left(\frac{T'}{t'}\right)^2 :: D^3 : D'^3; \text{ or, by Kepler's third law,}$$

$$\frac{d^3}{t^2} : \frac{d'^3}{t^2} :: D^3 : D'^3;$$

whence

$$t^2 : t'^2 :: \left(\frac{d}{D}\right)^3 : \left(\frac{d'}{D'}\right)^3 \qquad (3)$$

or, assuming the truth of proportion (1),

$$t^2 : t'^2 :: \left(\frac{d}{\Delta}\right)^3 : \left(\frac{d'}{\Delta'}\right)^3 \qquad (4)$$

It is seen on the slightest examination that in the solar nebula, as well as in each of the gaseous planets, the ratio of the revolving or equatorial radius to the radius of gyration varied throughout the entire process of condensation: in other words, that the rate of variation of density from surface to center was constantly changing. Were the solar mass expanded so as to fill the earth's orbit, the rate of variation of density remaining the same, the period of rotation would not be 365 days, but 5457. The diameter corresponding to one revolution in a sidereal year would be only 12,720,000 miles. If we compute the values of the principal radius of gyration when the spheroid extended to the present planetary orbits respectively, we find that the solar mass had reached a high degree of central condensation before the epoch of Neptune's separation. We find, moreover, that the condensation advanced much more rapidly about the center than near the surface of the contracting mass. Probably it became so great as to produce chemical action, thus forming a nucleus in a state of igneous fluidity by the precipitation of the denser portion of the nebula, long before the exterior parts had passed from their original gaseous condition.

It is not necessary to suppose, as has been generally done, that if the nebular hypothesis be true the outer planets of the system must have an immensely greater antiquity than the interior. The formative processes in the different cases may have been contemporaneous.[4] In this view of the case, it seems probable that the remoter planets are less advanced in their physical history than some nearer the sun. The formation of a single planet from matter diffused around the circumference of a larger circle, would undoubtedly require more time than the aggregation of a ring of smaller dimensions. Possibly there may be

[4] Since writing the above, we have been favored with the reading of some highly interesting researches on the nebular hypothesis, by DAVID TROWBRIDGE, Esq., of Perry City, N. Y. These mathematical investigations sustain the probability of the synchronous formation of different members of the solar system. We are gratified to know that Mr. T. is preparing a treatise on this subject for the press, and trust it may soon be given to the public.

rings exterior to Neptune still in the nebular state, or at least not yet collected about a single nucleus.

It has been shown that, according to the nebular theory, a planet's time of rotation ought to be *some* function of the ratio of the radius of its orbit to the diameter of its sphere of attraction. Those ratios are very nearly equal in the case of Jupiter and Saturn; the periods of rotation are also nearly identical: the ratio, however, is somewhat greater in the case of Saturn; so also is the time of rotation. The ratios again are not very different in the cases of Mercury, Venus, the Earth and Mars; and again *in each instance* a greater ratio corresponds to a slower rotation. The *form* of the function as expressed by the equation,

$$\frac{T}{tD^{\frac{3}{2}}} = \text{a constant},$$

was found by a tentative process.

This analogy indicates, as we have stated, a longer period of rotation for Uranus than had been *conjectured* by some astronomers. It assigns, however, a physical *cause* for this slow revolution; while the short period of nine hours and a half, assumed by some writers, has no such basis. The best observers have failed to detect any such polar compression of the planet as would indicate a rapid revolution. "Professor Mädler," says Mr. Hind, "thinks he has detected a very considerable ellipticity in the form of the planet, and makes the ratio of the equatorial to the polar diameter as ten to nine, the axis being inclined at an angle of 15° 26′ to the circle of declination (1843, September 28). Other astronomers, with more powerful telescopes, have not succeeded in gaining any certain evidence of an appreciable difference in the diameters. Mr. O. Struve has informed me orally that the grand refractor at Pulkova affords no indications of ellipticity."[5] The Rev. Robert Main, of the Royal Observatory, Greenwich, states that at his request "Professor Challis obligingly measured the planet, some years ago, with a double image micrometer attached to the telescope of the great Northumberland equatorial, for the purpose chiefly of discovering whether it had any sensible ellipticity, which the author suspected from some measures of his own made with a far inferior telescope. The result was that the ellipticity is too small to be measurable."[6] These measures were made at nearly the same time with those of Mädler. The disk of Neptune, also, even when viewed through the most powerful telescopes, appears perfectly circular. Now the degree of ellipticity correspoding to a short period of rotation, nearly equal to that of Jupiter or Saturn, would undoubtedly be so great, especially in the case of Uranus, as to be easily recognized. The preponderance of evi-

[5] Hind's Solar System, p. 121.

[6] Main's Rudimentary Astronomy, p. 130.

dence, therefore, apart from the reason assigned by my analogy, is unquestionably in favor of a long period of rotation.

It is easily shown that the equality between the angular velocities of rotation and orbital revolution, which obtains in the secondary systems, is not incompatible with the law of rotation. When the volumes of the primary planets had the same ratio to their spheres of attraction as those of the satellites now have to theirs, the former were still in a state of vapor, their masses extending beyond the present orbits of the secondaries, and not having reached, in all probability, the limits of equality between the two angular velocities in their respective cases. Had the satellites, at the corresponding epoch in *their* history, been equally rare, so that any increase in rotary velocity would not have been prevented or arrested by solidification, the same law would doubtless have obtained in the secondary systems.

I have believed, however, almost from the time of its first announcement, that the statement of my analogy requires some modification. If it be the expression of a physical law, it must depend on the relation between the primitive momentum of rotation and that of orbital revolution. Now the time of rotation of any planet having satellites is evidently greater than it would have been had the entire mass condensed in a single body. In order, therefore, to find the proper constant of rotation for the earth, we must determine the time in which its axial revolution would be performed were the matter of the moon diffused over its surface; the momentum of the satellite's orbital motion being converted into momentum of rotation.

The earth's momentum of rotation cannot be known with accuracy, because the rate of variation of density from surface to center has not been determined. It is probable, however, that the mean density is nearly attained not far from the surface, and that no very important error would result from considering the radius of gyration to be that of a homogeneous sphere. Adopting this hypothesis, we obtain, by an easy calculation in mechanics, about 22h. 48m. as the rotation period of the united mass. The corresponding number of days in a year would be 384·4.

The sum of the masses of Jupiter's satellites is only about $\frac{1}{6000}$th, the mass of Jupiter being 1. The sum of the masses of the Saturnian satellites is still less as compared with their primary. In a calculation of this nature, therefore, the acceleration of the rotary velocities of these planets from the precipitation of their secondaries may be wholly neglected. The mass of Saturn's ring, however, according to Bessel, is $\frac{1}{118}$th, that of the primary being 1. A calculation similar to that which we have made for the earth and moon shows that the precipitation of even this mass upon the planet would shorten the period of rotation only about 17 minutes. But with Bond's estimate of the thickness of the ring, this value of the mass would indicate a density more than three times that of Saturn—a greater density than has been found for any planet, primary or secondary, exterior to Mars. This result seems too improbable to be admitted without confirmation. We have thought it best, therefore, in this state of the case, to adopt, without alteration, the received value of Saturn's period of axial revolution, viz: 10h. 29m. 17s.

If we use the masses of Jupiter, Saturn and Uranus, adopted in the *American Nautical Almanac*, we find the diameter of Saturn's sphere of attraction = 8·5478. Hence the constant of rotation

$$C = \frac{n}{D^{\frac{3}{2}}} = 985{\cdot}161\,;\ \log C = 2{\cdot}993507.$$

The diameters of the spheres of attraction for the other planets are then found from the formula

$$\log D' = \tfrac{2}{3}\,(\log n' - \log C).$$

The results, tabulated, are as follows:—

Diameter of the sphere of attraction of	Mercury,	0·1992
" " " "	Venus,	0·3801
" " " "	Earth,	0·5340
" " " "	Mars,	0 7731
" " " "	Jupiter,	4·8342
" " " "	Saturn,	8·5478

Let us now find the corresponding masses of Mercury and Mars, together with the mass and distance of the asteroid planet. We will employ the mass of the earth given in the *American Ephemeris*. Leverrier's value of the mass of Venus there adopted is probably somewhat too large, as seems to have been more recently admitted by Leverrier himself. We will, therefore, take Encke's value, $\frac{1}{401837}$. The calculation is obvious and need not be repeated. We find

The mass of Mercury = $\frac{1}{2997700}$.
" " Mars = $\frac{1}{3097100}$.
" " Asteroid Planet = $\frac{1}{1255868}$.
Distance of the Asteroid Planet = 3·1116.

Remarks.

1. This value of Mercury's mass is considerably greater than that found by Encke, but is very nearly identical with Leverrier's second value.

2. Our mass of Mars is somewhat less than Burckhardt's. Mr. Airy's researches on the theory of the sun have led him to the conclusion that a considerable diminution of that value is actually demanded.[7]

[7] Grant's Hist. of Phys. Astr., p. 129.

3. The resulting mass of the asteroid planet is less than one-third that of the earth; about the limit assigned by Leverrier for the mass of the zone of asteroids.

Researches of Professor Hinrichs.—The learned and interesting paper on the density, rotation, and relative age of the planets, in the American Journal of Science and Arts, for January, 1864, by Professor Hinrichs, would seem to enhance the difficulty of developing any order either in the *present* arrangement of the members of the solar system, or their revolutions on their axes. The conclusions of Professor H., however, depend upon the existence of a resisting medium, the only indication of which is found in the shortening of the period of Encke's comet. Encke himself regards this resistance as insensible exterior to the orbit of Venus. Now if the zodiacal light consists of solar rings, the sensible resistance of the comet is limited to the time of its passage through them. May we not thus find a sufficient cause for the diminution of the period?—and ought not all speculations in regard to the effect of the medium on the motions of the remoter members of the planetary system, to be received with caution, until we have some further evidence of its existence?

With respect to the want of harmony in the present arrangement of the planetary orbits, Prof. Hinrichs remarks as follows:

"The present configuration of the planetary system is without that harmony and order everywhere else observed where matter is aggregating (e. g. in crystals, etc.); we must therefore suppose that the original harmonious configuration has been altered by the action of some general cause, displacing the celestial strata (orbs) according to the individual mass, size, and position of each body; the same we know to have occurred in the case of the earth's figure, being at first ellipsoidal, but now to some extent irregular—or the terrestrial strata of rocks, which were at first continuous, but are now greatly dislocated. This cause has been, and is, *the resistance of the ether* filling the heavenly space in which the celestial globes are moving; for the mathematical investigation of the effects of such a resistance agrees perfectly with the phenomena observed." [8]

Without entering upon any special discussion of this interesting subject, we respectfully submit the following considerations:

1. The hypothesis adopted by Encke in regard to the medium which causes the acceleration of the mean motion of his comet, is that the density varies inversely as the square of the distance from the sun, and that the resistance is proportional to the density of the medium and the square of the velocity of the moving body.

[8] Am. Jour. of Sci. and Arts, Jan., 1864, p. 55.

2. Granting the existence of an ethereal medium, it would seem unphilosophical to ascribe to it one of the properties of a material fluid—the power of resisting the motion of all bodies moving through it—and to deny it such properties in other respects. Its condensation, therefore, about the sun and other large bodies must be a necessary consequence.

3. This condensation existed in the primitive solar spheroid, before the formation of the planets: the rotation of the spheroid would be communicated to the ether co-existing with it: *hence, during the entire history of the planetary system, the ether has revolved around the sun in the same direction with the planets.*

4. This condensed ether must participate in the progressive motion of the solar system.

5. Even if we reject the doctrine of the development of the planetary system from a rotating nebula, we must still regard the density of the ether as increasing to the center of the system. The sun's rotation, therefore, would communicate motion to the first and denser portions; this motion would be transmitted outward through successive strata, with a constantly diminishing angular velocity. The motion of the planets themselves through the medium in nearly circular orbits would concur in imparting to it a revolution in the same direction.

6. Whether, therefore, we receive or reject the nebular hypothesis, the resistance of the ethereal medium to bodies moving in orbits of small eccentricity and in the direction of the sun's rotation, becomes an infinitesimal quantity.

7. The doctrine of a resisting medium is not generally accepted by astronomers as an established fact. "It is manifest," says an eminent writer, "that more extensive indications of such a medium must be discovered before the problem of its existence can be considered as having received a definitive solution. It has not yet affected to a sensible extent any of the other celestial bodies, and, until such is found to take place, the question relative to it must remain in abeyance."[9]

II.—The Planetary Distances.

As long ago as the commencement of the seventeenth century, the celebrated Kepler observed that the respective distances of the planets from the sun formed nearly a regular progression. The series, however, by which those distances were expressed, required the interpolation of a term between Mars and Jupiter—a fact which led the illustrious German to predict the detection of a planet in that interval. This conjecture attracted but little attention till after the discovery of Uranus, whose distance was found to harmonize in a remarkable manner with Kepler's order of progression. Such a coincidence was of course regarded

[9] Grant's Hist. of Phys. Astr., p. 135.

with considerable interest. Toward the close of the last century, Professor Bode, who had given the subject much attention, published the law of distances which bears his name, but which, as he acknowledged, is due to Professor Titius. According to this formula, the distances of the planets from Mercury's orbit form a geometrical series of which the ratio is two. In other words, if we reckon the distances of Venus, the earth, &c., *from the orbit of Mercury*, instead of from the sun, we find that—interpolating a term between Mars and Jupiter—the distance of any member of the system is very nearly half that of the next exterior. The series is usually expressed as follows:—

$$\begin{array}{ll} 4 & = 4 \\ 4 + 3 \times 2^0 & = 7 \\ 4 + 3 \times 2^1 & = 10 \\ 4 + 3 \times 2^2 & = 16 \\ 4 + 3 \times 2^3 & = 28 \\ \&c. & \&c. \end{array}$$

The numbers 4, 7, 10, &c., represent approximately the relative distances of Mercury, Venus, the earth, &c., from the sun. The ninth term, however, which corresponds to Neptune, is 388, instead of 300. It was, moreover, remarked by Gauss that "the member which precedes $4 + 3$ should not be 4; i. e. $4 + 0$, but $4 + 1\frac{1}{2}$. Therefore, between 4 and $4 + 3$, there should be an infinite number; or, as Wurm expresses it, for $n = 1$, there is obtained from $4 + 2^{n-2} \times 3$, not 4, but $5\frac{1}{2}$."

Professor Challis has applied a modification of Bode's empirical formula to the secondary systems of Jupiter, Saturn and Uranus;[10] his coincidences, however, are far from exact. The formula itself, as well as the modifications by Wurm and Challis, may be expressed—commencing with the greatest distance and proceeding toward the center—by the series,

$$a + br^n,\ a + br^{n-1},\ a + br^{n-2},\ \&c.$$

In the scheme of Bode and Titius, $a = 4$, $b = 3$, and $r = 2$; in that of Wurm, $a = 387$, $b = 293$, and $r = 2$. Both fail to represent, even approximately, the relative distances of Mercury and Neptune.

I have never doubted that the planetary distances were arranged in *some* discoverable order. These failures, however, in the series of Titius have seemed a sufficient cause for its rejection, or, at least, some considerable modification. I have, for many years, been devoting such thought and attention to the subject as circumstances would permit, and I now propose to submit my results to the public.

In the American Journal of Science and Arts, for September, 1852, the fact was noticed that, "if we commence with Neptune, the most remote planet known, we shall find that the primary

[10] Cambridge Philosophical Trans., vol. viii, p. 171.

planets are arranged in *pairs*, the members of which are nearly equal in diameter." Neptune and Uranus constitute the first pair; Saturn and Jupiter, the second; the asteroids and Mars, the third; the earth and Venus, the fourth; finally, Mercury is without a *known* companion. It was also remarked that in each of the three complete pairs, the first, second and fourth, the densities of the members are to each other very nearly as their volumes; and that the facts seemed to indicate "a similarity in the original constitution of the members of each pair, and an intimate mutual dependence or connection in their primitive condition." It appeared not improbable that in the first stages of their history, Neptune and Uranus constituted a system of closely associated rings; Saturn and Jupiter, another, &c., and that the law of planetary distances might be found in *the relative situations of the centers of gyration of those binary rings.* In short, my researches on the subject led to the hypothesis that *the differences of the radii of gyration of the primitive rings form a geometrical series;* or that

$$d_{(1)} = d_{(2)}k = d_{(3)}k^2 = d_{(4)}k^3 = \ldots = d_{(n)}k^{n-1};$$

where k = a constant;

$d_{(1)}, d_{(2)}, d_{(3)} \ldots$ &c., $= r_{(1)} - r_{(2)}, r_{(2)} - r_{(3)}, r_{(3)} - r_{(4)}$, &c., respectively;

$r_{(1)} = \left(\frac{D_{(1)}^2 + D_{(2)}^2}{2}\right)^{\frac{1}{2}}$ = the radius of gyration of the first pair, Neptune and Uranus;

$D_{(1)}, D_{(2)}$, &c. = the distances of Neptune, Uranus, &c., from the sun;
$r_{(2)}, r_{(3)}$, &c., = the radii of gyration of the successive binary rings;

Examination of this Hypothesis.

The values of $r_{(3)}$, k, and $D_{(5)}$ are thus computed: Having found

$$r_{(1)} = 25{\cdot}20061$$
$$r_{(2)} = 7{\cdot}68305$$
$$r_{(4)} = 0{\cdot}87270$$
$$d_{(1)} = 17{\cdot}51756$$

we have the equations

$$r_{(3)} = \left(\frac{D_{(5)}^2 + (1{\cdot}5236923)^2}{2}\right)^{\frac{1}{2}} \qquad (1)$$

$$(7{\cdot}68305 - r_{(3)})\,k = 17{\cdot}51756, \qquad (2)$$
$$(r_{(3)} - 0{\cdot}87270)\,k = 7{\cdot}68305 - r_{(3)}, \qquad (3)$$

whence

$k = 3{\cdot}34189$ $\qquad$ $\log = 0{\cdot}5239916$
$r_{(3)} = 2{\cdot}44123$ $\qquad$ $\log = 0{\cdot}3876087$
$D_{(5)} = 3{\cdot}09800$

This value of the distance of the asteroid planet is almost exactly identical with that found by my analogy.

The radii of gyration of the primitive rings, together with their differences, are as follows:—

$r_{(1)} = 25{\cdot}20061$	$d_{(1)} = 17{\cdot}51756$	$\log = 1{\cdot}2434736$
$r_{(2)} = 7{\cdot}68305$	$d_{(2)} = 5{\cdot}24182$	$\log = 0{\cdot}7194821$
$r_{(3)} = 2{\cdot}44123$	$d_{(3)} = 1{\cdot}56852$	$\log = 0{\cdot}1954906$
$r_{(4)} = 0{\cdot}87270$	$d_{(4)} = 0{\cdot}469350$	$\log = \bar{1}{\cdot}6714991$
$r_{(5)} = 0{\cdot}40335$	$d_{(5)} = 0{\cdot}140455$	$\log = \bar{1}{\cdot}1475076$
$r_{(6)} = 0{\cdot}26289$	$d_{(6)} = 0{\cdot}042026$	$\log = \bar{2}\,6235161$
$r_{(7)} = 0{\cdot}22086$	$d_{(7)} = 0{\cdot}012575$	$\log = \bar{2}{\cdot}0995246$
$r_{(8)} = 0{\cdot}20828$	$d_{(8)} = 0{\cdot}003763$	$\log = \bar{3}{\cdot}5755331$
$r_{(9)} = 0{\cdot}20451$	$d_{(9)} = 0{\cdot}001126$	$\log = \bar{3}{\cdot}0515416$
$r_{(10)} = 0{\cdot}20338$	$d_{(10)} = 0{\cdot}0003369$	$\log = \bar{4}{\cdot}5275501$
&c. &c.	&c. &c.	&c.
$r_{(\infty)} = 0{\cdot}20299$	$d_{(\infty)} = 0{\cdot}0000000$	$\log = -\infty$

Remarks.

1. The radius of gyration ($r_{(5)}=0{\cdot}40335$) of the fifth primitive ring corresponds very closely with the mean distance of Mercury. This planet, therefore, according to the hypothesis, ought to be an exception to the binary arrangement. Such, in fact, appears to be the case. Or, if the planet originally existed as a binary ring, the mean distances of the members having the same ratio to each other as those of Venus and the earth, both must have been included between the present limits of Mercury's orbit. The union of the two rings and the formation of a single planet may thus have resulted from the eccentricity of the primitive annuli.

2. The sum of the infinite series

$$d_{(1)}+d_{(2)}+d_{(3)}+\text{\&c.},=\frac{d_{(1)}k}{k-1}=\frac{17{\cdot}51756\times 3{\cdot}34189}{2{\cdot}34189}=24{\cdot}99762;$$

and $25{\cdot}20061-24{\cdot}99762=0{\cdot}20299=$ the distance from the sun's center to the limit at which the separation of solar rings must have ceased. This is immediately exterior to the present limit of equilibrium between the centripetal and centrifugal forces.

3. The fifth radius of gyration, according to this hypothesis, exceeds the present mean distance of Mercury by the quantity 0·01626, or about 1½ millions of miles. The corresponding difference in the period is about 5 days, or twice the amount by which the period of Encke's comet has been shortened in the last half century. Have the period and mean distance of Mercury been diminished, during the immensity of past time, from the same cause? Or may this slight and only exception to the strict accuracy of the law be referable to zones or groups of asteroids in the vicinity of Mercury's orbit, the existence of which has been indicated by the researches of Leverrier?[11]

[11] Runkle's Math. Monthly, vol. ii, p. 240.

4. The radius of gyration of the sixth primitive ring is 0·26289. This distance is nearly equal to that of Mercury's perihelion. Between this and the limit, 0·20299, the formula indicates the abandonment at the solar equator of an indefinite number of rings in close proximity to each other. The appearance of such zones or rings of nebular matter would be similar to that of the *zodiacal light.* This phenomenon was ascribed by Cassini to the blended light of an innumerable multitude of extremely minute asteroids revolving round the center of our system. Recent observations, it is true, have suggested the hypothesis that the appearance is produced by a *terrestrial* ring; but in opposition to this view, Prof. Barnard has adduced a number of weighty, if not insuperable objections. The greatest elongation of these rings of nebular or meteoric matter, when abandoned at the solar equator, would certainly be much less than that of the vertex of the zodiacal light; but is it not possible that, as in the case of several comets, the orbits of some may have been greatly changed by planetary perturbations?

Application to the Secondary Systems.

I.—THE SATELLITES OF SATURN.

The distances of the satellites of Saturn, in radii of the primary, are as follows:—

I.	Mimas,	3·1408
	Enceladus,	4·0319
II.	Tethys,	4·9926
	Dione,	6·399
III.	Rhea,	8·932
	———	
IV.	Titan,	20·706
	Hyperion,	25·029
V.	———	
	Iapetus,	64·359

This table is taken from *Loomis's Practical Astronomy.* The distances, we are informed by the author, "were derived chiefly from Mädler, modified in some instances by comparison with Herschel's Astronomy and Hind's Solar System." A chasm in the order of distances is observed between Rhea and Titan, and another between Hyperion and Iapetus—that is, immediately interior to the largest two members of the system. Now, as in the primary system the zone of asteroids occurs just within the powerful mass of Jupiter, supplying the missing term noticed by Kepler and Bode, is it not probable that similar *secondary* zones exist in those intervals? By interpolating these two satellites or asteroid zones, we obtain a series of ten terms, in which the eight known distances are represented *with perfect accuracy.* It is also remarkable that the limit of the ring-forming process, according to this series, is precisely where Bond's ring is situated, between the body of the planet and the inner bright ring. The interpolated distances are 12·43 and 35·32 respectively. The radii of gyration of the pairs, together with their differences, are as follows:—

Names of Satellites.	Rad. of Gyr	Differences.
I. Mimas and Enceladus,	3·61	2·13
II. Tethys and Dione,	5·74	5·08
III. Rhea and ——	10·82	12·13
IV. Titan and Hyperion,	22·95	28·96
V. —— and Iapetus,	51·91	

These differences form a geometrical series whose ratio is 2·385. The sum of the series = 50·59, which substracted from 51·91 gives 1·32 as the limit. The series indicates the abandonment of an indefinite number of satellites or rings in close proximity to each other, between Mimas and this limit. Now Professor Vaughan has shown that neither a gaseous nor a liquid satellite, of considerable magnitude, would be stable so near the primary.[12] Hence the probable origin of Saturn's rings.

II.—THE SATELLITES OF JUPITER.

If we suppose the third and fourth satellites of Jupiter to have originally constituted one ring, or system of associated rings, the first and second, another, and that the ratio of the differences of the radii of gyration was the same as in the primary system, we shall find

$r_{(1)} = 21{\cdot}960$ equatorial radii of the planet,
$r_{(2)} = 8{\cdot}036$ " " "
$d_{(1)} = 13{\cdot}924$ " " "
$d_{(2)} = 4{\cdot}166$ " " "
$\frac{d_{(1)}k}{k-1} = 19{\cdot}869$ " " "

and $21{\cdot}960 - 19{\cdot}869 = 2{\cdot}091$ = the distance from the center of the primary, within which no rings could have been formed.

This again is nearly equal to the distance (2·299) of the point of equilibrium between the centripetal and centrifugal forces.

But we discover no decided indications of the binary arrangement in the Jovian system. Is a similar relationship then to be found between the respective distances of the satellites themselves? "The four satellites of Jupiter," says Humboldt, "present a certain regularity in their distances, forming nearly the series, 3, 6, 12. The distance of the second from the first, expressed in diameters of Jupiter, is 3·6; the distance of the third from the second, 5·7; and that of the fourth from the third,

[12] Proc. Am. Assoc. for the Adv. of Sci., 1856, p. 111.

11·6." This would indicate a value of k for the Jovian system equal to 2; $r_{(\infty)} = 3$; and the possible separation of secondary asteroids between the limit 3 and the distance 4·5.

Our knowledge of the Uranian system is perhaps too imperfect to justify any conclusion in regard to the prevalence of a similar order.

In researches of this nature, the want of *exact* numerical verification ought not to be regarded as decisive evidence against the truth of an hypothesis. A partial deviation may be produced by the interference of some other law or arrangement. Of this we have innumerable instances in nature. Even the celebrated laws of Kepler, as commonly stated, are not strictly true; the mutual attractions of the planets producing endless perturbations.[13]

III.—The Mean Distances of the Periodic Comets, and their Relation to the Solar System.

The celebrated Laplace remarked that, according to the nebular hypothesis, "the comets do not belong to the solar system." He regarded them as small nebulæ which, wandering through space till they come within the sphere of the solar influence, enter our system from *without*, pass around the sun, and, unless influenced by the attraction of the planets, or the resistance of the ethereal medium, again pass off in parabolas or hyperbolas. Other astronomers believe them to have originated within the solar system. Perhaps each view may be partially correct. Several comets, among which we may instance that of June, 1861, have moved in hyperbolic orbits. These, together with many whose orbits seem to be parabolas, have probably entered the system *ab extra*. On the other hand, a large majority of *periodic* comets are believed to have originated *in* the system, and to belong properly to it. The author several years since called attention to the fact that there is an approximate coincidence between the planetary and cometary periods.[14] There are 13 known

[13] The idea of employing the radii of gyration of the planetary pairs, occurred to the writer in 1852. Various hypotheses involving this element were tested in the search for the law of distances; but the fact that the preceding formula gives the radius of gyration of the fifth planetary pair *greater* than the mean distance of Mercury, seemed, it was thought, sufficient ground for its rejection. At the Albany Meeting of the American Association for the Advancement of Science, in 1856, a paper was read by Professor Stephen Alexander, "On some Special Arrangements of the Solar System, which seem to confirm the Nebular Hypothesis." In this memoir, Professor A. likewise employed the radii of gyration of the planetary pairs; his method of using them, however, was different from the author's. This paper induced the writer to resume the subject. The applicability of a similar formula to the Saturnian system was regarded as confirmatory, and, finally, as it seemed possible to account for the slight inaccuracy in the case of Mercury, the striking conformity of the theory with facts has determined the author to submit his results to the public.

[14] Proc. Am. Assoc. for the Adv. of Sci., 1858, p. 10.

comets whose periods are included between those of Mars and Jupiter. Their motions are all direct; their orbits are less eccentric than those of other comets; and the mean of their inclinations is about the same as that of the asteroids. The perihelia of 5 are exterior to the earth's orbit, and the nearest approach of Faye's to the sun is several million miles beyond the orbit of Mars. In fact, there is less difference between the eccentricity of the orbit of Faye's comet and that of some of the asteroids, than between the latter and that of some of the old planets; so that this body may be regarded as a connecting link between planets and comets. These facts appear to indicate some connection in their origin with the zone of asteroids.

Since the commencement of the present century, five comets have been discovered, which form, with Halley's, an interesting and remarkable group. The first of these was detected by Pons, on the 20th of July, 1812; the second by Olbers, on the 6th of March, 1815; the third by DeVico, on the 28th of February, 1846; the fourth by Brorsen, on the 20th of July, 1847; and the last by Westphal, on the 27th of June, 1852. The periods of these bodies are all nearly equal, ranging from 68 to 76 years; their eccentricities are not greatly different; and the motions of all, except that of Halley, are direct. The existence of these two cometary groups was noticed several years since both by Hind and Alexander. The latter supposes the cluster whose times of revolution are nearly equal to the period of Uranus, to have had a common origin. He infers from various facts that in the early part of the fourteenth century a large comet approached very near to Mars, if indeed there was not an actual collision between the two bodies. This ancient comet he supposes was thus separated into fragments. That most, if not all, of this cometary group have had a common origin, we regard as highly probable: we doubt, however, whether the true explanation of that origin has yet been proposed.

Again: the comet discovered by Peters on the 26th of June, 1846, has a period, according to the discoverer, of about 13 years; and Tuttle's comet (1858, I.) completes its revolution in 13·6 years. The perihelion of each is exterior to the earth's orbit, and their motions are direct. The periods of these bodies are a little greater than that of Jupiter. It may also be remarked that the comet which passed its perihelion on the 28th of November, 1793, has, according to Burckhardt, a period of 12 years. The period of the great comet of 1843 is probably nearly the same with that of Neptune. Other coincidences might be pointed out, but the periods in most cases are too doubtful to be relied upon. Those which we have adduced seem to point to an approximate coincidence between the mean distances of the planets and those of the periodic comets.

May not the exterior secondary rings, thrown off by the planets, have been at too great a distance to form *stable satellites?* and in such case would not the detached portions of matter revolve round the sun in very eccentric orbits, the degree of eccentricity depending on the direction of their motion at the epochs of separation from the secondary system? If so, the approximate coincidence between the periods of planets and comets would follow as a consequence.[15]

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