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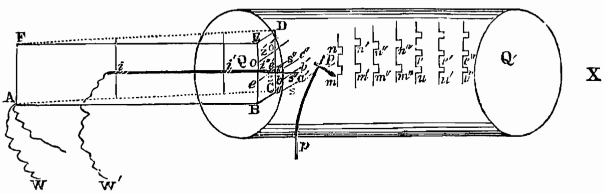
ART. XXV.—*On the Variability of Personal Equation in Transit Observations;* by WILLIAM A. ROGERS, Director of Alfred University Observatory.

THE personal equation of an observer is the interval of time which habitually intervenes between the actual and the observed transit of a star over a given thread of the transit instrument. This may be termed the absolute personal equation.

The relative personal equation of two observers is the interval of time by which one habitually observes a transit either earlier or later than the other; or, it is the difference between their absolute equations.

It is well known that personal equation is one of the most uncertain elements in chronographic determinations of longitude. It has been the custom, at least in this country, in observations for longitude, to assume that the value of this function of the time remains unchanged during the entire series, even though the comparisons for the determination of this element were delayed for several weeks. Now if the personal equation of the observers is not a constant quantity, if it can be shown that from any cause it varies from week to week, from day to day, and even from hour to hour, between limits nearly as large as the usual value of the function itself, it will be evident that some uncertainty yet remains in the accepted values of longitude, especially since in almost every instance, the observations for longitude and personal equation have been separated by a considerable interval of time. The investigation of the variability of personal equation is, therefore, not an idle inquiry.

The results on which this discussion is founded depend upon about 8,000 observations of artificial stars. In the present instance, the artificial stars were made of paper and centered upon fine steel wires placed in a vertical position. Now it is evident that if the wire to which the star is attached, could be made to pass a stationary vertical wire with a uniform motion, and if the exact time of opposition could be automatically recorded, we should have a standard with which the observed time of passage could be compared. The following device was employed to accomplish this object.



Let $Q\,Q'$ in the figure represent the cylinder of a Bond chronograph. At one end of the cylinder and attached to the table upon which the chronograph rests, is placed a wire frame work ABCDEF, in such a position that the vertical face BCDE shall be parallel with the plane of the end of the cylinder and in close proximity to it. ee', oo' are two vertical threads of fine wire. aa', bb', cc', are fine steel wires placed vertically at the circumference of the base Q. Upon these wires are centered the stars $s\,s'\,s''$. $i\,i'$ is a wire insulated with respect to the frame work at i and i' but in contact with it at z. To the end of this wire is attached a fine needle $i''\,i'''$ projecting beyond the plane of the wires on the cylinder just far enough to come in contact with them as they revolve. The wires $w\,w'$ run to a battery after passing through a coil (not represented in the figure) with which the pen $p\,p'$ is connected.

As the cylinder revolves, the instant $b\,b'$ arrives opposite $e\,e'$ it breaks the connection at z and the circuit continues broken till the needle springs back upon ee'. During this interval a spring attached to $p\,p'$ moves it horizontally, making a break at m. The beginning of the break is the instant of opposition. At another revolution another break is made at m', and we thus have the instant of opposition automatically recorded upon the line $m\,m'''$. Suppose an observer standing at X with a break-circuit key, to *observe* through a small telescope, the instant of conjunction between the fixed wire $o\,o'$ and the movable one $b\,b'$. By breaking the circuit at that instant, a break is made at n, depending only on the judgment of the observer. *Now, if the measured interval between the two fixed wires* $e\,e'$ *and* $o\,o'$ *is* *equal to the interval between m and n, expressed in the same unit, there is no absolute personal equation. If m n is less than z z′ the observation is too early, if greater, it is too late, in either case by the difference between z z′ and mn.*

In order to find the common unit of measurement, the galvanic connection is made through the clock; then every alternate swing of the pendulum gives the constant spaces *u r*, *u′ r′*, *u″ r″*, each equal to two seconds of time. In the present instance this space was divided upon mica into twenty equal parts by the aid of a filar micrometer screw. By placing the scale upon *m n*, the distance from *m* to *n* is measurable to $\cdot1^s$ and by estimation to $\cdot01^s$. The value of *z z′* adopted in this discussion after numerous and careful measurements upon different parts of the scale, is $\cdot77^s + \cdot015^s$. Hence whenever the interval between the beginning of the automatic break and that made by the observer is less than $\cdot77^s$ the observation is too early, and when greater, it is too late. The difference, either way, is the absolute personal equation. If there are two observers, their relative personal equation is easily found, from the absolute equation of each.

In entering upon this discussion the first and the main question is:

Does the personal equation vary from any cause?

As the basis of investigation, I give below the mean of the values found for each date, with explanatory remarks concerning the conditions under which the observations were made. By an abnormal position of the body, I mean as painful a position as I could assume. The wires were illuminated by placing a light nearly in front of them. Faint illumination was produced by reducing the volume of the flame.

The observations were made by Prof. Edward M. Tomlinson, Mr. Herbert E. Babcock, and myself.

Absolute Personal Equation.

ROGERS.

Date. 1867.	Equation.	Remarks.
Nov.		
19	=−·118s	
20	·093	
21	·057	—Read the record for Nov. 19
22	·022	and 20. First knowledge of
23	·088	the value of my personal
24	−·047	equation. Observations from
25	+·025	Nov. 19 to Nov. 29 were
26	·019	made under a normal condi-
27	·034	tion of the body.
28	·019	
Nov.		
29	+·017	—Read up all the preceding records.
Dec.		
5	−·067	Normal position.
5	·032	Abnormal position.
6	·038	Normal.
6	·037	Abnormal.
7	·042	Normal.
7	·040	Abnormal.
8	·042	Normal.

Date. 1867.	Equation.	Remarks.
Nov.		
8	—·040[s]	Abnormal.
12	·047	42°=read'g of thermometer.
Dec.		
12=	·083s	−4°. Hands protected with cotton gloves.
12	·030	−4°. Hands unprotected.
12	·080	−3°. Hands protected.
12	·072	−3°. After warming; hands unprotected.
12	·061	42°. Regular beat of chronograph pendulum.
16	·024	34° " " "
16	·040	35°. Irregular beat.
16	·061	35°. Regular beat.
16	·054	9°. Normal.
16	·039	38°. Normal.
16	·036	38°. Abnormal.
22	·034	Bright wires.
22	·061	Very faint wires.
24	·032	Bright wires.
24	·053	Moderately faint wires.
24	·023	Bright wires.
28	·040	Bright wires.
28	·039	Faint wires.
28	·023	Bright wires.
1868.		
Jan.		
13	—·057	Bright wires.
13	·118	Very faint wires.
13	·061	Bright wires. Abnormal.
13	·065	" " Normal.
13	·078	Dark field.
13	·087	Normal.
Feb.		
10	·075	Thermometer =38°.
10	·067	" — 3°.
22	·069	" 34°.
22	·039	" — 8°.
22	·035	" 34°.
April.		
27–8	—·040	Observed from 8 P. M., Apr.
27–8	·045	27, till 5h A. M. Apr. 28, at
27–8	·077	irregular intervals; the in-
27–8	·069	tervening time being occu-
27–8	·049	pied with observations in the prime vertical for latitude.
27–8	·041	Very tired and sleepy.
27–8	·029	Time of observation, 8h A.M. Slept from 5h till 8h A. M.

Date. 1868.	Equation.	Remarks.
April.		
27–8	—·033[s]	Time, 9h A. M.
27–8	·051	Time, 9½h A. M.
27–8	·011	Time, 2h P. M. Slept from 12h till 2h.
27–8	·051	Time, 7h P. M. Engaged in severe exercise from 4h till 7h.
May.		
26	—·042	Observed by day-light.
26	·041	" " lamp-light.
26	·050	Severe physical exercise between these observations.
26	·032	Very tired.
31	·051	Day-light.
31	·040	Lamp-light.
31	·019	After observation with the equatorial for relative equation. Tired.
June.		
2	—·045	Day-light.
2	·019	Lamp-light.
2	·034	Day-light.
2	·045	Lamp-light.
4	·033	Observed with the equatorial from 8h P.M. till 11h. Slept
4	·040	from 11h P. M. till 1h 30m. A. M. Watched with the sick
5	·027	till 5 A. M. Time of observation, 5h 30m A. M.
9	·033	Normal.
15	·024	Slept very little June 14. Slept from 1h P. M. till 3h.
15	·018	Time of observation 3h P. M.
17	-·055	After severe exercise.
22	·053	Normal.
26	·045	Normal.
26	·046	Normal.
26	·019	*Assumed* that I observed too late. No knowledge of the
27	·008	value of my equation.
27	·028	Normal.
28	·040	Normal.
28	·013	Hungry.
28	·010	
28	·018	
28	·007	Ate nothing for 30 hours.
28	·007	
28	.006	
28	·007	
28	—·001	*Very* hungry.

Tomlinson.

Nov. 19=	+·096[s]	Nov. 24=	+·149[s]
20	·116	25	·180
21	·116	26	·204
22	·139	27	·179
23	·167	28	·163
29	·213	June 15	·027

Babcock.

Nov. 21=	+·035[s]
23	+·013
24	−·007
25	·000
26	+·011
27	+·018

Feb. 10	=+·036s	June 15	+·047s	Nov. 29	—·030s
10	—·037	17	·055	Feb. 10	·035
Apr. 27	+·047	22	·032	10	·143
May 31	·062	26	·036	Apr. 27	·071
June 2	·034	27	+·047	27	—·137
4	·054				
9	+·070				

Relative Personal Equation.

ROGERS minus TOMLINSON=
R—T.

Nov. 19	=—·214s	Apr. 27-8	=—·098s
20	·209	May 31	·113
21	·163	31	·102
22	·161	31	·081
23	·255	June 2	·079
24	·196	2	·053
25	·155	2	·068
26	·185	2	·079
27	·145	4	·087
28	·144	4	·094
29	·196	4	·081
Feb. 10	·111	9	·103
10	·030	15	·061
Apr. 27-8	·087	15	·065
27-8	·092	17	·110
27-8	·124	22	·085
27-8	·116	26	·081
27-8	·096	26	·082
27-8	·088	26	·055
27-8	·076	27-8	·055
27-8	·080	27-8	·075
27-8	·098	27-8	—·087
27-8	—·058		

ROGERS minus BABCOCK=
R—B.

Nov. 22	=—·112s
23	·101
24	—·040
25	+·025
26	+·008
27	·016
29	+·047
Feb. 10	—·040
10	+·076
Apr. 27-8	·031
27-8	+·026
27-8	—·006

It will be evident from an examination of the values given above, that personal equation is a varying quantity, if it can be shown that the variation exceeds the probable error of observation. Without going into details, I give below the value of the probable error from each source, depending upon a sufficiently large number of observations:

I. Probable error of observation for each star—
ten revolutions, R=±·013s
T=±·017
B=±·016

II. Probable error of each reading on the scale =±·02

III. Total error derived from a change in the common unit of measurement as affected by a variable beat of the chronograph pendulum, (estimated), =±·02

IV. Error of centering a single star,	$=\pm$·02 (estimated).
V. Error arising from the condition that the revolving pins may not have been in a vertical plane with the stationary ones at the instant of conjunction,	$=\pm$·005 (estimated).
VI. Parallactic error, arising from the parallax of the two wires at the instant of observation.	Inappreciable, the revolving wires almost touching the fixed one at their transit.

With regard to these errors it is to be observed:

(*a.*) Since there were an average of 8 artificial stars attached to the cylinder, and the observations were on the average continued through 10 revolutions, each result depends on about 80 observations. The probable error of observation and reading for the final result must therefore certainly be less than $\pm$·01^s. No allowance has been made for the third source of error. While it was not difficult to detect the error itself by a change in the measured length of the comparison unit, it was so variable in amount between the extreme limits +·02^s and −·02^s, that I did not find it possible to deduce a definitive mean value. I am confident, however, that the final result for any date cannot be affected with so large an error as $\pm$·005^s.

(*b.*) Errors IV and V affect the absolute equation; but in considering the *variability* of this function, they are to be disregarded, since the wire frame work, the wires and the stars, remained absolutely in the same position from day to day, unless purposely disturbed. So also, these errors are eliminated, if the observations are for relative personal equation.

It is therefore obvious that the change manifest in the value of the personal equation, whether absolute or relative, cannot be accounted for either as instrumental errors, or as errors of observation, but must be due to the external conditions under which the observations were made. Having determined the fact of the general variability of the personal equation, let us now consider the variations due to certain given physical conditions.

I. *Does the personal equation vary between a normal and an abnormal position of the body during observation?*

Normal position minus Abnormal.

R.—Dec.	6	= −·031^s	
	7	−·002	
	8	−·002	
	16	−·003	
Jan.	13	−·004	
	13	−·026	Mean = −·011^s

Thus, while the change is not large, every series of observations gives the same sign. It will not answer, however, to assume, either that the mean value remains constant, or that another observer would find the same value, since several conditions contribute to the result found. In reading up the records, the rather curious fact was noticed, that the probable error of observation was less for an abnormal than for a normal position of the body.

II. *Does a change of temperature affect the personal equation?*

Absolute.

Date		Observer / Value
		R.
Dec.	12	$42° - (-4°) = +{\cdot}036^s$
	12	$42° - (-4°) = -{\cdot}014$
	12	$42° - (-3°) = +{\cdot}019$
	12	$42° - (-3°) = -{\cdot}011$
Jan.	13	$35° - (-9°) = -{\cdot}007$
	13	$38° - (-9°) = +{\cdot}015$
Feb.	10	$38° - (-3°) = -{\cdot}008$
	22	$34° - (-8°) = -{\cdot}030$
		T.
Feb.	10	$38° - (-3°) = +{\cdot}073$
		B.
Feb.	10	$38° - (-3°) = +{\cdot}107$

Relative.

Feb. 10.

	Temp.	Value	Change.
B.—T.	$38°$	$= +{\cdot}071^s$	
	$-2°$	$+{\cdot}105$	$\cdot 034^s$
R.—T.	$38°$	$= +{\cdot}108^s$	
	$-2°$	$= +{\cdot}028$	$\cdot 080$
R.—B.	$38°$	$= +{\cdot}037^s$	
	$-2°$	$= -{\cdot}077$	$\cdot 114$

The observations for ordinary temperature were made in the clock-room. For the low temperature observations, an aperture about 1·5 inch in diameter was made through a pane of window glass and the theodolite was placed on the outside. The first and third set of observations for Dec. 12 were made with gloved hands, the second and third with hands unprotected. It will be seen that in my own case, the change is slight, while with B. and T. it is large. The values depend upon very careful observations continued through 20 revolutions of the cylinder. The probable error for high and low temperature did not sensibly vary with myself and B., but was about $\pm{\cdot}005^s$ larger for low temperature in the case of T.

III. *Does an exhausted state of the system produce a variation of the personal equation?*

It will be seen from the observations of Apr. 27–8, May 26–31 and June 4–5–15–17, that no decisive change resulted from extreme weariness. The mean effect was a slight tendency to diminish the equation by an amount hardly measurable with certainty. This result was contrary to my anticipations, and if confirmed by other observers, establishes a fact of much importance, inasmuch as astronomical observations are usually carried far into the night.

In every instance, the equation was quite largely diminished when the observation was made directly after waking from a sleep preceded by extreme exhaustion. Here, as before, I found that an abnormal condition rather improved the probable error of observation.

IV. *Does hunger affect the value of the personal equation?*

Normal state minus a state of hunger.

June 27–8 =	—·027^s
	·030
	·022
	·032
	·034
	·034
	·034
	·034
	·033
	—·039

There is thus a decided and quite regular change, the mean being —·032^s.

V. *Does the mental state of the observer have any influence on the personal equation?*

Normal state minus a state in which the observation is *assumed* too late.

Nov. 20–1 =	—·036
June 26	—·026
27	—·038

I have already remarked that I obtained the first knowledge of the value of my personal equation, Nov. 21, 1867, and that after Nov. 29 I had no farther knowledge of its value, till after all the observations were completed, not in fact till I read up the records in July, 1868. As the value for Nov. 29 was positive, I *arbitrarily assumed* it negative in order to ascertain the effect of this assumption upon it. The result confirms the suspicions which I have for some time entertained, that the simple knowledge of the value of one's personal equation induces a tendency to reduce its value. Since constancy in value is more desirable than this reduction, which is uncertain and variable, it follows that it may not be best for an observer to have a knowledge of his personal equation. I ought to remark, however, that the *expectation* of having my suspicions on this point confirmed, *may* have had something to do with the results found, though I endeavored to free my mind from every bias except the single one assumed.

Let us now consider certain variations of the personal equation in which the condition of the observer is not taken into account.

I[1]. *Does a change in the character of the illumination of the wires affect the personal equation?*

Bright wires minus faint wires.

Dec. 22=	+·027^s;	very faint.
24	+·024;	faint.
28	{ —·001 +·016 ;	moderately faint.
Jan. 13	+·060;	extremely faint.

II[2]. *Does the personal equation vary between a natural and an artificial illumination of the wires?*

Natural minus artificial.

May 26=	—·002^s
31	—·011
June 2	—·026
2	—·011
2	—·000

These observations were made during the day time. The artificial illumination was produced in the way already described, the clock-room being darkened.

III[2]. *Does the size of the stars observed affect the personal equation?*

From Nov. 20, till Nov. 29, only five stars were employed, the first one being larger than the others.

R.—T. from Nov. 20 till Nov. 29.

Large star.	Small stars.
—·233^s	—·144^s—·167^s—153^s—166^s

Large star minus mean of small stars.

—·233^s—(—·152^s)=—·081^s.

First wire minus each of the following wires from Nov. 20 till Nov. 29.

$$\text{T.}\quad \frac{+\cdot052^s+\cdot049^s+\cdot034^s+\cdot053^s}{4}=+0\cdot47^s \qquad \text{R.}\quad \frac{-\cdot040^s-\cdot022^s-\cdot049^s-\cdot033^s}{4}=-\cdot036^s$$

Difference =R − T=—·083^s.

I must remark, however, that I did *not* find a well-defined corresponding difference between bright and faint stars in the observations for relative personal equation with the equatorial.

IV[2].—*Does a variation of the interval between the wires affect the personal equation?*

From May 26 till June 27, ten stars were attached to the cylinder, the first three being separated by an interval of from ten to fifteen seconds, and the last seven by an interval varying between two and three seconds.

R—T.

	Mean of the first three wires.	Mean of the last seven wires.
May 26	=—·201^s	—·068^s
June 2	·080	·075
4	·086	·088
9	·187	·059
15	·116	·052
17	·155	·091
22	·152	·051
26	·122	·064

	Differences.	
May 26	=—·133^s	
June 2	·005	
4	·000	
9	·128	Mean =—·069^s
15	·064	
17	·064	
22	·101	
26	·053	

While these differences do not agree well with each other they all have the same sign. The disagreement is without doubt partly due to certain local disturbances on some of the wires, which will be noticed under the next head.

V.—*Does the shape of the stars observed affect the personal equation?*

By comparing the values R.—T. for each star, I found in several instances that two and sometimes three stars gave results widely differing from the rest, for two or three days at a time, after which the difference would disappear. As these differences did not occur in the values R.—B., it is evident that the variations were mainly due to the observations of T. I can attribute this to no other cause than the influence which the shape of the stars had upon the judgment of the observer in determining the time of transit. In actual observations of bright stars there is no doubt but that projecting wisps of light affect the personal equation. Something analogous to this may have occurred in the present instance. I can assign no reason for the recurrence of the disturbances.

I close this investigation with the following inquiry:

Does the relative equation derived from artificial stars agree in value with that derived from actual observations with the transit instrument?

This inquiry is an important one; for if this agreement is found to exist, it will then be easy to free longitude from the error arising from the variability of personal equation, by determining the value of this function by means of artificial stars, the observations for this purpose being nearly coincident with those of longitude.

The observations of actual stars were made with the equatorial, four, of four wires, constituting a complete set. Thus:

R	observed	the	first	four	wires.
T	"	"	last	"	"
R	"	"	"	"	"
T	"	"	first	"	"

Reducing each observation by the interval from each wire to the mean of the wires, and taking the arithmetical mean of the results, the relative equation was found free from the error of wire intervals.

R.—T. from the equatorial.		R.—T. from artificial stars.
May 31	=—·08s	—·113s
June 2	·11	·079
4	·15	·087
9	·16	·103
15	·09	·071
17	·06	·110
22	·16	·085
Mean from 880 comparisons,	=—·116s.	Mean from 1,000 comparisons, =—·093s

I do not positively assert that this agreement will exist in every case, because—

(*a.*) The result might have been different had the observations been made with the transit instrument.

(*b.*) The same agreement might not exist with other observers.

I do not consider that the results which I have found, settle definitively any point except the general variability of the personal equation. The conditions of the problem are so complex that it is impossible to assume one condition and reject the consideration of all others. The last inquiry is worthy of further investigation, as affording the means of obtaining the most probable value of the relative equation.