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ART. XXX.—*On the Musical Ratios, and our Pleasure in Harmonious Sounds;* by HENRY WARD POOLE.

UTILITY, in a liberal sense, may be assumed to be the goal of scientific investigations. We should all rejoice if the discoveries in heat should lead to material results in making available those immense forces which are said to result from the burning of a pound of hydrogen or of coal. It would be gratifying if the laws of light shall be so far understood, that we can have photographs fixed with the colors shown in nature. In the department of sound, utility seems to be almost entirely confined to the musical department, where complicated vibrations and relations are excluded, and only the simple and regular are admitted. And here, strange to say, in all times, the practice has been in advance of the science.

Singers and players have used, and still are using, intervals which please themselves and their auditors, while the scientists have not known that they used them, and have declared that really harmonious sounds are discordant and harsh.

To illustrate this assertion, I take the last and most complete work on Sound* that I have seen. The excellence of other parts of this work, and the high merits of the author, make it noteworthy that after such minute experiment with metal and wooden rods, disks, singing flames, etc., the higher department should have received so little attention.

* Sound. A course of eight lectures delivered at the Royal Institution of Great Britain, by John Tyndall, LL.D., F.R.S. London. 1867.

But although every singer and musician may have his inner revelation of what musical intervals really are, it is important that they should be understood and acknowledged by the scientific, and the teachers of music.

It is not difficult to find a pair of children, or a quartette of friends, who will sing in perfect tune. But in nearly every church and music room is set up an organ with but twelve pipes in each octave, which can not make a perfect chord, or one which would be tolerated by the vocalists referred to. What is the fault of these instruments? Can instruments be made and played in perfect tune? First of all, what are the notes, the scales, the harmonies, which composers write and artists execute? These are some of the questions whose resolution leads to useful results. The ignorance of teachers leads to the mischief, that they spoil good singers, by compelling them to take intervals from imperfect instruments. An example will show what I mean. Nothing is more common than to sing a seventh, ratio 4 : 7, on the dominant harmony, concordant, and as any one would wish to give it. The theoretical books would have this given as 9 : 16, or about a seventh of a tone too high. A teacher by the rude instruments now in use, would require the pupil to sing the interval with something like this error. The effect on a good singer is to deaden his sensibility and consequently spoil him. It is remarkable that this interval of 4 : 7 is not recognized by Professor Tyndall, who has, apparently, aimed at giving what is known up to the present time. That such an interval is used and is harmonious has been published in this Journal, eighteen years since, and elsewhere.

Euler gives a good theory, which Professor Tyndall quotes as follows.

> Euler analyses the cause of *pleasure*. We take delight in *order*. It is pleasant for us to observe 'means co-operant to an end.' But then the effort to discern order must not be so great as to weary us. The simpler the terms in which the order expresses itself, the greater is our delight. Hence the superiority of the simpler ratios in music over the more complex ones. Consonance, then, according to Euler, was the spiritual pleasure derived from the perception of order without weariness of mind.

I approve of Euler's explanation but not of Professor Tyndall's statement. I understand that the musical ratios collectively are the simplest of the mathematical ratios, and that these are *equally and entirely perfect and agreeable when duly used in music.*

But our author states that the unison, 1 : 1, is the most perfect. Next comes the octave, 1 : 2, still very harmonious.

Next, the fifth, 2 : 3, is "very pleasing, but the consonance is hardly so perfect as in the last instance; there is a barely perceptible roughness here. Next to the octave, this is the most pleasing combination." The "roughness of the fourth, 3 : 4, is a little more pronounced." The harmony of the major third 4 : 5, is declared "less perfect," and that of the minor third, 5 : 6, "usually [?] less perfect still." And he declares the law that "*the combination of two notes is the more pleasing to the ear, the smaller the two numbers which express their vibrations.*"

No ratio higher than 5 : 6 is mentioned, except 13 : 14, which is declared "altogether discordant." As a reason, it might be stated that the prime 13 is far beyond the limit of musical ratios. The ratios 6 : 7, 5 : 7, 7 : 9, and all reference to the chords derived from seven, are omitted by Professor Tyndall in this course of lectures. But he says that musicians have chosen the simple intervals "empirically, and in consequence of the pleasure they gave, long before any thing was known regarding their numerical simplicity."

If musicians have chosen the most simple intervals as most pleasing, it is not true that they have considered fifths as inferior to octaves, or the latter as less pleasing than unisons. All the intervals named, and those of the harmony of the seventh with their inversions and derivatives, within due limits and properly combined, are entirely perfect and equally pleasing, one with the other. Beyond these limits, all are incomprehensible and alike unharmonious. Theorists have improperly classed fifths as "perfect concords," thirds, as "imperfect concords," and worst of all, the beautiful concord of the seventh, 4 : 5 : 6 : 7, is called a "discord." It is difficult to say why such distinctions were made, unless they came from the imperfect tuning of the keyed instruments which these theorists used.

It appears that Professor Tyndall treats the coincidences of harmony and those of discordant vibrations under the same term; viz., "beats." And, following Helmholtz, he says that beats of 33 per second, are in their "condition of most intolerable dissonance;" and that "with higher rates than 33 the roughness lessens until at 132 it totally disappears." He then examines the several chords, as follows, beginning with the octave:—

Here our rates of vibration, are

512—256; difference=256.

It is plain that in this case we can have no beats, the difference being too high to admit of them.

Let us now take the *fifth*. Here the ratios of vibration, are

384—256; difference=128.

This difference is barely under the number 132, at which the beats vanish; consequently the roughness must be very slight indeed.

Taking the *fourth*, the numbers are

384—312; difference=72.

Here we are clearly within the limit where the beats vanish, the consequent roughness being quite sensible.

Taking the *major third*, the numbers are

320—256; difference=64.

Here we are still further within the limit, and, accordingly, the roughness is more perceptible.

Thus we see that the deportment of our tuning forks is entirely in accordance with the explanation which assigns the dissonances to beats. pp. 299–300.

There is an evident error in the numbers assigned to the fourth. As four tuning forks are mentioned, they are probably those of the key-note, its octave, fifth, and major thirds. The interval of the fourth, then, would be obtained by sounding the fifth with the octave; giving

512—384; difference =128

which would give to the fourth the same "roughness" as to the fifth.

If coincidences in the vibrations of simple chords be what is meant by "beats," it would be easy to show that 33 such beats per second are not necessarily disagreeable. It is stated that 33 vibrations give a "perfect musical tone." If we sound with this its *octave* with 66 vibrations, we shall have a still more agreeable musical effect, as always results from such addition to a very low note—and there will result just 33 coincidences, or so called beats. But if we should add a fifth, with 49·5 vibrations, the effect would not be so good. A major third, with 41·25 vibrations would be intolerable, yet the "beats" would be just 33 in each case. Why is this difference? The answer is found by examining the harmonic series:

$$1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 : 10.$$

It is found that the several prime chords, fifth, third, etc., are most sonorous and agreeable when taken in the relative position as above. Assuming that the lowest sound has 33 vibrations per second, its octave follows with 66, the fifth to the last 99, the major third, to the double octave, with five times 33, or 165, and the seventh with seven times 33 or 231. The rule being as follows:

The vibrations of the lowest notes which it is permissible to use in the relations of fifth, major third, or seventh, are in the relations respectively, as the numbers 3, 5, 7. The lowest audible sound may be the limit for sustained or final chords, and if so, the fifth must have at least three times its number of vibrations, the third five times, and the seventh seven times. At all events these different chords will always keep these relative positions, or a fifth can always be sounded lower than a third, and both lower than a seventh; which accords with the practice of musicians. For certain melodic movements, it would seem as if all these intervals can be taken about an octave lower than the limiting note of 33 vibrations would permit. Can not the vibrations of the octave below, or $16\frac{1}{2}$, be *felt* in union with the octave and other harmonics above, and in this way be considered as audible? For the musical ear hears only what is plain enough to be understood, and relations too remote are often made clear and agreeable by the addition of other sounds. So a note in combination may be audible, which alone is not so.

Composers know the variety and advantages which result from taking chords in their various inversions, positions and distributions; but it is certain that the harmonic series shows the best order and the proportion of each element admissible. Thus, before the seventh is sounded, we have the fundamental note in three octaves, the fifth in two, and the major third only in one.

Professor Tyndall says much about the harmonics, which he calls "overtones." He says:

> And be it remarked, that the overtones are indispensable to the character of musical sounds. Pure sounds, without overtones, would be like pure water, flat and dull.

I believe that pure water is good, either to use in its purity, or to mingle with other elements at pleasure, and that pure sounds are most desirable in singers and in instruments. It was a triumph when the piano-forte was made to give less of the jangling harmonics, and more of the pure fundamental tone of the string. He says that in the organ, the overtones are so much felt to be a necessity, that they are introduced by small pipes, that "in fact, a good musical clang requires the presence of several of the first overtones."

It is asserted by Professor Tyndall that the vowel sounds are due to accompanying harmonics, and that the quality of sounds is also owing to their distinctive harmonics. That a flute sounds differently from a violin is owing to the fact that every flute has certain harmonics, while a violin has others. But one violin sounds different from another, and the material of a tube alters its quality of tone. Each instrument has the characteristic quality of its class, and besides, something distinctive of itself. There does not appear enough evidence to show that this varied *quality* of tone is owing to such simple cause as what is understood by harmonic tones. If this were true, the perception of the distinctive qualities would be easy, and in the department of a musical ear. But quality of tone in general, is not especially noticed by the musician, who only deals with certain simple and regular forms of vibration. The unmusical ear recognizes as quickly the tone of a voice, or that ring which tells that a dollar is made of silver. One can imagine that the infinite variety in the quality of tones is owing to some difference in the conditions and forms of their vibrations, which the ear distinguishes by some wonderful faculty which is not the higher perception of music. Such power of distinguishing by the different senses is possessed even by the lower animals; in some cases beyond the power of man.

Quality of sound is so little understood at present that distinctive terms are wanting. We speak indeed, of a loud sound, which is one that can be heard at a long distance; of a pure sound, called also smooth, sweet, and even soft, although the tone may be powerful. Any term expressing agreeable qualities to any of the the senses is applied to pleasant sounds. For example we have a *brilliant* tone, borrowing the word from the sense of sight, a *harsh* or *rough* sound, as if to the sense of touch. When, in the theatre, music is desired, very pure in quality and in tune, and soft and even in power, it is technically known as " aerial music." We shall be glad to have more definite knowledge concerning the qualities of tone, but it depends on something outside of the harmonic sounds.

In extension of the theory that beats of 33 per second are the most offensive possible, and that those of 132 are harmless, Professor Tyndall gives the several chords from the octave to the minor third, accompanying each note by its harmonic to the tenth degree. The octave is as follows:

		1 :	2
Fundamental Tone,		264	528
Overtones,	1	528	1056
	2	792	1584
	3	1056	2112
	4	1320	2640
	5	1584	3168
	6	1848	3696
	7	2112	4224
	8	2376	4752
	9	2640	5280

Comparing these tones together in couplets it is impossible to get out of the series a pair whose difference is less than 264. Hence as the beats cease to be heard when they reach 132, dissonance must be entirely absent from the combination which we have just examined. The octave therefore is an absolutely perfect consonance.

There is something strange in calling a combination harmonious because the rapidity of the discordant beats is so great that they are inaudible. The fact is, that a chord is nearly in tune and begins to give agreeable effects when its beats are very slow. When the chord is more out of tune and the beats become rapid, and mingle in a flutter, whether of 132 a second or more, the ear ceases to regard the sounds as a chord; but it cannot be said that "dissonance must be entirely absent;" on the contrary it is terribly present.

Whether the twenty sounds given above be harmonious or not, it can be shown that this does not depend on the circumstances that there are no coincidences less than 132. For twenty other sounds might be added at random utterly discordant, but not beating slower than 192. For example, 265 : 529, 266 : 530, etc., all having the difference of 264. And in all the examples given, the addition of the overtones does not effect the case, for if the fundamental tones have the desired difference of 132, the first harmonic is sure to have twice this difference, and the others still more. But a point might have been made, which has been overlooked by Tyndall, in the circumstance that these overtones make a "rough" chord more harmonious. See the major third, 4 : 5, 330−264=difference 66, and the minor third, 5 : 6, 396 − 330=difference 66. These are called rough, because their differences are less that 132. But their first harmonics have a difference of 132, and all the following differences are far beyond the assumed limits of dissonance. But the whole theory is untenable.

If the effect is good when each note is accompanied by harmonics, the reason will be found in the simple relations which exist, and in the circumstance that what would be complicated alone, may be clear when auxiliary sounds are present.

After asserting the equality of the musical ratios in composition, I admit the superiority of the simpler ratios for modulations. A change of key is best made to the fifth above or below, by the ratios 2 : 3, or 3 : 4. If the change by an octave, 1 : 2, be considered as a modulation, it is an extremely simple one. But modulations do not seem practicable by a third or seventh, or the primes 5 and 7. Such a change would sound like an abrupt transition.

In rhythm again, no other primes than 2 and 3 have been found available. A single composition has been made with *five* beats, or supposed equal divisions, in the bar, but no one desires to sing it or hear it a second time. It may be said that double time, in general, is more satisfactory than triple; and because it is simpler. Yet the waltz movement is perfectly clear to most lovers of music. The rhythm which results from joining three doublets or two triplets in a single measure gives an agreeable variety.

Although Professor Tyndall remarks that it is not his "vocation to lead into the musical portion of Acoustics," it is to be wished that so able a man would do so. The physical portion of musical acoustics, and the definition of musical intervals have also been omitted by a late lecturer on Harmony at the Royal Institution (G. A. Macfarren, 1867). Unless the intervals be defined, nothing is to be learned from a lecture on harmony, beyond what can be seen by studying the works of musicians directly. Let the syren be improved so as to give chords with the number 7, and let reed-pipes give the various chords and melodies. Let us know the exact pitch of every note. Musical notation is a sort of short hand, which may not be understood by a mechanical player. One who understands harmony knows what the composer meant in all cases where the composer himself knew it.

South Danvers, Mass., Feb. 1868.

Since writing this paper with regret at the neglect musical science has received, I have had the pleasure of learning that they are doing better in Germany. First, I read the admirable book on the training of the "Voice in Singing," by Mrs. Emma Seiler, late of Heidelberg, and co-laborer with Dr. Helmholtz, in this department, and now established in successful teaching in Philadelphia. Mrs. Seiler has studied the action of the vocal organs from life, and describes the meaning of the "registers" or the several methods of producing tones. She shows how these are to be developed, protected from injury by straining under bad instruction, and made to sing easily and truly. For want of such knowledge we have few good singers. Soprano voices are undeveloped or ruined, and the falsetto register of tenors is no longer used. Yet the great Italian masters used to train tenors with a full and strong falsetto, which united with the chest voice imperceptibly, and continued upward easily and purely. Now the tenors go as high as the chest voice can be painfully urged, and consider it something great if they get up to the *a*. In old times good singers used to preserve their voices to old age. We had here such trained tenors when the English Glees were sung. The following shows that Mrs. Seiler deserves her high reputation as a scientific teacher, and insists on singing in perfect tune.

Purity in the art of Singing is, however, such a primal condition of its beauty, that a piece of music purely executed, even by a weak and slightly cultivated voice, always sounds agreeably, while the most sonorous and practised voice offends the hearer when it is out of tune or forced upward. The training of our singers by pianos, as they are now tuned, by equal temperament, is altogether unsatisfactory. The singer who practices with the piano has no safe principle by which he can measure the height of his tones with any exactness. But persons of good musical talent, made aware of this disadvantage by a competent teacher and practicing accordingly, can nevertheless overcome this difficulty caused by our present method of tuning, and learn to sing correctly and purely.*

Dr. Feyjoó, who, in his time did most to enlighten Spain, was learned in music, and explained what the "enharmonic" meant. In his convent at Oviedo, in 1734, he thus exalts pure intonation, which is the "unknown charm" of singing, the "*no sè què*," which one has and another has not.

"Could I hear this voice," says he, "I would tell you what is the attractiveness which you call occult. * * * *Perfection of intonation* is a beauty which is occult to the very musicians. Let us not mistake each other. The musicians distinguish very well errors in intonation up to a certain degree; those of a comma, a half comma, or, I will allow you, a quarter comma; so that those who have a delicate ear, although the error is so small, perceive that the voice does not strike with complete exactness, though they cannot tell the amount of the error. But when the error is much less, as the eighth of a comma, no one thinks the voice is not true. But for all that this defect is so small as to escape the reflection of the understanding, it makes a sensible effect upon the ear; and the composition does not please as much as if it were sung by another voice, which should give the intonation more justly. * * * This unknown something, when deciphered, is the just, the perfect intonation."—*Theatro Critico*, tom. vi, p. 432.

I have also just received the large work of Dr. Helmholtz,† of which I can only notice, among much that is highly interesting, that which relates to pure Intonation. He also declares that musical harmonies should be given purely. He has even invented and built a "Harmonium" with perfect tuning. It has 24 notes, and 4 pedals or registers, each controlling 3 notes of a group *a*, and 3 of a group *b*. The key-notes are in a true series of fifths, like my own. Gen. Thompson's enharmonic organ is noticed, and others made in Europe,—nothing from the American side. Notes a comma apart are distinguished by large and small letters as I have done,—originally as I thought,—in this Journal of July last; which plan seems to have been used, previously, by Hauptmann. The harmonic seventh, 4 : 7, is discussed, and in some degree admitted, after ages of constant service in the art, to a place in German musical science; my long neglect of the German language and shortness of time, have not yet allowed me to learn all I would wish about the treatment of this favorite protégé of mine. But there are signs of general progress regarding pure vocal music and its auxiliary instruments, and a rational theory of Harmony.

I am obliged to omit a description of a new "*Enharmonic Vocal Intonator*" for training singers in pure intervals, and for accompanying. It gives full harmonies to the tenth degree—1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 : 10—plays these chords altogether, or any part of them, and in *arpeggios;* is more easy to play than the guitar and as portable; plays melodies, but chords most easily, being peculiarly constructed for that purpose; has all the harmonies for the major and minor, as in my enharmonic table (this Journal, July, 1867); plays very softly, or loud enough to accompany a choir; and, finally, can be made for the cost of a good guitar. H. W. P.

March 20, 1868.

* The Voice in Singing, translated from the German of Emma Seiler. [By Dr. W. H. Furness.] Philadelphia. 1868.

† Die Lehre von den Tonempfindungen als Physiologische Grundlage für die Theorie der Musik. Von H. Helmholtz. Braunschweig, 1865. 2d ed., 605 pp. 8°.