ART. XXXIII.—*The Measurement of the Optic Axial Angle of Minerals in the Thin Section;* by FRED EUGENE WRIGHT. (With Plates I and II.)

DURING the past few decades the development of the petrographic microscope and petrographic methods has been exceedingly rapid and has now attained a state of such high efficiency that nearly every optic constant of a rock mineral in the thin section can be ascertained with ease and some degree of certainty. In many instances, in fact, the petrographer has at present at his disposal two and even more methods for the determination of a single optical constant under the microscope. The results obtained by these different methods are, however, often not of the same value, nor are all methods equally applicable to a given mineral section. Comparative and critical studies of the relative merits and accuracy of such methods under different conditions are not abundant in petrographic literature. It has therefore seemed to the writer desirable that such a study should be undertaken, and the microscopic examination of the artificial mineral aggregates produced in the Geophysical Laboratory has furnished a favorable opportunity. It has been found necessary in this work to use most of the available petrographic microscopic methods and to test them under different conditions.

Minerals are determined under the microscope by means of their crystallographic and optic properties; the more accurately such properties or constants can be ascertained for any given mineral, the more reliable and satisfactory is the determination. Of the several optical constants which are thus made use of in the identification of minerals, the optic axial angle, and with it the optical character, whether positive or negative, are among the most important. Different methods have been suggested, from time to time, by means of which the optic axial angle of any given biaxial mineral can be measured with greater or less accuracy and more or less rapidly; certain of these methods are of general application and can be employed on all mineral sections, while others can be used only on sections of fixed orientation. In microscopic work, however, all possible sections are to be encountered, and in each particular case, that method is the best and should be selected which not only gives the most accurate results, but also requires the least time. This is particularly true of extremely fine-grained preparations where it is frequently difficult to obtain results of any value whatever. In the following pages, the methods for measuring optic axial angles of minerals in thin section are to be described briefly, together with several novel methods which have been found serviceable, and a summary given of the results of comparative tests of all these methods on mineral sections of known and measured optic axial angles. As many of these methods have not been described in English, it has seemed desirable to introduce the general discussion by a mention of their underlying principles and an outline of their mode of application to different sections.

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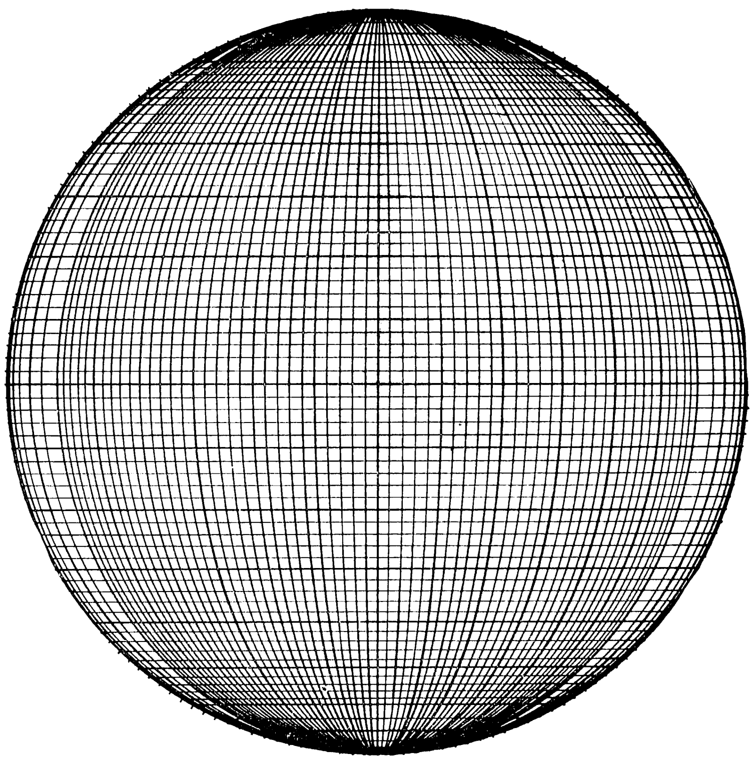

Several of the methods are in part graphical methods and involve the use of stereographic, orthographic or gnomonic projection plats. (Plate I and fig. 1.)* A few paragraphs,

* Plate I is a photolithographic production of the stereographic projection plat by Professor G. Wulff (Zeitschr. f. Kryst., xxxvi, 14-15, 1902). Both great circle and small circle arcs are located two degrees apart in the projection plat, the great circle arcs having a radius R_1 for a given angle ρ, $R_1 = \frac{r}{\sin \rho}$, r being the radius of the projection plat; the small circle arcs, a diameter, $R_2 = r \cot \rho$. In the projection plat, these arcs have been con-

therefore, on these different methods of projection, their essential characteristics, and their application to microscopic work, are inserted at this point because of the important rôle they play in the theoretical consideration and elucidation of the optical methods to be given below.

The spacial relations between the optic and morphologic properties of a crystal are frequently complicated and extremely difficult of correct conception and description without the aid of special models or projections of the same. Both observation and theory have shown that for any crystal the optical phenomena which it presents can be ascertained for a particular color of light, both in direction and length, by reference to an ellipsoid, either triaxial ellipsoid or ellipsoid of rotation, or sphere in the limiting case. Having once determined the exact position and character of this ellipsoid within the crystal and the relative lengths of its axes, it is possible to figure mathematically and to represent graphically the optical properties of the crystal for light waves of the specified length transmitted in any given direction. In the general treatment of the optical properties of crystals, the source of light is considered to be located at a central point within the crystal and the light waves to emerge in all directions from that central point along the radii of an inscribed sphere of unit radius. In space any radius can be represented accurately by its intersection with the surface of the unit sphere, and like any point on the earth's surface, its position can be fixed accur ely by two angles equivalent to those of latitude and longitude.

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structed with great accuracy and measurements can be made directly by their use with errors of less than $\frac{1}{4}°$. Having given the stereographic and orthographic plats of Plate I and fig. 1, the process of plotting angles from them graphically and measuring angular distances is rendered easy and certain by the use of tracing paper, as first suggested by Professor Wulff. With the plat as a base, the measured angles, obtained from the microscope, are plotted directly on tracing paper from the projection plat underneath, and the various necessary operations of passing great circles through given points and measuring angular distances, etc., accomplished by revolving the sheet of tracing paper about the center of the projection plat as a pivot, until the required great or small circle of the projection plat beneath is found. The circle is then sketched with sharp pencil point on tracing paper and is sufficientiy accurate for practical purposes, the slight errors produced by using tracing paper and stereographic plat being of the same order of magnitude as those of observation and therefore negligible. The amount of time and energy, moreover, expended in this method of free-hand sketching, is much less than that necessary for constructing accurately the required small and great circles. In actual practice it has been found expedient to draw with colored ink, on both stereographic and orthographic projection plats, radii and circles 2° apart, and thus greatly facilitate the ease with which observed data can be plotted. To incorporate these radii and circles in the original plats did not seem advisable because of the great complexity of lines which arises when radii, circles and great and small projection arcs are all superimposed in black ink. By the use of colored ink, no confusion arises and the radii and circles can be added at any time with little effort by the aid of rule and compass.

As a general rule, it is neither convenient nor feasible to work with actual models in the study of optical phenomena. Several different types of projection have been devised to overcome this difficulty by representing the spacial relations on a plane. In all cases the relation of the object to its projection plot is one of definite construction, and is dependent on the method of projection adopted.

In optical work, any one of three different methods of projection, the gnomonic, the orthographic, and the stereographic, may be used, each of which possesses certain favorable and certain unfavorable features. In each projection, the points on the sphere are pictured in a fixed plane.

In the *gnomonic* projection,* the plane of projection is the horizontal tangent plane passing through the crest of the unit sphere. (Fig. 2.) The point of intersection of the radius

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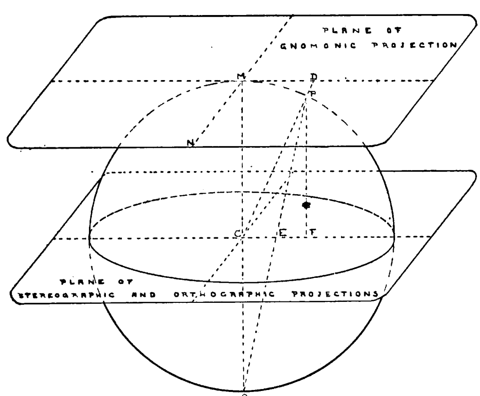

In this figure, the point D is the gnomonic projection point of the point P on the sphere, or of the direction CP in the crystal. The distance MD is evidently $r.\ \tan \rho$, r being the radius of the sphere and ρ the angle MCD. Similarly, the distance CE in the stereographic projection is $r \tan \frac{\rho}{2}$ and CF in the orthographic projection, $r \sin \rho$.

drawn through any given point on the sphere with the tangent plane is the projection point for that direction. In this projection, the great circles of the sphere become straight lines in the plat, and are small circles, hyperbolae. In the following pages, however, the gnomonic projection will not be used to any extent and will not therefore be described here in greater detail. It is particularly well suited to crystallographic work,

* For a comparative study of projections, see V. Goldschmidt, Ueber Projektion und graphische Krystallberechnung, Berlin, 1887.

since by its use all crystal faces are reduced to points and zones to straight lines.

In the *orthographic* (also called orthogonal or parallel or ocular) projection, the eye of the observer is supposed to be at an infinite distance above the plane of projection and to look directly down upon the sphere. The lines of sight are then parallel and the points on the sphere are vertically above their projection points on the central diametral plane (figs. 2 and 3). In this projection, great circles appear as ellipses and small circles as straight lines. This projection is especially impor-

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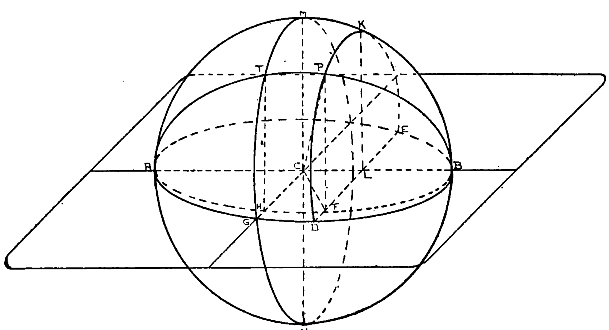

In fig. 3 the point P of the sphere, which is located in this case by the intersection of the great circle ATP and the small circle DPK, in the orthographic projection F and is there located by the ellipse AHF, the orthographic projection of ATP, and the straight line DFL, the projection of DPK. F is also the point of intersection of the diametral plane CGB with the line PF, normal through P to that plane.

tant since all interference phenomena observed in convergent polarized light under the microscope appear to the eye of the observer as they would were the actual interference phenomena plotted in this projection. The serious drawback to its general application in optical work lies in the rapid decrease of its sensitiveness to differences in angular distances near its outer margin. The interference figures below are all represented in orthographic projection, although their construction was accomplished in part by using the stereographic projection. In fig. 1, the ellipses represent great circles with a common diameter of intersection and drawn at intervals of 2° apart; while the straight lines are the projections of small circles, also 2° apart and corresponding to small circles of latitude. As on the sphere itself, the angular distance between any two points in projection can be found by passing through the two points the common great circle (ellipse in projection) and counting directly the distance in degrees by means of the small circles.

The actual modus operandi of this and the stereographic projection will appear more clearly later when actual data of observation are plotted.

In the *stereographic* projection,* the eye of the observer is considered at the lower extremity of the vertical radius, the lines of sight being directed toward the points of intersection of given radii with the surface of the sphere. The intersection of the line of sight for a given radius or direction of light wave propagation with the central horizontal plane determines then the stereographic projection point of that radius. (Plate I and figs. 2 and 4.)

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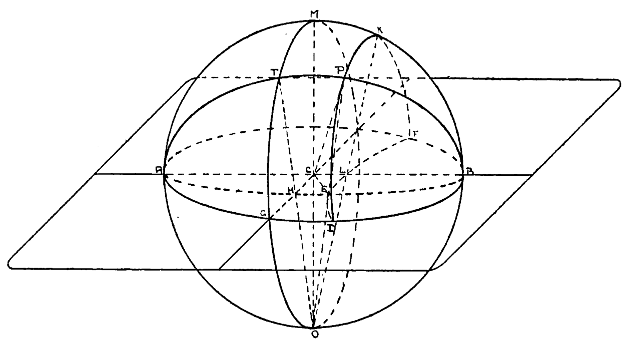

In this figure the point P of the sphere, corresponding to the direction CP within the crystal and located in this instance by the great circle ATP and the small circle DPK, becomes E in the stereographic projection plat and is there located at the intersection of the great circle arc AHE, the stereographic projection of ATP, and by the small circle arc DEL, the stereographic projection of DPK. E is also the point of intersection of the line OP with the horizontal diametral plane CGB.

The stereographic projection is unique in that all circles, whether great or small, appear in the projection as circles instead of ellipses, as might be supposed at first thought. The angle, moreover, which two great circles make with each other, is preserved unaltered in the projection. The projection is thus angle-true. In Plate I, the portions of great circles of the upper half of the sphere are represented by the circular arcs of which the horizontal radius is the limiting case, and the small circles by the arcs of which the vertical radius is the limiting case.

Biot's or Fresnel's rule.—In several of the optic methods to be described, frequent use is made of a rule first formulated

* S. L. Penfield, this Journal (4), xi, 1,115; E. Fedorow, Zeitschr. Kryst., vols. xxvi, xxvii, xxix, and G. Wulff, Zeitschr. Kryst., xxi, 249, 1893; xxxvi, 14–15, 1902, have given complete descriptions of the stereographic projection.

by Biot* by which the directions of extinction for any section of a birefracting mineral can be found. Some ten years before Biot announced this general rule, Malus† had found that the light waves emerging from a calcite rhomb were plane polarized and that for any given section of calcite the lines of extinction were parallel and at right angles with the trace of the plane containing the optic axis and the normal to the section; in other words, the orthogonal projection of the optic axis on any given section of a uniaxial mineral determines its lines of extinction which are parallel with and normal to this projection line. By modifying the wording of this rule slightly, it is possible, as Biot proved experimentally and Fresnel‡ demonstrated theoretically, to make it of general application to all birefracting substances; thus, the directions of extinction of a biaxial mineral§ cut after any section are parallel to the traces, on that section, of the planes bisecting the angles between the two planes containing the normal to the section and the optic axes (or optic binormals); in other words, the lines bisecting the angles between the lines of orthogonal projection of the optic binormals on any given section of a biaxial mineral are the directions of extinction for that section for the particular color of light employed. It should be noted that this rule applies to optic phenomena within the crystal itself, and that for oblique incidence of light, as in convergent polarized light, the apparent angles observed in air must be reduced to true angles by means of the average refractive index of the mineral in question; by the formula $\sin V = \frac{\sin E}{\beta}$, E being the observed angle, V the required true angle, and β the mean refractive index of the mineral.

The above methods of projection, together with the Biot-Fresnel rule of construction for finding the directions of extinction on any plane, constitute the basis on which several of the methods described below rest. With practice, these projections become working models for the observer and aid him very materially in grasping and picturing the optical phenomena presented by different minerals under varying conditions.

* Biot, J. B., Mém. de l'Acad. de l'Inst. de France, iii, 228, 1820.

† Malus, E. L., Théorie de la double réfraction de la lumiére dans les substances cristallisées, Mém. prés à l'Inst. Sc. math. et phys., ii, 303, 1811.

‡ Fresnel, Second Mémoire sur la double réfraction, Pogg. Ann., xxiii, 542, 1831.

§ The uniaxial minerals may for the moment be considered the limiting case of biaxial minerals for which $2V = 0$.

Part I. Theoretical.

In this part of the paper, the attempt has been made to describe briefly the methods, at present known to the writer, for measuring the optic axial angle of mineral sections under the microscope and to emphasize particularly their modus operandi and the principles on which they are founded. In the preparation of these descriptions constant use has been made of the Mikroskopische Physiographie, I, 1, by Rosenbusch and Wülfing, and of the original papers in which the methods were first mentioned, especially those by Becke and Fedorow. Several of the text-figures below have, in fact, been derived, with slight modifications, from these sources.

Convergent Polarized Light.

Optical phenomena in general can be observed and be measured by use of either convergent or plane polarized light. The study of optic axial interference figures has, however, until recently, been effected solely in convergent polarized light, and the methods applicable thereto are better known and more fully developed than those for plane polarized light. We shall, therefore, begin with the methods for measuring the optic axial angle of minerals in convergent polarized light.

There are several different lens combinations which can be used to advantage for obtaining and observing interference figures under the microscope in convergent polarized light; and of these, the one suggested by Amici* in 1830 has been found to be the best suited for optical measurements. With this method, the primary interference image, which is formed directly above the high power objective, is magnified and reproduced as a secondary image in the upper part of the microscope tube, where it can be observed either with magnifying glass or ocular with cross hair and micrometer scale. The small Amici-Bertrand lens by means of which this change of microscope to conoscope is effected must be inserted at such a point between ocular and objective that the secondary interference image observed through the ocular is as sharp and clear as possible.

Both theory and observation show, however, that all parts of the interference figure thus formed are not in perfect focus† at one and the same time. Fig. 5 illustrates the path of a light beam from condenser lens to eye of observer. From this figure, it is evident that the surface of focus for light waves entering in all possible directions is not a plane but roughly

* Ann. Chim. Phys. (3), xii, 114, 1844.

† Rosenbusch-Wülfing, Mikroskop. Phys., I, 1, 215, 1904.

a sphere. It is necessary, therefore, to focus sharply on points about midway between the center and margin of the

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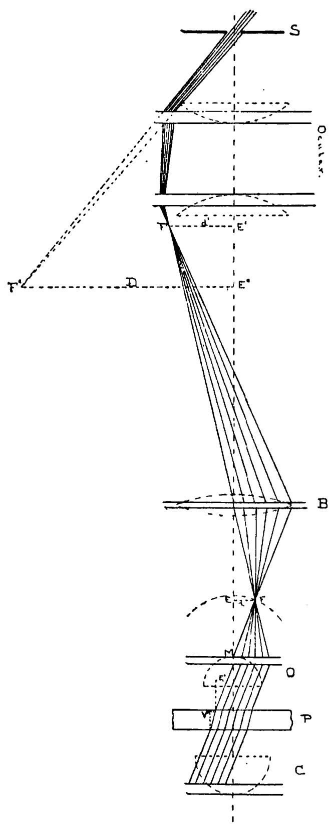

FIG. 5. In this figure, C represents the condenser lens system; O that of the objective, B, the Bertrand-Amici lens, P, the mineral plate and Oc, the ocular of the Ramsden-Czapski type. The primary image EF (d) is reproduced as a secondary image at E′F′ (d') and later still more magnified by the ocular at E″F″ (D), where it is observed by the eye back of the small aperture S. In the writer's microscope with Fuess No. 9 objective, of equivalent focal length, $2{\cdot}7^{mm}$, and numerical aperture 0·97, the diameter of the primary interference figure measures about 5^{mm} and this is magnified very considerably by the Bertrand lens and by the ocular in the secondary interference figure as shown in the figure. By choosing a Bertrand lens of suitable focal length and inserting the same at the proper point between objective and ocular, it is possible to increase the magnification of the secondary interference figure and thus to enhance the accuracy of the measurements, providing the interference figure is sufficiently sharp and distinct to permit magnification to such an extent.

field (F of the figure) and to adjust the lenses for these points in order that the remainder of the field be as distinct as possible under the circumstances.

In order to minimize the error due to the slight parallax of rays near the margin of the field, a small stop, S, may be placed above the ocular as shown in the figure.

The exact position of any point in a given interference figure can be determined with considerable accuracy by using either a micrometer ocular with fixed scales or movable screw micrometers, or by means of a projection by camera lucida on paper. The actual position in space of any point thus fixed in the projection can then be stated if the relation between projection and object be known.

In 1882, Mallard* proved theoretically that the interference figure observed in the microscope is approximately an orthographic projection of the optic phenomena in space. To take the simplest case, let both condenser lens and objective consist each of a simple hemispherical plane convex lens, C and O, fig. 5. The rays which in the objective lens are parallel come to a focus at F, where they can be viewed directly by the unaided eye. Assuming the radius of the lens to be r, the radius R (MF, fig. 5) of the focal circle is evidently $\frac{r}{n-1}$, where n is the refractive index of the objective lens glass. It should be noted that under the microscope the optic phenomena are observed as they appear in the objective itself, i. e., modified by their refraction in the glass. The distance

$$\mathrm{EF} = d = \mathrm{R} \sin \sphericalangle \mathrm{EMF} \tag{1}$$

Replacing $\sphericalangle$ EMF by its corresponding angle in air, E, the relation between which is expressed by $\sin \mathrm{E} = n. \sin \sphericalangle$ EMF, equation (1) can be written

$$\mathrm{EF} = d = \frac{\mathrm{R}}{n}. \sin \mathrm{E}. \tag{2}$$

By fixing rigidly the relative positions of objective, Amici-Bertrand lens and ocular (fig. 5), the distance d for all angular values E, which can be measured in the microscopic field, bears a constant relation to D of the secondary interference figure

$$\mathrm{D} = k\, d \tag{3}$$

This distance D can be measured accurately with the movable screw micrometer ocular. Substituting D for d, equation (2) becomes

* E. Mallard, Bull. Soc. Min. Fr., v., 77-87, 1882.

$$D = \frac{R}{n \,.\, k} \,.\, \sin E,$$

an equation which can be simplified to

$$D = K \sin E \qquad (4)$$

by replacing $\frac{R}{n \,.\, k}$, an expression consisting entirely of constants, by K, the Mallard constant of the microscope for the given fixed position of microscopic lens system. The constant K can be determined once for all by ascertaining the D for any known angle E. The method usually employed to determine this constant K is to measure, on a section of a biaxial mineral of known optic axial angle and cut normal to the acute bisectrix, the distances between the apices of the dark hyperbolic bars of the interference figure in the diagonal position; this distance is 2D, and the optic axial angle measured is 2E.

This method, however, does not furnish a check on the value of K, thus obtained, and its validity verified for other angles E, unless many similar sections of different biaxial minerals be taken and the K of the microscope for each angle E be ascertained. The formula, $D = K \sin E$, of Mallard assumes that the loci of the focal points of waves entering in all directions lie on a perfect spherical surface, an assumption which actual microscopic objective lens systems in the strict sense of the word do not fulfill.

To test the validity of the formula for the microscopic field of a given objective (No. 9, Fuess, with Amici-Bertrand lens, system 7–9, and movable screw micrometer ocular with Ramsden ocular), two polished plates of calcite were used, the one cut after 0001 and the second at an angle of 23° 17′ with 0001 as measured on a two-circled goniometer, the adjustment on the goniometer having been accomplished by means of fresh cleavage faces along the edge of the plate. These plates were placed successively on a carefully adjusted Fedorow-Fuess universal stage (large model) and the positions of the optic axis measured on the micrometer ocular screws as the stage was turned about its horizontal axis from degree to degree on both sides from the horizontal position. By means of the two plates, a continuous set of readings was obtained for D for angles E ranging from 0° to 47°. These values were ascertained for both scales (horizontal and vertical) of the double screw micrometer ocular described on page 336, and are listed in Table I.

The values for D listed under the heading "calculated" in this table were figured by Mallard's formula by taking the average value of K for both the horizontal and vertical scales of the double micrometer ocular; for the vertical micrometer

TABLE 1.

	Observed		Calculated			Observed		Calculated	
E	Dv	Dh	Dv	Dh	E	Dv	Dh	Dv	Dh
0°	0	0	0	0	26°	181	186	----	----
1	8·5	8	----	----	7	187·5	192·5	----	----
2	16·5	16	----	----	8	194·5	199·	----	----
3	23	22·5	----	----	9	199	206·5	----	----
4	29	30	----	----	30°	205·5	212·5	206·1	211·2
5	36·5	37·5	35·9	36·8	1	213·5	218·5	----	----
6	44	44	----	----	2	219	225	----	----
7	52	51	----	----	3	225	230·5	----	----
8	58·5	57·5	----	----	4	230·5	237·5	----	----
9	65	65·5	----	----	35	236·5	243·5	236·5	242·3
10°	71·5	73·5	71·6	73·4	6	243	250·5	----	----
1	78·5	80	----	----	7	249	256·5	----	----
2	85	87·5	----	----	8	254	261·5	----	----
3	92·5	94·5	----	----	9	259	266·5	----	----
4	100·5	102	----	----	40°	264	272	265	271·5
15	107·5	109·5	106·7	109·3	1	269	277	----	----
6	113·5	117	----	----	2	273	283	----	----
7	119·5	124·5	----	----	3	279·5	288	----	----
8	125·5	131·5	----	----	4	286·5	293·5	----	----
9	132	138	----	----	45	290	299	291·5	298·7
20°	139·5	144·5	141·0	144·5	6	295	303·5	----	----
1	146·5	152·5	----	----	7	300	308	----	----
2	154	158·5	----	----	8	----	----	----	----
3	160	164·5	----	----	9	----	----	----	----
4	166	171	----	----	50°	----	----	316·6	323·6
25	173	179	174·2	178·5					

scale, $K_v = 421{\cdot}3$; for the horizontal, $K_h = 422{\cdot}4$. The curves of fig. 6 express these relations graphically, the zero line of each succeeding curve being one division above the preceding curve, to prevent confusion. For the objective in question (No. 9, Fuess), and with the precautions observed to avoid parallax by placing a fine stop diaphragm S (fig. 5) above the Ramsden ocular, the agreement between theory and practice is remarkable.* The screw of the horizontal (H, fig. 11, p. 336) micrometer

* In the Mikroskopische Physiographie I, 1, p. 330, by Rosenbusch and Wülfing, the latter gives a series of measurements with an α-monobromnaphthaline immersion objective of R. Winkel and finds disagreements as high as 8° between observed and calculated values, as indicated in the following table :

	H	D	K	H_1
Aragonite	11°33′	0·325	1·623	10°59
Muscovite	24°43	0·700	1·674	24°14
Topaz	39°05	1·075	1·705	(39°05)
Calcite	60°51	1·590	1·821	68°50′

The angles under H are half the axial angles for these minerals obtained scale registered ·005^{mm} for each division on the head, while the vertical (V, fig. 11) scale screw, which was constructed at a later period and on a different lathe, was a trifle coarser and registered a slightly greater movement for one division on its head. For this reason the values K_v and K_h are slightly

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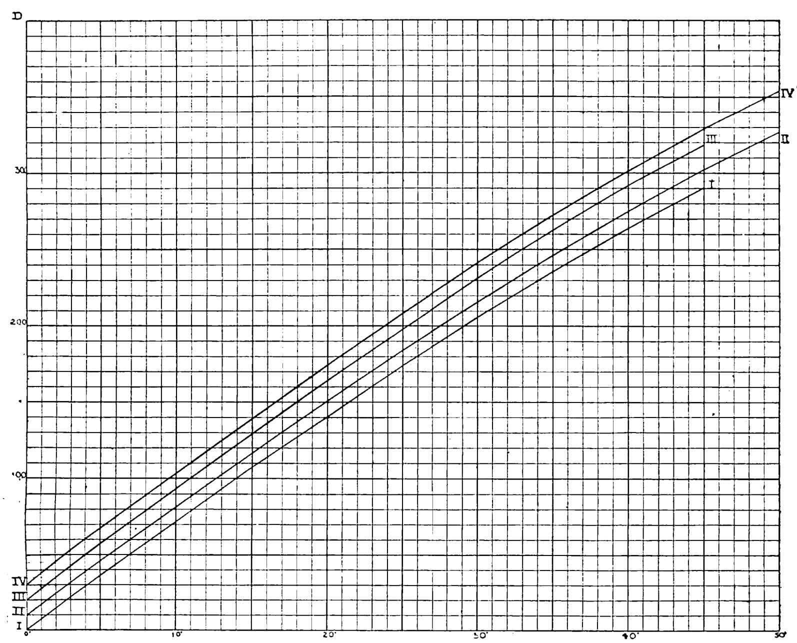

Fig. 6. Curve I of this figure contains the observed values of D_v for the vertical (V) micrometer screw of the double micrometer ocular (fig. 11, p. 336) for the different values of E from 0–47°; the values of D in Curve II were calculated from Mallard's formula for the same micrometer screw scale; Curve III in like manner was plotted from observed values of D_H, the horizontal micrometer screw for values of E from 0–47°; while Curve IV was plotted from values of D_H calculated from Mallard's formula, $D_h = K_h . \sin E$.

from measurements on an optic axial angle apparatus while the angles under H_1 were calculated by Mallard's formula on the assumption that the K obtained for topaz (1·075) is valid for all angles. The differences between observation and calculation are large and indicate that the determination of the positions of optic axes near the periphery of the field is less accurate than that for more centrally located points; on comparison of this series of results with those obtained by Fuess, No. 9 objective, it is evident that objectives vary considerably in this particular and that for accurate work the constants K of each microscopic objective and lens system should be determined for a number of directions, either by using minerals of known optic axial angles or a uniaxial mineral as calcite in conjunction with the universal stage.

different. On an average, a movement of 6 divisions or $\cdot 03^{mm}$ corresponded to one degree, so that with this method of special refinement, the probable error for E remains at least $\pm$ 10′, and in wide hyperbolic bars, differences of 1° and over should be expected.

If only a single screw micrometer ocular be used, the section should be cut very nearly normal to the acute bisectrix, otherwise the values become much less certain. With a double screw micrometer ocular, however, this error can be eliminated directly and equally good values obtained on sections only approximately normal to the acute bisectrix, as will be shown later (page 336).

In place of solving the above equations $D=K \sin E$ and $\sin E=\beta \sin V$ by logarithms, it is possible to use a graphical method which is sufficiently accurate for the purpose and which Fedorow appears to have been the first to use.* An accurate drawing (Plate II) is made once for all which serves for all possible angles and all refractive mineral indices to be encountered. To solve the equation $D=K \sin E$, or $\sin E=\frac{D}{K}$, draw the circle with radius K (Plate II, preferably in colored ink); the intersection of the ordinate D with this circle makes then the angle E in degrees, as is evident from the right angled triangle. To solve the equation $\sin E = \beta \sin V$, or $\frac{\sin E}{1} = \frac{\sin V}{\frac{1}{\beta}}$, find the intersection of radius E° with the circle for the given refractive index and pass horizontally from this point to intersection with outer circle of drawing, which point indicates V in degrees.

Examples.

(1) $K = 54\cdot 0$ $D = 21\cdot 1$

Intersection of ordinate D with K-circle is at radius 23°.

(2) $E = 42°$ $\beta = 1\cdot 65$

Pass along radius 42° to intersection with circle labelled $\beta = 1\cdot 65$, and then horizontally to outer circle and read $V = 24°$.†

* Fedorow, Zeitschr. f. Kryst. xxvi, 225–261, 1896. F. E. Wright, this Journal, xx, 287, 1905.

† The drawing of Plate II can also be used to solve the birefringence formula of Biot, $\frac{\gamma' - \alpha'}{\gamma - \alpha} = \sin \alpha_1 \sin \alpha_2$, which is very approximately correct and has been used frequently in optical work and in which $\gamma' - \alpha'$ denotes the measure of birefringence for any given section of a mineral, $\gamma - \alpha$, that of the maximum birefringence of the mineral, α_1 and α_2 the angles included between the normal to the section and the two optic axes respectively. This formula can be solved graphically at once by noting the length of the ordinate of the point of intersection of the radius α_2 with that circle whose radius is equal to the ordinate of the point of intersection of the radius α_1 with the outer circle. This graphical solution gives directly the relative birefringence of the section in per cent of the absolute birefringence $(\gamma - \alpha)$ as represented by the radius of the outer circle.

Mallard's method for measuring the optic axial angle is one of the most satisfactory of the microscopic methods and if available sections are at hand which show the required phenomena, Mallard's method should be adopted without question, especially if the measurements can be made with a double screw micrometer ocular. The limits of error of measurements of 2V by the Mallard method should not exceed 1°–2° on clear interference figures.

Methods of F. Becke.—In place of the single screw micrometer ocular which in itself is of very limited application, F. Becke* has substituted a graphical method in which the observed optical phenomena are projected by a camera lucida on a revolving drawing table fixed in position relative to the microscope. Accurate drawings of the interference phenomena are thus prepared and serve in place of the actual interference figure. This method has been fruitful in its results and with practice the necessary manipulative skill can be acquired to obtain trustworthy axial angle values. The accuracy of the method is dependent on several factors—the accuracy with which the drawing is prepared, the exactness with which the drawing table is centered, and the care with which measurements are made on the finished drawing.

The actual field of the projection does not measure over 25^{mm} in diameter, and a difference of 1° of E corresponds to a difference in D of about $\cdot 25^{mm}$, a distance which is easily measurable. With unusually sharp axial bars and nice adjustment of the optical system, it is theoretically possible to obtain an accuracy of about $\frac{1}{8}$–$\frac{1}{2}$°; in practice, however, a greater accuracy than 1–2° cannot be claimed for the method.

The writer has not seen the revolving drawing table described by Professor Becke, and has used in his work a small revolving disk D, graduated in degrees and supported by an arm E attached to the microscope stand by the collar F, as shown in fig. 7. This device was constructed in the mechanical workshop of the Geophysical Laboratory, after specifications furnished by the writer. The results obtained with it have proved satisfactory and the manipulation with the same convenient.

Having once fixed the position of this table so that its axis of revolution coincides after reflection in the camera lucida with the optical axis of the microscope and is also at the proper distance from the eye for distinct vision, its constant K, corresponding to the K of the microscope in the formula $D = K \sin E$, must be determined by one of the methods described above.

With the drawing of an interference figure thus properly prepared, it is possible to determine the angular distance,—polar

* F. Becke, Tschermak's Min. petr. Mittheil., xiv, 563, 1894; xvi, 180, 1896.

angle ρ and longitude angle ϕ, of any point in the projection,—and to plot the same in stereographic or orthographic or gnomonic projection and thus to measure the angular distance between any two points, as those between optic axes occurring in the field of vision.

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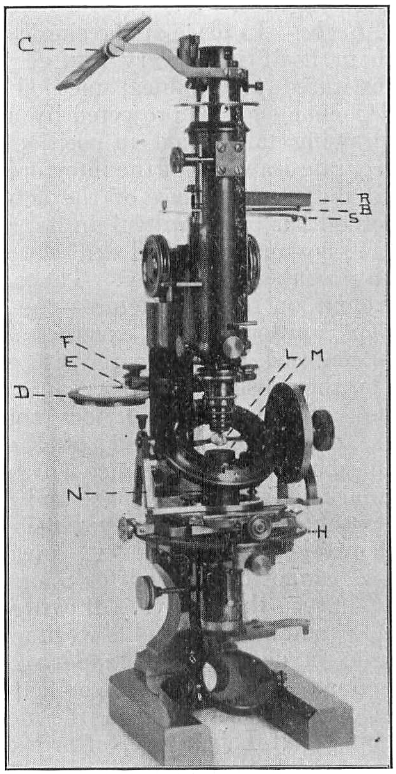

FIG. 7. In this figure, C is the camera lucida: D, revolving disk of drawing table graduated into degrees and supported by the arm E, which in turn fits into the collar F clamped to the stand of the microscope at the proper distance from C; R, axial angle reflector; B, Bertrand-Amici lens; S, sliding stop diaphragm; L, upper lens of condenser system fitted in brass ring in horizontal circle H_3 of universal stage (page 343); N, brass cylinder extension to hold lower portion M of condenser lens system (page 342).

In a recent article,* Professor Becke has described an ingenious method by which any section, in which only one optic axis appears in the field, can be used for the measurement

*F. Becke, Tschermak's Min. petr. Mitth., xxiv, 35–44, 1905.

of the optic axial angle, although the values obtained are only close approximations to the true value of the 2V. He utilizes the fact that sections of biaxial minerals, cut approximately normal to an optic axis, exhibit, in convergent polarized light, dark axial bars which resemble hyperbolas in the diagonal position and whose degree of curvature is dependent on the optic axial angle 2V. For any given position of the stage, the points along the dark bar of the interference figure correspond to those directions of light wave propagation in space whose planes of vibration coincide with the principal plane of the lower nicol (polarizer) and for which the extinction angle is zero.

To measure graphically the optic axial angle of a given mineral from the degree of curvature of its dark axial bar (zero isogyre) on a section about normal to an optic axis by this method, the axial bar is first drawn when in a position parallel to the horizontal cross hair (fig. 8, the straight line A_1 C in this position being the trace of the plane of optic axes); the microscope stage and drawing table are then revolved in the same direction about some convenient angle 30° or 45° and the axial bar drawn in the new position (A_1 P of fig. 8).* These drawings are repeated after revolution of the microscopic stage or drawing table alone through 180° (A_1' C′ and A_1' P′).

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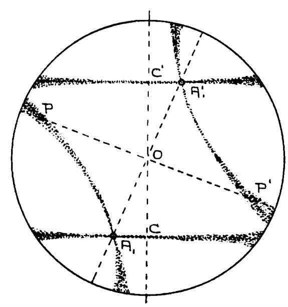

The point P in the projection is any convenient point on the dark curve or zero isogyre, and is therefore a direction in the crystal in its given position relative to the nicols along which light waves are propagated without changing their original plane of vibration. The plane of vibration for the point P is thus known, and the law of Biot can be applied directly to find by construction the second optic axis A_2.

A convenient form of construction is shown in fig. 9, the details of which are the same in every case. After plotting the observed points P and A_1 on tracing paper above the projection plat, the great circle $A_1'CA_2'$, of which P is the pole, is first found by revolving the tracing paper about the center O until P coincides with the vertical diameter of the under-

* In his work the writer has found it more convenient and accurate to revolve the nicols instead of the stages, which remain stationary except for revolutions of 180°.

lying plat, and then finding and sketching that great circle whose intersection C with the vertical diameter is at 90° from P (to be counted from P, each of the great circles of the projection being 2° apart). Similarly, the great circle A_1A_2, containing A_1 and the extremities of the horizontal diameter DE, is located and drawn. The great circle which indicates the plane of vibration for P, can be found by either one of two methods: (*a*) it is that great circle containing P and tangent at P to the small circle, GPL (fig. 9), which parallels

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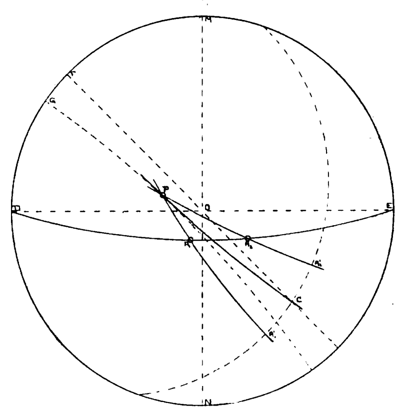

FIG. 9. In this figure the points P and A_1 are the two points obtained from the original drawing. The plane of vibration for the point P in its position of darkness is indicated by the great circle PC tangent at P to the small circle GPL, which is parallel to FOC, the trace of the plane of vibration of the polarizer when the zero isogyre passes through P. Further details of construction are added under fig. 10.

the trace GOF in projection of the principal plane of the lower nicol; (*b*) it is that great circle passing through P and the intersection C of the polar great circle $A_1'A_2'$ C with the trace of the principal plane FOC of the lower nicol.* In actual work, however, it is not necessary to draw this great circle PC, since the point C is the point sought and determines at once the direction of extinction for the given section. The simplified construction is illustrated in fig. 10, where C is

* In his paper, Professor Becke determines this great circle as the one which is tangent at P to the straight line parallel to the trace in projection of the principal section of the lower nicol. This method does not facilitate the finding of the great circle in any degree, and introduces an error, as Professor Becke himself recognizes, which decreases the accuracy of the method to just that extent. It seems advisable, therefore, to use one of the new methods described above which are theoretically correct and equally simple.

the intersection of the great circle $A_1'A_2'$, and the diameter OC, the trace of the plane of vibration of the lower nicol.

Having thus determined the point A_1' and C, the projection of the second optic axis A_2 is found by making $A_2'C = A_1'C$ (Biot's law). The intersection of the great circle PA_2' with the plane of optic axes A_1A_2 determines then the position of A_2, and the angle A_1A_2 in projection is 2V, the angle between the optic axes.

The actual time consumed in this operation is not great, and the values obtained are approximately correct. The objection

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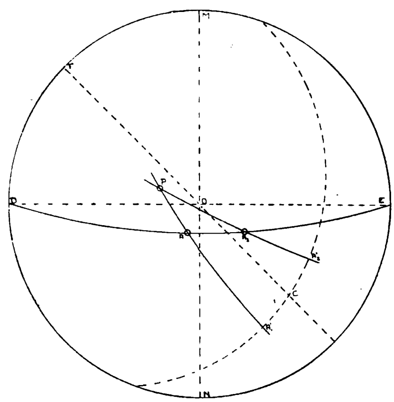

FIG. 10. In this figure the operations of actual construction are given which are required to measure A_1A_2 from the data in the drawing. The points A_1 and P are first located accurately in the drawing, reduced to angular values and plotted directly on tracing paper in stereographic projection; the great circle $A_1'A_2'$ polar to P_1, and the horizontal great circles A_1A_2 are then sketched; the point A_1' is the intersection of the great circle, containing P and A_1 with great circle $A_1'A_2'$, while the line OC is the trace of the plane of vibration of the lower nicol (polarizer) as it appears on the drawing after the revolution of 30° or 45°; the angle MOF indicating directly the angle of revolution. By definition $A_2'C$ is equal to $A_1'C$, and the intersection A_2 of the great circle PA_2' with the great circle A_1A_2 determines A_2. the second optic binormal, the angle A_1A_2 being the desired optic axial angle.

to its use lies chiefly in the subjective factor involved, namely, the skill required in drawing accurately the phenomena observed, and also in the nice adjustment of all parts of the instrument.

The location of the optic axis A_1 is at all times more accurate and trustworthy than that of P, owing to the indistinctness and width of the black axial bars near the margin of the field in consequence, chiefly, of elliptic polarization.

New Methods with Double Screw Micrometer Ocular.— In seeking for more accurate and at the same time simpler methods than those of Professor Becke described above, the writer has substituted in place of the usual single screw micrometer ocular, with a movement in one direction only, a double screw micrometer ocular with movements in two directions normal to each other. By the use of this double screw micrometer ocular, which was constructed in the workshop of the Geophysical Laboratory (fig. 11)* it is possible to determine

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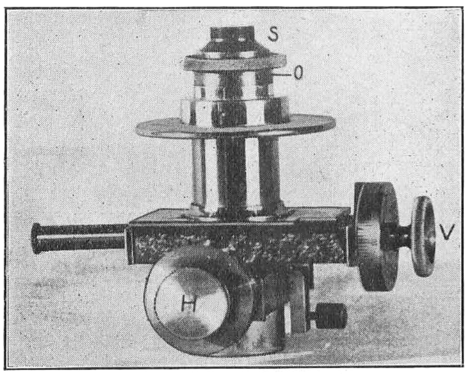

the position of any point in the interference figure accurately by means of two micrometer screw readings, which correspond to rectilinear coördinates in the orthogonal projection and small circle coördinates in the stereographic projection. By means of the constant K of the microscope for each of these micrometer movements, K_V and K_H, which must have been determined previously by means of known angle values (table 1, page 328), each of these readings is reduced as usual to its angle value for the crystal by means of K and the average refractive index of the crystal,

$$\sin V = \frac{D}{K \cdot \beta}$$

Having given the interference figure from a section of a biaxial mineral, cut so that one axial bar is visible, the course

* The double screw micrometer ocular is fitted to microscope of fig. 7. The two movements, H horizontal and V vertical, are effected by fine micrometer screws, reading accurately to $\cdot 005^{mm}$. The construction of this ocular is similar to that of the single screw micromoter oculars with the exception that here two screws with corresponding movements are used in place of the single screw. O = Ramsden ocular; S, small stop aperture to reduce errors of parallax.

of procedure in measuring the optic axial by means of the double screw micrometer ocular consists in: (*a*) revolving the microscopic stage until the dark axial bar is parallel to the horizontal cross hair of the ocular; (*b*) moving the horizontal cross hair by means of the vertical micrometer screw V until it coincides precisely with the center of the dark axial line (fig. 12, A_1C_1); (*c*) revolving the nicols (not the stage as may be done in the Becke method) about a suitable angle (usually 30 or 45°), the exact position of the optic axis A_1 then being the intersection of the axial bar with the horizontal cross hair (fig. 12, A_1C_1 with A_1P_1); (*d*) moving the vertical cross hair by means of the horizontal micrometer screw until it coincides with this intersection and recording both vertical and horizontal micrometer screw readings; (*e*) the stage is then revolved about an angle of 180°, and similar readings for A_1 taken in its new position, A_1'. This last step is necessary in order to locate accurately the center of the field O. The position of A_1 is thus fixed accurately and can be plotted directly, after proper reduction to true angles within the crystal, in stereographic (small circles) or orthographic (coördinates) projection. Any point P of the dark curved axial bar can now be determined by two micrometer readings (coördinates from the center) and, after reduction to angles within the crystal, plotted in the projection. From this point on the method does not differ from the foregoing methods. The optic axial angle is determined by construction from the projection plat thus obtained.

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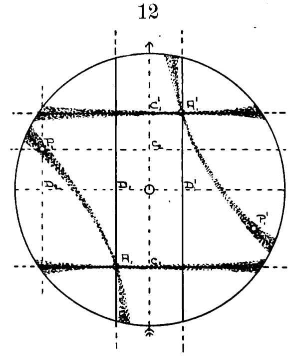

In plotting the angles corresponding to the coördinate readings of the double screw micrometer ocular, it should be noted that these angles apply to small circles, the angle for each micrometer screw indicating the position, from the center of the projection, of a certain horizontal or vertical small circle. The intersection of the horizontal and vertical small circles thus obtained from the two micrometer screw readings for a particular point of the interference figure determines the location of that point in the projection.

Measurements with the double screw micrometer ocular are more delicate, and therefore more accurate, than those with the Becke drawing table, and the values for 2V correspondingly more trustworthy. This method is of general application to all sections cut in such a way that either one or both optic axes appear in the microscopic field. By using projection plats, either stereographic or orthographic, results of a fair degree of accuracy can be obtained in a very few minutes.

On sections in which both optic axes of the interference figure are visible, the exact position of A_1 and A_2 can be measured directly, and after plotting, the value 2V obtained from the projection by direct reading. Such values should be accurate to $\pm 1°$.

To form an idea of the relative degree of curvature of the axial bar in the diagonal position for sections of biaxial minerals cut at different angles with an optic axis (binormal) and for the optic axial angles $2V = 0°$, $15°$, $30°$, $45°$, $60°$, $75°$ and $90°$, the writer has constructed by graphical methods the following figures:

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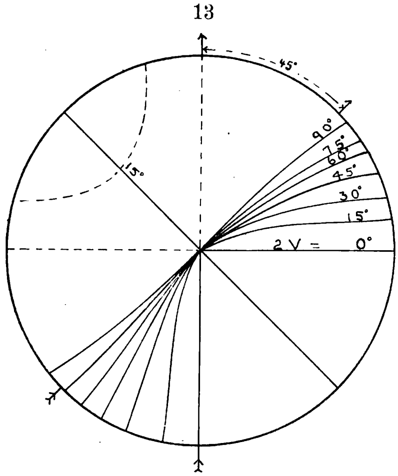

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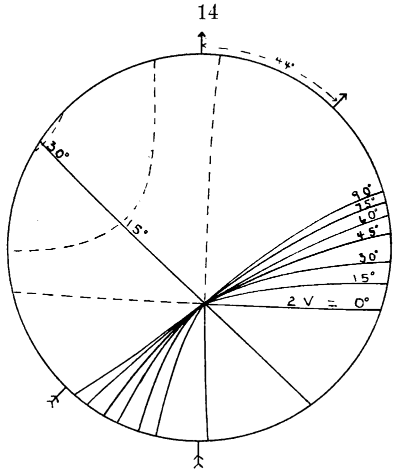

Fig. 13. This figure illustrates the positions in the interference figure of the dark curves of no extinction (axial bars) as they would appear in the field were observations made in air on a series of biaxial minerals having a mean refractive index of 1·60 and the optic angles indicated, and cut normal to one of the optic axes (binormals). From the figure it is evident that the radius of curvature of the axial bar increases with the optic axial angle, so that for $2V = 90°$, the axial bar is practically a straight line.

Fig. 14. The conditions of construction for this figure were similar to those of fig. 12, except that the section is considered cut at an angle of $\lambda = 10°$, $\mu = 10°$ (small circle coördinates from the center) with one of the optic binormals.

In these figures, the dark lines represent the curves of 0° extinction (zero isogyres or axial bars) of the interference figure in orthographic projection, the plane of the optic axes making angles of about 45° with the plane of vibration of light waves from the lower nicol.

The curves of figs. 13, 14 and 15 are constructed for minerals with a refractive index $\beta = 1{,}600$, while in those of figs. 16–19 orthographic projections of the actual positions of the directions of $0°$ extinction within the crystal are given (refractive index of mineral and medium in which the phenomena are observed, being considered equal). It is evident from the figures that the differences in curvature of the axial bars for the different values of 2V are sufficient to warrant their use in measuring optic axial angles approximately. The accuracy of the method depends on the accuracy with which the points A_1 and P (fig. 12) can be determined. The positions most favor-

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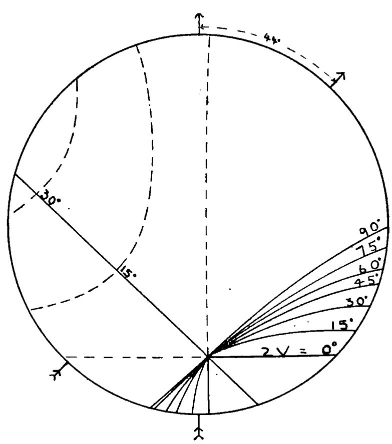

FIG. 15. This figure differs in construction from figs. 12 and 13 only in the fact that the section is considered cut at an angle $\lambda = 20°$, $\mu = 20°$ with one of the optic binormals.

able for these points are located one-half to two-thirds the distance from center of field to its margin. Near the center of the field, the errors of construction increase rapidly, while near the margin, errors due to imperfections in the lenses and to elliptical polarization tend to modify the interference figures, and decrease the accuracy to be attained.

The actual diameter of the field covered by the micrometer screw movements of the writer's microscope measures about 600 micrometer screw divisions. The distance covered by the extremes of the curves for $0°$ and $90°$ is less than 200 divisions, or on an average, about 2 divisions for one degree. Taking into consideration the indistinctness and width of the axial bars, it is easily possible to make an error of three or four divisions of the micrometer scale in these readings, so that an

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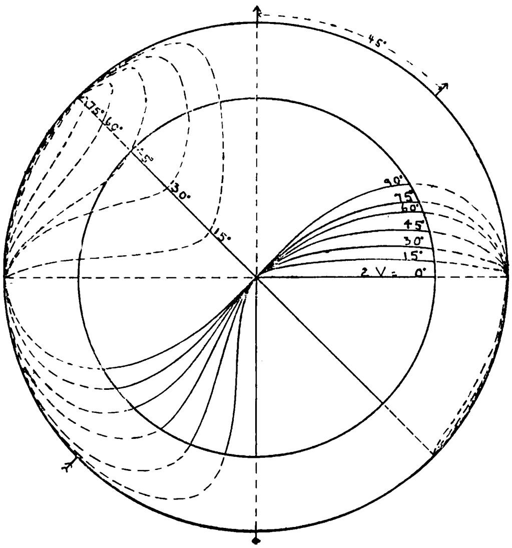

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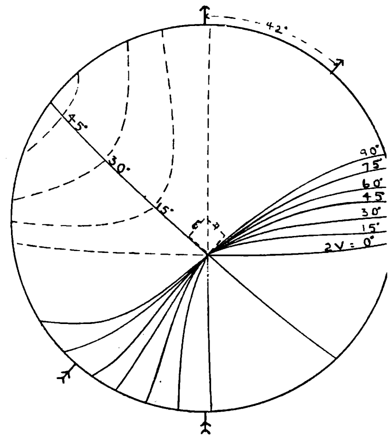

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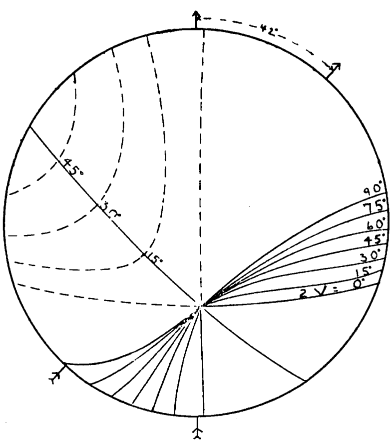

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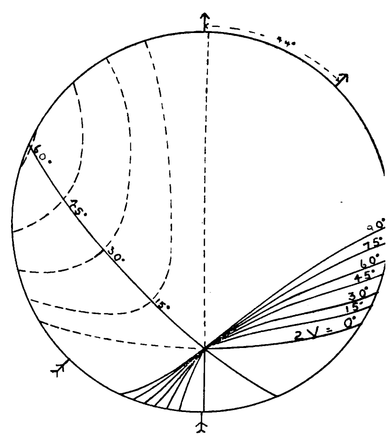

FIGS. 16, 17, 18, 19. In figs. 16–19 the axial curves (zero isogyres) are constructed under the assumption that the mean refractive index of both the mineral and the medium in which its interference phenomena are observed, are identical; in short, an orthographic projection of the phenomena as they appear within the crystal is given. In fig. 16, the section is normal to an optic axis; in fig. 17, the section makes angles, $\lambda = 5°$, $\mu = 6°$ (small circle coördinates) with the optic binormal; in fig. 18, the angles are $\lambda = 12°$, $\mu = 13°$, while in fig. 19, the section makes angles $\lambda = 20°$, $\mu = 20°$ with the optic axis; the angle of revolution of the stage and consequent new position of the trace of principal plane of lower nicol in each case is indicated by the arrows. The area included by the inner circle of fig. 16 indicates the relative field of vision of ordinary microscopes.

accuracy of more than $\pm 2°$ cannot be claimed for this method. With the drawing table, this probable error increases about $\pm 5°$ under the same circumstances.

In place of the expensive double screw micrometer ocular, the writer has had constructed a simpler, although slightly less accurate ocular, consisting of a Ramsden ocular with finely divided cross section scale in its focal plane (arrangement similar to that in ocular of Czapski). (Ocular of fig. 5.) After insertion of the Amici–Bertrand lens, the secondary image of the interference figure is brought to focus in the focal plane of the ocular, where the location of any point can be read off directly in coördinates, which in turn are to be reduced, just as the readings of the double micrometer screw ocular, to angle directions within the crystal, and then plotted in suitable projection. The coördinate scale employed in this ocular is a photographic reproduction on thin glass of a greatly reduced drawing.

Klein's lens, which was first described by Becke, can also be changed to fit the new conditions by simply introducing the above fine cross-grating or coördinate micrometer scale in place of the single micrometer scale.

By the use of such oculars with fine coördinate scales, one has the entire field of the interference figure under control, and, by use of projection plats, can readily measure optic axial angles on all sections which are so cut that one optic axis at least is in the field. If two optic axes appear within the field of vision, their positions can be read at once from the coördinate scale of the ocular and after proper reduction plotted in stereographic or orthographic projection where their angular distance can be determined directly.

Michel-Lévy Method.—For sections normal to an acute bisectrix of a mineral with large optic axial angles, Michel-Lévy has suggested a method which, although interesting theoretically, is not of great practical value, owing to the indistinctness of the phenomena to be observed. His method consists in reading the angle of revolution of the stage necessary to bring the interference figure from the crossed position to that in which the emerging axial bars of the interference figure are tangent to a given circle.* Actual practice with this method has shown that it is not sufficiently accurate and of such general application as to warrant a more detailed description at this point.

Method with Universal Stage.—In practice, it frequently happens that a given section is not favorably cut to show the

*Michel-Lévy, et Lacroix, Les Mineraux d. Roches, 94-95, 1888. For a modification and simplification of his formula, see F. E. Wright, this Journal, xxii, 289, 1905.

optical phenomena to the best advantage, and that by tilting it a certain angle, the interference figures can be improved materially. This is particularly the case with fine-grained artificial preparations where, although individual crystals and cleavage can frequently be obtained, they do not rest in the section in the most advantageous position. Such crystals and crystal plates can be tilted either by means of an axial angle apparatus for the microscope, as that described by Bertrand* many years ago, or by use of the glass hemisphere of Schroeder van der Kolk,† or by the new upper condenser lens of ten Siethoff.‡ The last two methods are qualitative methods only, while that of Bertrand, although quantitative, permits of revolutions only in one plane. To supply the want of a universal condenser lens on which angular movements can be accomplished and measured in any direction, the writer has modified the Fedorow-Fuess universal stage by having a brass disk, L, constructed in the workshop of the Geophysical Laboratory to fit in the Fedorow-Fuess stage (large model) in place of the inner ring bearing the glass with cross hair (see fig. 7, page 332). Into this ring the upper lens of the ten Siethoff condenser lens system is inserted. The partially bevelled upper surface of this condenser lens has a radius of $1{\cdot}5^{mm}$, and permits, even with a No. 9 Fuess objective, angular movements of about 30° on either side of the normal. By means of a proper cylinder, N, of brass (fig. 7, page 332) resting on the cylinder containing the lower nicol, the remaining lenses of the condenser system are raised to the required distance from the upper lens. This type has proved extremely useful for work with artificial preparations, since by its use sections may be so placed that the most favorable measurements possible can be accomplished with the double micrometer ocular, and in certain cases even, where the optic axial angle is small, the same can be measured directly by means of the universal stage angles in convergent polarized light.

Although the measurements accomplished by the universal stage methods of Fedorow are made in parallel polarized light and with low power objectives, the same objectives can be used for weakly convergent polarized light with Bertrand lens and the position of the optic axes thus determined if it be possible to bring them within the field of vision and provided they are sufficiently distinct for accurate location under these conditions.

For general work, however, with thin sections in convergent polarized light, the methods requiring the double screw micrometer ocular are the most accurate and easy of application.

*E. Bertrand, Bull. Soc. Min. Fr., iii, 97-100, 1880.

† Schroeder van der Kolk, Zeitschr. f. wiss. Mikroskopie, viii, 459-461, 1891, and xii, 188-189, 1895.

‡ E. G. H. ten Siethoff, Centralblatt f. Min , 657, 1903.

The methods for determination of the plagioclase feldspars, rich in lime, which have been developed by F. Becke* by using Klein's lens and Czapski ocular, also by means of revolving drawing table, can be applied directly to the two screw micrometer ocular and more accurate data obtained by its use.

Parallel Polarized Light.

The introduction of the universal stage methods by Fedorow in 1893† and succeeding years placed a powerful instrument

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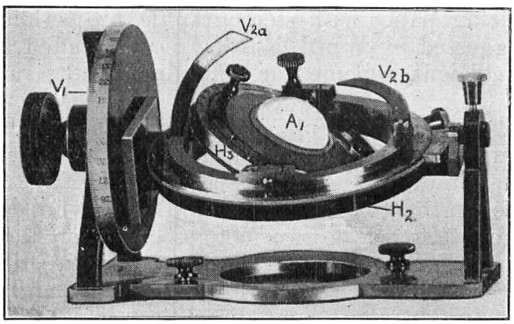

Fig. 20. In its present form the universal stage comprises, when attached to the microscope stage, five graduated circles; H_1, the horizontal circle of the microscope stage, H_2, the large horizontal circle of the universal stage, with H_3, the inner and thin section bearing circle, V_1, the large vertical circle, and V_2, an inner circle consisting of two segments V_{2a} and V_{2b}, and placed to measure the angles of revolution of the inner disk H_3 about a horizontal axis. Two glass hemispheres (A_1 being the upper) are usually employed with the stage to increase the angle of vision of the microscopic field.

of attack in the hands of petrologists. With his methods, it is now possible to obtain the optic properties of mineral sections which before were considered practically useless. The universal stage (fig. 20) can be attached securely to any suitable petrographic microscope; parallel polarized light only is used. By means of horizontal and vertical axes of revolution, a crystal section can be brought to any given position and revolved about any axis for optical examination.

In plane polarized light an optic axis is recognized by the fact that when placed parallel to the axis of the microscope it remains uniformly dark during a complete revolution about

*F. Becke, Tschermak's Min. petr. Mitth., xiv, 415-442, 1895.

† E. Fedorow, Zeitschr. Kryst., xxi, 574-678, 1893; xxii, 229-268, 1894; xxv, 225-261, 1895; xxvii, 337-398, 1897; xxix, 604-658, 1898.

that axis. By plotting these directions graphically in projection, and by determining extinction angles in given zones, it is possible not only to measure the optic axial angle, but also to determine the exact position of the optic axes with reference to the crystal plate, even though it may happen that neither optic axis appears within the field of vision.

The values for 2V thus obtained on different sections, however, are not all of the same order of exactness, as will appear later in the more detailed discussion of the different sections. It should be noted that in the Fedorow methods, as in the preceding, the measured angles are reduced by means of the average refractive index β to true angles within the mineral before plotting in stereographic projection. Here also the combination of tracing paper with stereographic projection plat as a base, as suggested by Wulff, is to be recommended as the best and most efficient scheme for obtaining results rapidly and accurately.

In these methods the rule of construction of Biot-Fresnel, that the planes of vibration of light waves propagated in any given direction bisect the angles between the two planes containing one of the two optic axes and the given direction, is used constantly, since the two factors, on which the universal stage methods are practically based, are the directions of the optic axes, as they may be determined directly, and extinction angles for certain zones and directions. Fedorow has also shown how it is possible with his methods to measure the refractive indices and also the birefringence approximately of a mineral from any section. These methods are not, however, germane to the purpose of this paper, and will not be discussed further. It may be stated that, although the methods of Fedorow involve the use of a stereographic projection plat and are in part graphical in nature, they are not difficult of application and often furnish results where other methods fail. In ordinary microscopic work, it frequently happens that one method will yield more accurate data in a shorter time than a second, and that particular method should then be chosen in preference to all others.

In general, the Fedorow methods are indirect methods and frequently involve a large expenditure of time to complete the observations on a single plate. For these reasons chiefly, petrologists have not adopted them so rapidly and generally as might have been anticipated, particularly as the old tested methods accomplish about what is desired by the busy petrologist who uses the microscope simply as a means to an end—to aid him in interpreting geological phenomena and relations.

When attached to the microscope, the Fedorow-Fuess stage (fig. 20) possesses, when in the 0° (primary) position, three horizontal circles, H_1 (microscope stage), H_2 and H_3, each circle graduated into degrees with verniers attached to H_1 and H_2; each of these circles is revolvable about a vertical axis; the horizontal axes of revolution and equivalent vertical circles are V_1 and V_2 also divided into degrees and V_1 with vernier attached. On the original stage described by Fedorow and made by Fuess, the partial scales V_2 are wanting and have been attached by the writer. These scales were constructed in the workshop of the Geophysical Laboratory, and have been found of practical service in several methods, especially those involving the principal sections of the triaxial ellipsoid of any mineral (page 353). Each partial scale of V_2 is accurately divided and carefully adjusted to the instrument. When not in use, the scale segments of V_2 can be inclined to a horizontal position V_{2b} and are then entirely out of the way. Measurements given below will be referred to this modification of the Fedorow-Fuess universal stage.

To increase the angle of vision of the field, two glass hemispheres, A_1 and A_2 (in fig. 20 A_1 only appears, A_2 being hidden by H_2), are usually employed; between these the preparation is placed, either cedar wood oil or glycerine being used to stick the same together and to reduce the effects of total reflection. For general work with the universal stage, it is advisable to follow the suggestion of Fedorow and use special circular (2^{cm} diameter) object glasses on which to mount the preparations in place of the ordinary rectangular (26×46^{mm}) thin section object glasses.

With the universal stage of this type, it is possible not only to bring a crystal section in any given position, but also to revolve that section about any axis; in short, by its use one has control over all possible directions and zones or axes of revolution of a crystal.

The Determination of the Crystal System of a Given Mineral by Means of the Universal Stage Method.—The fact that the universal stage allows the observer to study the different effects of a given mineral section on light waves transmitted through it in different directions, enables him to determine at once the crystal system to which the crystal belongs. This is accomplished most readily by means of extinction angles along certain directions, since the term extinction angle implies a definite relation between a given crystallographic and a given optical direction in any mineral. These relations vary with the crystal system of the mineral, and in fact are such definite functions of the same that, as Brewster* was the first to show, it is possible from extinction angles alone to determine definitely the crystal system of a given mineral. Briefly, an iso-

* Brewster, D., Phil. Trans., 1814, 187-218; 1818, 199-272.

metric mineral is isotropic for all directions of light wave propagation. Uniaxial minerals (hexagonal and tetragonal) appear isotropic for light waves passing along the principal crystallographic axis. For all other directions, they are anisotropic, but even then can generally be distinguished from biaxial minerals at once by the fact that every section of a uniaxial mineral contains the ω ellipsoidal axis, parallel with and normal to which it extinguishes. If the section be placed, therefore, in the position of darkness between crossed nicols and be revolved about a horizontal axis, V_1, it will continue to remain dark, if the ellipsoidal axis ω coincides with the axis of revolution, while if the ellipsoidal axis ω be normal to the latter, the crystal will exhibit interference colors of polarization on revolution except for sections of the prism zone. Biaxial minerals, on the other hand, do not in general remain dark for either axis revolution, and only do so for sections in the principal zones of the optical ellipsoid. Biaxial minerals show, moreover, two directions of apparent isotropism, those of the optic axes or optic binormals. To trace out the relations obtaining for orthorhombic, monoclinic and triclinic minerals and their distinguishing features, is not a difficult matter, but one for which space is not here available. They are in effect those which are used for the same purpose with ordinary methods.

The Accurate Determination of the Position of an Optic Axis when in the Field of Vision.—Although the underlying principles of determination by means of the universal stage are the same for all sections of a mineral, it has been found by experience that for certain sections, certain courses of procedure for measuring the optic axial angles are best adapted to produce the best results. Fedorow has divided the possible sections of any biaxial mineral into four convenient groups, each of which has its special characteristics and to each of which certain methods are best suited. The relative positions of the optic axes to and in the field of vision have been made the criteria for distinguishing these different groups; thus, in group (1) both optic axes are within the field of vision; (2) one optic axis is within the field of vision and makes an angle of less than 20° with the normal to the section; the second optic axis cannot be brought within the field of vision by any revolution of the stage; (3) one optic axis only appears in the field and makes an angle of over 20° with the normal to the section, the second optic axis lies entirely outside the field; (4) both optic axes lie outside of the microscopic field, the section in question being cut more or less nearly perpendicular to the optic normal, or about parallel to the plane of the optic axes; or about normal to the obtuse bisectrix of a mineral with small optic axial angle.

In case one or both optic binormals of a biaxial mineral section can be brought by revolution to coincidence with the axis of the microscope, it is necessary to determine these angles of revolution with the greatest possible accuracy. In all cases, an approximate determination is first effected by revolving the section about V_1 and H_2 until it is dark and remains dark during a complete revolution of the microscope stage H_1. In weakly convergent polarized light the optic axis can be seen in the center of the field. In ordinary microscopes, where absolutely plane parallel polarized light cannot be obtained, the section in such a position will not be perfectly dark, owing to

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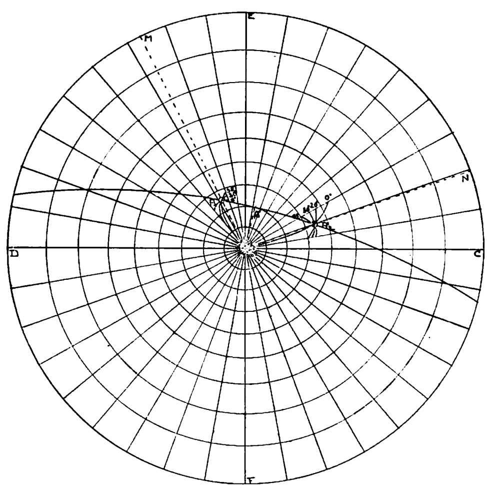

FIG. 21. In this figure, the method for locating the position of the optic axes by means of optical curves is illustrated. The figures 0°, 20°, 30° and 45° opposite the curves indicate the angles which the plane of vibration of the polarizer at the time of observation made with the plane of symmetry of the microscope.

internal conical refraction, but will preserve the same degree of slight uniform illumination for all positions of the microscope stage.

More accurate determinations of the position of an optic axis can then be made by means of extinction angles along definite directions, which, when plotted in projection, give rise to curves all of which pass through the optic axis. The average point of intersection of a set of such curves is then the true position of the optic axis in projection. (Fig. 21.)

Such curves have been called optical curves by Fedorow and are obtained most readily by first placing the crossed nicols in any given but fixed position, then turning H_2 through angles of 5° respectively, and for each position of H_2 determining the angle of inclination about V_1 for which the section is in the darkest position (0° extinction) (fig. 21); the same results can also be attained by first turning the preparation about V_1 a specified angle and then about H_2 until darkness ensues. By this method, those directions in the crystal are obtained (after proper reduction of observed angles to crystal directions by means of the refractive index) for which the extinction is zero for a given position of the nicols. The curve uniting these directions in projection is the optical curve for the particular position of the nicols to the axes of revolution. Analogous curves for other and different positions of the nicols are to be obtained and plotted in similar manner. All such curves pass through the optic axes and also the center of the projection. Their points of intersection in the projection determine, therefore, with considerable accuracy, the exact position of the optic axis or of both axes, in case both axes can be brought within the field of vision. Since such optical curves are intended solely to increase the accuracy of the determination of the positions of the optic axes, their approximate positions being known from the preliminary determination, it is necessary, in actual practice, to take readings of H_2 only 5° or 10° on either side of the approximately correct position of the optic axis as determined by the preliminary direct observations. Convenient positions of the nicols for optical curves are at 0°, 45°, 15° and 30° from the V_1 axis of revolution. If both optic axes appear within the microscopic field of vision, the most satisfactory method of measuring the optic axial angle by means of the universal stage is to determine the exact position of each axis by the above method and to plat the same in stereographic projection, in which the angle can be measured directly by graphical methods rather than by calculation, from the cosine formula, $\cos 2V = \cos V_{1a} \cdot \cos V_{1b} + \sin V_{1a} \sin V_{1b} \cos (H_{1a} - H_{1b})$ in which $2V$ designates the optic axial angle; V_{1a}, H_{1a}, the readings for the one optic axis; and V_{1b}, H_{1b}, those for the second.

The results obtained by the use of optical curves can be checked and verified by several of the methods described below, which are of general application and can readily be applied to this special case.

Fedorow has shown that in actual practice with minerals of weak to medium birefringence, the errors can be disregarded which are due to the reduction of all observed angles by means of the average refractive index of the crystal in place of the true refractive indices for each given direction; and likewise those errors which may arise from slight deviations in planes of vibration (extinction angles) due to refraction from steeply inclined plates and consequent elliptical polarization, are small quantities of a low order of magnitude and can be disregarded in general.

The method of measuring the optic axial angle by means of opti-

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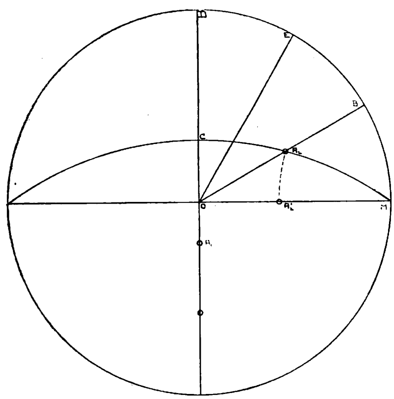

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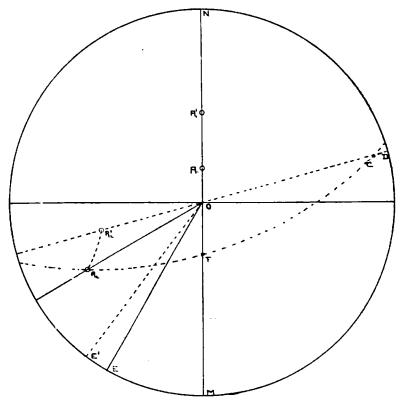

FIG. 22. In the method illustrated by this figure, the visible optic axis A_1 is brought to coincide with the plane DCO and the extinction angle DOE measured while the stage is in the horizontal position. A_2 must lie then in the plane OB, the angle BOE having been made by construction equal to DOE. The section is then revolved about OM (axis V_1) until the extinction angle becomes 45°, in which case the plane OM contains A_2, since A_1 has remained during this revolution in the plane DCO; on turning the specimen back to its original position, the line OM becomes the great circle CA_2M and the intersection of this great circle with the plane OB fixes A_2 definitely in the projection. In practice, the great circle CA_2M need not be drawn, since on placing the tracing over the plat it is only necessary to find that small circle A_2A_2', the arc of which intercepted between OB and OM is equal to the angle of revolution.

FIG. 23. The general method of extinction curves shown in this figure is applicable to all sections in which one optic axis A_1 can be brought to coincidence with the axis of the microscope. After the determination of the exact position of A_1 by means of optical curves the specimen is revolved about H_3 until A_1 coincides with the plane NO normal to the axis V_1 of the universal stage. The extinction angle MOE of the specimen in horizontal position is then determined; by construction EOA_2 is made equal to MOE; the specimen is then revolved about V_1 a convenient angle (apparent angle observed to be reduced to true angle), and the new extinction angle MOE′ ascertained. In the new position, the optic axis is contained in the plane OA_2', angle $E'OA_2'$ having been made equal to MOE′. The exact position of A_2 is then determined on the drawing on tracing paper by noting the small circle of the underlying projection plat, whose arc A_2A_2' intercepted between OA_2 and OA_2' is equal to the angle of revolution. This determination can be checked by drawing the great circle CF, which marks the position which the plane OA_2' would assume were the specimen turned back to its original position. In practice the position of A_2 is determined for different angles of revolution about V_1 and the mean position of all determinations taken as the most probable and correct location of A_2.

cal curves can be used only when both optic axes appear within the field of vision. In other cases, other methods are to be employed which involve either the measurement of extinction angles in zones or the determinations of the position of the principal planes of the ellipsoid, these latter to be plotted in appropriate projection. In most cases, however, one optic axis can be determined directly by optical curves, while the second optic axis makes a large angle with the normal to the section, and must be determined indirectly. A simple but comparatively accurate method to accomplish this consists in first turning the stage about H_2 until the known optic axis comes to lie in the plane normal to the axis V_1 (OCD, fig. 22), and in determining the extinction angle ($\sphericalangle$EOD) when the stage is in horizontal position and also at such an inclination about V_1 that the extinction angle is 45°; this can be recognized most readily by placing the nicols in the 45° position and then revolving the preparation about V_1 until darkness ensues. By thus ascertaining the angle of revolution necessary to attain the required 45° extinction angle, the great circle CA_2M is fixed with reference to the horizontal diameter, the plane in which the unknown optic axis A_2' must rest when the extinction angle is 45°. The intersection A_2 of the great circle CA_2M with the radius OB drawn at an angle, with the vertical line, of twice the angle of extinction ($\sphericalangle$EOD) for the plate in the horizontal position, fixes the position of the second optic axis in the projection. This method, however, is not always applicable owing to the indistinctness of extinction phenomena in steeply inclined sections (effect of elliptical polarization), and a second method of extinction curves, of which the above is only a special case, can be used to advantage. Having first placed the known optic axis in the plane normal to the axis V_1 as in the above method, measure the extinction angles for different inclinations of the stage about V_1 (the angles, as usual, to be reduced to real angles within the crystal by means of its average refractive index), and plot these directions of extinction in stereographic projection. (Fig. 23.) Under these conditions the radii, which make an angle with the vertical diameter OM_1 equal to twice the extinction angles, are evidently the planes containing the second optic axis A_2 whose exact location can be readily found by noting for two given radii, as OA_2 and OA_2', the small circle, whose arc A_2A_2' intercepted between the radii is equal to the angle of revolution of the stage. In practice it is advisable to repeat the determinations of the extinction angles and to take as angles of inclination those equivalent to 0°, 10°, 20°, 30°, 40° and 45° in the crystal on both sides of the normal to the section.

In actual work with this method, it happens occasionally that the determination of the location of A_2 is not accurate because of the acute angle between the radius and the small circle A_2A_2'. In such cases the writer has been able to apply with favorable results one of the two following new methods, which, like the preceding method, are based on the measurement of extinction angles for different angles of inclination about one of the horizontal axes of revolution of the universal stage. The new circle V_2 may render hereby valuable assistance.

In the first of the new methods, the horizontal position of the section is exactly that of the above method (fig. 24); A_1,

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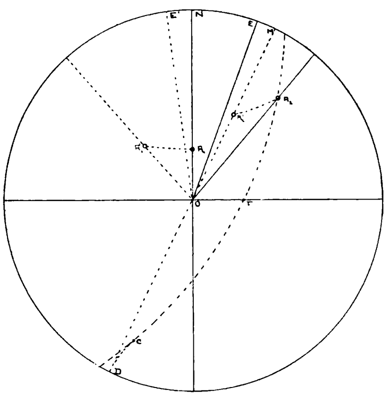

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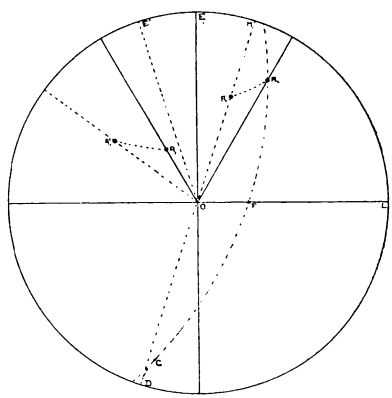

having been previously located accurately, is brought to coincidence with ON, and the extinction angle of the specimen in the horizontal position ascertained; and then instead of being revolved about the horizonal axis V_1 (the line OL in projection), it is revolved about V_2 (or ON in the projection) as an axis,* a given angle (apparent angle in air equivalent to true angle in crystal). A_1 travels during the revolution of stage to A_1' in the projection, the direction of extinction wanders from OE to OE′ and the plane OA_2 containing A_2, from OA_2 to OA_2', the angle $E'OA_2'$ being by construction $= E'OA_1'$. By recording the angle of revolution of the stage about ON (V_2) required to bring the section to its new position, it is not difficult to find in the projection that small circle, parallel to OL, whose arc A_2A_2' intercepted by the lines OA_2 and OA_2' is equal to the above angle of revolution and thus to locate A_2.

* The same effect can be produced by revolving the specimen 90° about H_1 and about V_1 as an axis.

To insure accuracy, this measurement should be repeated for several different angles of revolution and A_2 determined in each case. As in the first method, the great circle CF, indicating the original position of the plane OA_2', can be constructed and should pass through A_2 on the line OA_2.

The second new method differs from the first only in the fact that instead of placing the optic axis A_1 in the plane OE (fig. 25), and then measuring the extinction angle of the section in the horizontal position, the actual direction of extinction OE is brought to coincidence with the axis of revolution of the universal stage (V_1 or V_2); the section is then revolved a

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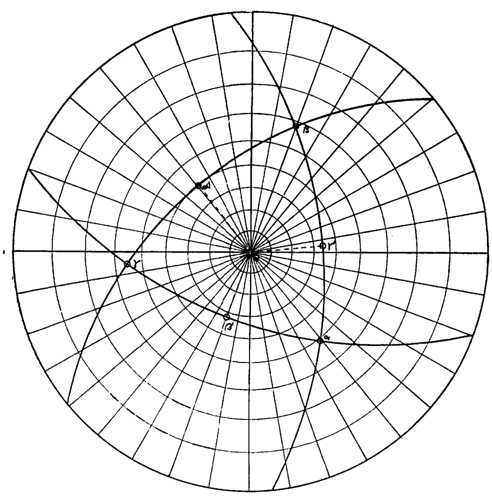

FIG. 26. In this figure, the great circles $\alpha\beta'\gamma$, $\alpha\beta\gamma'$ and $\alpha'\beta\gamma$ of the stereographic projection denote the traces of the principal planes of the optical ellipsoid within the crystal. They are fixed in position by determining the positions of H_3 and V_2 for which the section remains dark for all positions of inclination about the horizontal axis V_1 (V_2 being normal to V_1); the lines $O\beta'$, $O\gamma'$ and $O\alpha'$ are thus fixed both in direction and length and also the great circles $\alpha\beta'\gamma$, $\alpha\beta\gamma'$ and $\alpha'\beta\gamma$, the planes of symmetry of the ellipsoid, the intersections α, β and γ of which are in turn the ellipsoidal axes.

given angle about this axis and from the extinction angles the lines OA_2 and OA_2' determined whose arc is equal to the angle of revolution. The point A_2 is then the desired direction of the second optic axis.

In both new methods the determination can be varied by inclining the specimen first about V_1 as an axis and then determining a series of extinction angles for different angles of inclination about V_2 (V_2 in this case being normal to V_1) and thus locating A_2 afresh with each extinction. By establishing a set of observations about V_2 for each new position of V_1 it is possible to extend the number of observations indefinitely and thus to locate A_2 with the greatest possible accuracy. In fact, the position of A_1 in the projection is immaterial so long as this position be definitely known with respect to the axes of revolution (V_1 and V_2), since with A_1' located at any point in the projection it is still possible to locate A_2 by means of extinction angles for different angles of inclination about V_1 and V_2. This method, involving the use of both V_1 and V_2, is therefore a method of general application and is capable of furnishing reliable data on all sections so cut that one optic axis at least falls within the field of vision.

Still another method which furnishes trustworthy results and is of general application, consists in determining first the positions of the planes of symmetry and the axes of the ellipsoid within the crystal. (Fig. 26.) In this method, practically all of the graduated circles of the stage are brought into play, since not only must extinction angles be observed, but also the section revolved about the ellipsoidal axes and the exact position of each axis noted. The method of procedure consists in first placing the stage in the zero (primary) position, H_3, H_1, H_2, and V_1 in zero position, and V_2 normal to V_1; the section having any orientation and position. The section is then inclined about V_2 until darkness between crossed nicols ensues; if this be not the case, it is turned about H_3 a small angle, and the attempt made a second time, and so on until at a definite angle of inclination about V_2 darkness is observed. The preparation is then revolved about V_1, and if by chance the correct position be obtained, darkness will continue for every angle of inclination about V_1. This is usually not the case, and by repeated trial that position of H_3, H_2 is to be found for which the preparation remains dark for every angle of revolution about V_1. The angle of inclination V_2 and the directive angle H_3 determine then the position of one of the planes of symmetry of the ellipsoid within the crystal, e. g., the plane $\alpha\beta'\gamma$ of fig. 26, this being fixed by the line $O\beta'$; in similar fashion the planes $\alpha\beta\gamma'$ and $\alpha'\beta\gamma$ are located and plotted in the stereographic projection. This method of locating the planes of symmetry of the ellipsoid within the crystal is comparatively rapid and sensitive, and a fair degree of accuracy can be attained by its use. The new circles V_2 (fig. 20) which were attached in the Geophysical Laboratory to the large Fedorow-Fuess universal stage, have proved extremely serviceable and time savers in this method.

Having once determined the position of either α or γ by this method, and that of one optic axis A_1 by optical curves, the position of second optic axis A_2 is readily obtained, since the angle $A_1\alpha$ or $A_1\gamma$ is by definition equal to $A_2\alpha$ resp. $A_2\gamma$.

After some practice, the exact relative positions of H_2, H_3 can be found without difficulty for which darkness remains for all angles of inclination about V_1. To insure accuracy, however, the fact of remaining dark should be scrutinized very sharply, since the correct position is not always that of absolute darkness but rather that for which the same degree of darkness or intensity of uniform lighting obtains throughout.

From the complete determination by this method of the positions of α, β and γ, which should be mutually 90° apart, Fedorow has shown that the average refractive index of the mineral can be derived approximately, although the determination is not of sufficient accuracy to be of great practical value.

By this method of determining the positions of the principal sections of the ellipsoid, the distinction between uniaxial and biaxial minerals is greatly facilitated and the general problem solved for all possible sections. In case the position of neither optic axis can be determined directly, both optic axes lying outside the field of vision, the methods for measuring the optic axial are based solely on the determination of extinction angles along certain directions, and are of such a nature that by their use only very rough approximations to the true value of 2V can be obtained, errors of ± 10° and over being easily possible within the range of possibility. Fedorow has suggested one principal method applicable to such cases and the writer has had occasion to use several others. They are not so satisfactory, however, as the above methods, and are not of equal practical value. For the sake of completeness, they are described briefly in fine type below.

Section nearly perpendicular to the optic normal β.

In case the section of a mineral is so cut that it makes an angle of 30° or less with the plane of the optic axes, neither optic axis appearing, in consequence, within the field of vision, the above method places the observer in a position to measure the optic axial without even seeing either optic axis. The exact position of β can first be determined by this method, and then brought to coincidence with the microscopic axis, in which case the plane of the optic axis is horizontal. In this position the circles V_1 and H_1 are free and the section can be revolved about V_1 and extinction angles determined on H_1. (Figs. 27 and 28.)

Since the exact positions of α and γ have been determined and the two optic axes make equal angles with these bisectrices, it is possible by trial to bring one of the optic axes A_1 to coincidence with the normal to V_1 (fig. 27), and to test the accuracy of its position by means of extinction curves for different inclinations of the section about V_1. Thus let a be the acute bisectrix (fig. 27) and assume that one optic axis A_1 coincides precisely with the normal to axis V_1; A_2 is then the second optic axis and angle $A_1\beta a$ equal to angle $A_2\beta a$, and $A_1\beta a$ is the extinction angle. On revolving, now, the section about V_1, the optic axial point A_1 is brought to A'_1 and the extinction angle $A'_1\beta E$ for the new position of the section should bisect exactly the angle $A'_1\beta A'_2$. If this be not the case and the extinction angle be too large or too small, the section should be revolved about H_2 either counter clock wise (A''_1 to A_1) or clock wise, A'''_1 to A_1, through a small angle and a new set of measurements made, until after repeated trials the corrected position is to be found for which observation and construction agree precisely. The angle $A_1\beta a$ is then half the desired optic axial angle.

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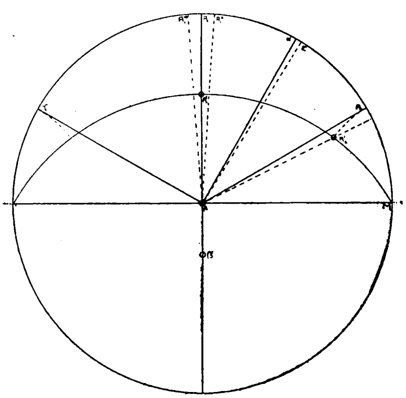

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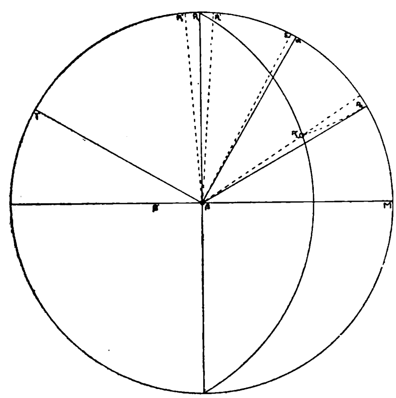

In certain cases this method of placing the one optic axis A_1 in the plane normal to the axis of revolution V_1 has been found unsatisfactory, and a new method used which consists in first bringing by trial the one optic axis to coincidence with the axis of revolution and then measuring the extinction angles for different angles of inclination about V_1 (or V_2) and testing the results of observation and construction until they coincide. The method is shown in fig. 28 and is so similar to the foregoing method (fig. 27) that further description is unnecessary.

For a section nearly normal to the obtuse bisectrix of a mineral, both optic axes lie again outside the field of vision and the optic normal β cannot be brought to coincidence with the axis of the microscope. The above methods do not apply, therefore, and new ones are required to meet the new conditions and of these the following has been found practicable by the writer.

Place the universal stage in the primary position, the axis of V_2 normal to that of V_1 and the circles H_1, H_2, and H_3, all in the horizontal position; determine the exact position of the obtuse bisectrix (α or γ, as the case may be) by the method of principal ellipsoidal planes (page 353), and bring it to coincidence with the axis of the microscope, the plane of the optic axes being then parallel to the vertical cross hair. (Fig. 29.) Revolve section some convenient angle about axis V_2 and then about V_1 (as shown in fig. 29), also through any suitable angle. Measure accurately

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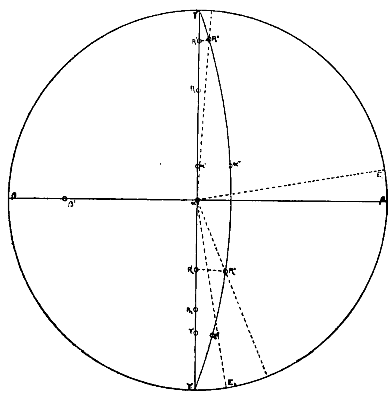


FIG. 29. To use the method indicated by this figure. turn the section so that its obtuse bisectrix coincides with the axis of the microscope (center of the projection plat), and the optic axial plane is parallel to the vertical cross hair; turn preparation about axis V_2 a convenient angle (reduce to true crystal angle equivalent to apparent angle observed in air or glass), and then about axis V_1 (normal to V_2) any suitable angle. and measure extinction angle of section in its new position. Plat these data in stereographic projection and find those two points A''_1 and A''_2 equidistant from the obtuse bisectrix and contained in the plane of optic axes in its new position, for which the observed line of extinction OE bisects the angle included between OA''_1 and OA''_2.

the extinction angle of the section in its new position. Plot data in stereographic projection after proper reduction of observed angles to true crystal angles; and find those two points A''_1 and A''_2 contained in the optic axial plane and equidistant from the obtuse bisectrix α'', which are so located that the observed extinction angle OE_1 bisects the angle $A''_2OA''_1$ (fig. 29). The angle $A''_1A''_2$ is then the desired optic axial angle, 2 V.

With the universal stage in its present form it is not always possible to execute the movements indicated in the above method, since when the obtuse bisectrix is brought to coincide with the axis of the microscope, the axis of V_2 is in general no longer horizontal and the revolution about V_2 is therefore along an inclined axis. In plotting the observed data, this fact should be carefully noted, otherwise errors may occur and nullify the results.

With the universal stage, it is thus possible to measure the optic axial angle of any grain of any transparent birefracting substance and to distinguish the biaxial and uniaxial minerals. The degree of accuracy of this measurement, however, is not of the same order of magnitude for all sections, but differs very materially with different sections. As a matter of experience it has been found that the most accurate results can be obtained on sections in which both optic axes appear within the field of vision; that good results can be had from sections which show only one optic axis within the field, while for sections in which neither optic axis appears within the field the determination is uncertain and at best only a rough approximation.

To summarize briefly the different methods best applicable to the four different possible cases cited above:

(1) The optic axes are both within the field of vision and inclined between 15°–55° with the normal to the section. Determine approximate position of the two optic axes by bringing each one, by means of H_2 and V_1, into the vertical position.

Determine position of each more accurately by means of optical curves in projection and check by means of extinction curves and exact location of principal planes of ellipsoid, especially the plane containing the optic axes.

(2) Section is nearly normal to an optic axis; one optic axis A_1 inclined less than 20° to section normal. Place stage in horizontal position,—H_2 and H_3 in horizontal position and V_1 normal to V_2—turn H_3 and incline about V_2 until optic axis coincides with the axis of the microscope; then revolve about V_1 and turn H_2 until darkness is attained, and thus determine plane of optic axes and β'. Incline V_2 back to 0° position, revolve about H_2 until the optic axis coincides with plane normal to V_1 and determine extinction curve, the intersection of which with plane of optic axes in projection fixes the position of the second optic axis accurately. Check by determining α and γ both from projection and observation; also by extinction curve for revolution about V_2.

(3) One optic axis inclined between 20°–55° within the crystal to the normal of the section, the second entirely out of the field of vision. Determine visible optic axis by optical curves and second optic axis by means of extinction curves, both about V_1 and V_2. Verify results by determination of α, β and γ.

(4) Both optic axes are entirely without the field of vision, i. e., are inclined at an angle of more than 65° in air with normals to the section. In such instances the location of the optic axes is accomplished by means of extinction angles alone and the values obtained are not accurate, since a slight error of 1° in the determination of the extinction angle may affect the

value of optic axial angle up to 30°. For accurate work, therefore, such sections are of little value in general at the present time for measuring the optic axial angle by the universal stage methods. In case, however, the section be about normal to the obtuse bisectrix, the measurement of the optic axial angle is much more certain and satisfactory.

As noted previously, experience has shown that the best and most rapid method of projection is that of Wulff, who uses an accurate stereographic or orthographic plat as a base and tracing paper on which to sketch the great circles and to execute the actual measurements.

Since the accurate measurement of the optic axial angle can be accomplished only on sections in which at least one optic axis is within the field of vision, it is of interest to note the probable relative frequency of occurrence of such sections in a rock section. The microscopic field of the universal stage fitted with glass segments includes an angle of about 60°, and the area on the surface of the unit sphere thus covered for a biaxial crystal is evidently $s = 4\pi.2\,(1-\cos\phi) = 4\pi.4\,\sin^2\frac{\phi}{2}$, 2ϕ being the angle of vision of the field reduced to the true value within the crystal; if the observed angle 2ψ be used, the average refractive index of the mineral β and that of the glass segments n should be introduced into the formula $s = 4\pi.2\frac{n^2}{\beta^2}\sin^2\frac{\Psi}{2}$. The probability, P_1, that a section showing an optic axis is evidently measured by the relative surfaces s to S, the surface of the sphere itself:

$$P = \frac{s}{S} = \frac{4\pi.4\,\sin^2\frac{\phi}{2}}{4\pi} = 4\,\sin^2\frac{\phi}{2} = 4\frac{n^2}{\beta^2}\sin^2\frac{\psi}{2}.$$

In case the areas covered by the two optic axes overlap, the formula should be changed, as Césaro has shown,* to

$$P = 4\,\sin^2\frac{\phi}{2} - \frac{2}{\pi}\left(\text{arc cos}\left(\frac{\sin V}{\sin\phi}\right) - \cos\phi\ \text{arc cos}\left(\frac{\tan V}{\tan\phi}\right)\right)$$

in which 2V denotes the angle between the optic axes.

Assuming an average refractive index of 1·65 for ordinary biaxial minerals, and 1·52 for the glass hemispheres, the probability of encountering a proper section ranges under these conditions from 4 to 10, in unaxial crystals, to 8 to 10 in biaxial crystals for which the fields for the optic axes do not overlap. The degree of probability is high and one should be able to find suitable sections in every slide for the measurement of the optic axial of each mineral present.

* G. Césaro, Mem. de l'Acad. Roy. d. Sci. d. Belgique, liv, 1895.

Fedorow* has also shown how it is possible to measure the birefringence $\gamma - \beta$ and $\beta - \alpha$ by use of the universal stage and the Fedorow mica-comparator and thus to ascertain the optic axial angle from the approximate formula $\cos^2 \frac{V_g}{2} = \frac{\gamma-\beta}{\beta-\alpha}$, either by graphical means or by calculation.

Lane† has also used the birefringence of different sections as a rough measure for the optic axial angle, but his methods are even less exact than those of Fedorow and can only give first approximations to the true optic axial angle of a given mineral. In cases of parallel intergrowths of different amphiboles and pyroxenes they have, nevertheless, rendered valuable service. Both his methods and the one of Fedorow will not, however, be discussed further in this paper.

Extinction angles of faces in zones whose axes lie in the plane of the optic binormals.

This method is particularly adapted to monoclinic minerals, as amphiboles and pyroxenes, and may be of service to secure a rough estimation of the optic axial angle of such a mineral. The underlying principle of this method is again the rule of Biot-Fresnel (page 322), and mathematical formulae suitable for its solution have been developed by Michel-Lévy,‡ Césaro,§ Harker,‖ Lane,¶ Daly,** and others. These formulae show that for the exact determination of the optic axial angle, the method of extinction angles on different faces in the same zone is not well adapted to optic axial determinations, especially when the optic axial angle of the mineral is small. In certain cases, it is possible to express this relation, as Lane has shown, in a slightly different form which is better adapted for measurements. Lane's method, as applied to the pyroxenes and amphiboles, consists in measuring the angle between the clinopinacoid and that face of the prism zone which shows the same extinction angle. For this case, in which the plane of the optic binormals contains the zonal axis, the formulæ of Césaro and Michel-Lévy reduce to the form,

$$\tan 2x = \frac{(\tan \lambda + \tan \mu) \cos v}{1 - \tan \lambda \tan \mu \cos^2 v} \qquad (1)$$

λ and μ being angles between the zonal axis and the two optic

* Feodorow, E. von, Zeitschr. f. Kryst., xxv, 349–356, 1896.

† A. C. Lane, this Journal (3), xliii, 79, 1892.

‡ Michel-Lévy et Fouquée, Minéralogie Micrographique, p. 368.

§ A. Césaro, Mem. de l'Acad. Roy. d. Sci. d. Belg., liv, 26, 1895.

‖ Harker, A., Miner. Magazine, x, 239, 1894.

¶ See Daly, Proc. Amer. Acad. Arts and Sci., xxxiv, 314, 1899.

** Daly, R. A., Proc. Amer. Acad. Arts and Sci., xxxiv, 314–323, 1899.

binormals respectively; and v the angle between a plane of the zone and plane of optic binormals. For the plane containing the optic axes $v = 0$ and the extinction angle becomes

$$\tan 2x = \frac{\tan\lambda + \tan\mu}{1 - \tan\lambda\tan\mu}.$$

For that plane which shows the same extinction angle, the equation

$$\frac{\tan\lambda + \tan\mu}{1 - \tan\lambda\tan\mu} = \frac{(\tan\lambda + \tan\mu)\cos v}{1 - \tan\lambda\tan\mu\cos^2 v} \quad (2)$$

or

$$\cos v = -\cot\lambda\,.\,\cot\mu \quad (3)$$

is evidently valid. In order that equation (3) be valid, $\cot\lambda\cot\mu$ must be less than unity, or

$$\lambda + \mu > 90^\circ.$$

But the values 2V, the optic axial angle, and $2x$, twice the extinction angle on the plane of optic binormals are related to λ and μ by the equations

$$\lambda + \mu = 2x$$
$$\lambda - \mu = 2\mathrm{V}$$

Snbstituting these values in (3), we find

$$\cos v = \frac{\cos 2x + \cos 2\mathrm{V}}{\cos 2x - \cos 2\mathrm{V}}. \quad (4)$$

or

$$\cos 2\mathrm{V} = \cos 2x.\ \frac{\cos v - 1}{\cos v + 1} \quad (5)$$

In table 2, the values of v are given for different optic axial angles (2V) and different extinction angles (x), the extinction angle being considered taken invariably to the acute bisectrix of the optic binormal angle. It is evident from this table that for small optic axial angles this method has no practical value for even rough measurements. The larger the axial angle, however, the more sensitive the method becomes.

Table II.

x_1	50	55	60	65	70	75	80	85
x_2	40	35	30	25	20	15	10	5
2V								
10	134°27′	118°58′	109°03′	102°08′	97°10′	97°37′	91°21′	90°00′
20	133 29	117 47	107 47	100 49	95 50	92 10	90 00	88 39
30	131 45	115 42	105 32	98 31	93 31	90 00	87 40	82 23
40	129 05	112 30	102 08	94 01	90 00	86 29	84 10	82 50
50	125 04	107 47	97 10	90 00	85 59	81 29	79 11	77 52
60	118 58	100 49	90 00	82 50	77 52	74 28	72 13	70 57
70	109 03	90 00	79 11	72 13	67 30	64 18	62 13	61 02
80	90 00	70 57	61 02	54 56	50 55	48 15	46 31	45 33

Measurement of the optic axial angle on the total refractometer.

Pulfrich,* Soret,† Viola,‡ Cornu§ and Wallérant‖ have shown that it is possible on a single section of a biaxial or uniaxial mineral to determine not only the three principal refractive indices α, β and γ, but also, by observing the planes of polarization of each wave corresponding respectively to $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$, to determine accurately the relative position of the principal planes of the ellipsoid to the given section; and from the accurate refractive indices thus ascertained to figure the optic axial angle with great exactness. These methods, however, require specially ground and polished sections and are not, in general, microscopic methods, although the total refractometer of Wallérant is attached directly to the microscope and is employed on thin uncovered and polished sections of rocks. Unfortunately, the writer has had practically no opportunity to work with the total refractometer of Wallérant, and is, therefore, not in a position to judge personally of its fitness for optic axial angle determinations. Viola and others have shown that on the Abbe total refractometer results of great accuracy and certitude can be obtained rapidly and without difficulty. The mineral plates should measure then $1^{\text{sq mm}}$ or over to furnish sufficiently intense reflexion signals for nice adjustment and measurement.

Measurements.

So much space has been devoted above to the theoretical considerations and descriptions of methods that in this section only a part of the available observational data can be enumerated and a brief résumé of the results presented. Enough data will be offered, however, to indicate certain inferences bearing on the relative accuracy and applicability of the different methods under test.

Different minerals, as aragonite, topaz, muscovite, etc., were first chosen and oriented sections cut to show the different phenomena required by the several methods. The correct optic axial angle for each mineral was then measured in sodium light on a Wülfing-Fuess axial angle apparatus, the angle obtained thereby being adopted as the standard of comparison for all methods. For each mineral a series of measurements of the optic axial angle for different sections and by the different methods was taken and the relative degree of accuracy of each method judged, not only by the results obtained, but also

* Pulfrich, C., Das Total reflektometer, Leipzig, 1890.
† Soret, Zeitschr. Kryst., xv, 43, 1899.
‡ Viola, C., Zeitschr. Kryst., xxxi, 40–48, 1889; xxxvi, 245–251, 1902.
§ Cornu, Compt. Rend., cxxxiii, 125; Bull. Soc. Min., xxv, 7.
‖ Wallérant, F., Bull. Soc. Min., xx, 12–26, 1898.

by the factors on which the method itself is dependent and their relative exactness under the conditions of observation.

Measurements with Axial Angle Apparatus.

The optic axial angles obtained in sodium light on the Wülfing axial angle apparatus varied slightly and the average of five determinations of each angle is given below:

Topaz, Willard Co., Utah.
$2E = 126° 13'$. $2V = 66° 42'$.
Aragonite, Bilin, Bohemia.
$2E = 31° 09'$. $2V = 18° 22'$.
Muscovite (*a*)
$2E = 71° 40'$
Muscovite (*b*)
$2E = 59° 42'$.

Measurements with the Becke drawing-table.—To economize space, the results are given below in their reduced form ready for plotting directly in projection, the angle θ denoting the longitude from the horizontal E–W line of the projection and ρ the polar distance; A_1 as usual denotes the visible axial point and P_1 any point on the dark axial bar.

Topaz.

(*a*)	ϕ	ρ
A_1	0°	5°·0
P_1	+65	20 ·5

In projecting these angles and performing the requisite mechanical operations, the optic axial angle thus determined on this section was $2V = 62°·5$. For a second section the values were:

	ϕ	ρ	
A_1	− 8°	3°·8	$2V = 70°$
P_1	−72	19 ·6	

For a third section:

	ϕ	ρ	
A_1	+27°	8°·8	$2V = 63°$
P_1	−58	23	

The average of these three values is 65°·2.

Aragonite.—In aragonite the optic axial angle is so small that both axial bars A_1 and A_2 are visible and the direct determination of $2V_{A_1A_2}$ should in all cases be accurate within one degree. The birefringence is so strong, however, that the measurements involving a point P_1 or P_2 on the dark axial bar and consequent introduction of the refractive index β for that point, may be decidedly incorrect. The use of the refractive index β presupposes only slight differences between the refractive indices of the mineral in order that the errors thus caused may not be too large.

(1)

	ϕ	ρ	
A_1	$-80°$	$17°\cdot5$	$2V_{A_1A_2} = 17°\cdot4$
A_2	-130	23	$2V_{P_1A_1} = 17\ \cdot5$
P_1	$+15$	$18\ \cdot4$	

(2)

A_1	$-10°$	$8°$	$2V_{A_1A_2} = 18°\cdot3$
A_2	$-17\ \cdot1$	$10\ \cdot6$	$2V_{P_1A_1} = 21$
P_1	$+42$	21	

(3)

A_1	$+44°$	$28°\cdot3$	$2V_{A_1A_2} = 18°\cdot0$
A_2	$+85$	$19\ \cdot5$	$2V_{P_2A_2} = 17\ \cdot6$
P_2	$-15\ \cdot5$	$18\ \cdot4$	

(4)

A_1	$-56°$	$11°\cdot2$	
A_2	-140	$14\ \cdot3$	$2V_{A_1A_2} = 17°\cdot5$
P_1	$+29$	$23\ \cdot4$	$2V_{P_1A_1} = 16$
P_2	$+158$	$26\ \cdot0$	$2V_{P_2A_2} = 23$

The values for $2V_{A_1A_2}$ do not differ over 1° from the true value, while those for $2V_{P_1A_1}$ differ as much as 5° from the true value.

Muscovite. (b)

	ϕ	ρ	
A_1	$0°$	$35°\cdot6$	$2V_{A_1A_2} = 71°\cdot2$
A_2	0	$-35\ \cdot6$	

Double Screw Micrometer Ocular.—The data given below appear also only in corrected form ready for plotting in projection, the actual scale readings having been reduced to equivalent angles in air and these in turn figured to true angles within the crystal, by means of the refractive index β. The errors observed above in aragonite sections because of strong birefringence apply equally well here. In the following tables H indicates the horizontal and V the vertical micrometer screw of the ocular.

Topaz.

(1)

	H	V	
A_1	$3°\cdot5$	$0°$	$2V = 64°\cdot5$
P_1	$5\ \cdot5$	$17\ \cdot5$	

(2)

A_1	$5°\cdot3$	$\cdot3$	$2V = 65°\cdot5$
P_1	$7\ \cdot6$	$17\ \cdot2$	

Aragonite.

(1)

	H	V	
A_1	7°	9°·8	$2V_{A_1A_2} = 18°·0$
A_2	−10 ·8	9 ·8	$2V_{P_1A_1} = 18 ·5$
P_1	17 ·8	6 ·8	$2V_{P_2A_2} = 14$
P_2	−17 ·5	6 ·8	

(2)

	H	V	
A_1	4°·6	20°	
A_2	−13 ·20	20	$2V_{A_1A_2} = 17°·9$
P_1	5	3	$2V_{P_2A_2} = 19$
P_2	−23 ·5	3	$2V_{P_1A_1} = 23$

(3)

	H	V	
A_1	7°·3	1°·8	
A_2	−11 ·5	1 ·8	$2V_{A_1A_2} = 18°·5$
P_1	20 ·6	18 ·8	$2V_{P_1A_1} = 18$

Muscovite. (*a*)

(1) *a*

	H	V	
A_1	35°·5	0°	2E = 71°
A_2	−35 ·5	0	

(1) *b*

	H	V	
A_1	0°	36°	2E = 72°
A_2	0	−36	

Muscovite. (*b*)

	H	V	
A_1	30°·1	0°	2E = 60°·2
A_2	−30 ·1	0	
A_1	0°	30°	2E = 60°
A_2	0	−30	

Measurements with the Fedorow-Fuess universal stage.—The angles given below were read directly on the different circles of the universal stage and before plotting in projection require reduction to true crystal angles by means of the refractive index β of the mineral and μ (=1·5239) of the glass hemispheres used. The letters H_1, H_2, H_3 and V_1, V_2 designate the different circles of the universal stage (see fig. 11) on which the angles were read. The angles after the letter N designate the angle made by the principal plane of the lower nicol with that of the microscope.

Topaz. Section after 001 (acute bisectrix).—A direct preliminary determination of the position of the optic axes in parallel polarized light was first made and the approximate location of each axis determined. These values were later corrected by means of optical curves.

Direct preliminary determination.

	H_1	H_2	H_3	V_1	V_2	
A_1	180°	90°	294°	35°	—·5°	2V = 66°·6
A_2	"	"	"	36°	—·5°	

Corrected by method of optical curves.

				N = 0°		N = 30°		N = 45°	
				V_1		V_1		V_1	
H_1	H_2	H_3	V_2	A_1	A_2	A_1	A_2	A_1	A_2
180°	80°	294°	1°	33·5	—33·5	37	—39·5	33	—37
"	85	"	"	35	—36	36	—38	33·5	—37·5
"	90	"	"	35·5	—36	34	—37	34	—36·5
"	95	"	"	34	—34·5	33	—36	34	—37
"	100	"	"	33	—34·5	33	—34	35	—37

After proper reduction of these angles, the corrected angle, obtained directly from the stereographic plat, is 2V = 66°·5.

Topaz. Section nearly normal to an optic axis.—The determination in this case can be most readily accomplished by first locating A_1 accurately by optical curves and then fixing the position of A_2 in projection by means of the principal ellipsoidal planes.

Optical curves for A_1.

				N = 0°	N = 30°	N = 45°
H_1	H_2	H_3	V_2	V_1	V_1	V_1
180°	80°	225·5	—1	4·5	4·5	5
"	85	"	"	5	5	5·5
"	90	"	"	6·5	6	5
"	95	"	"	7	6	5
"	100	"	"	8	5	4

After reduction to true angles, the position of A_1 in projecon was found to be: H_1, 180°, H_2, 90°, H_3, 225°·5, V_1, 6° and V_2, 1°. The $\gamma\beta$ ellipsoidal plane was located by: H_1, 180°, H_2, 0°, H_3, 315°, V_1 —, V_2, 26°, while for the $\alpha\gamma$ ellipsoidal plane, the readings were: H_1, 180°, H_2, 90°, H_3, 225°·5, V_1, —, V_2, —1°. The optic axial angle thus determined in projection plat is 2V = 64°. In such cases, where the section is nearly normal to an optic axis, the method of extinction curves is not of practical value, owing to the difficulty of determining extinction angles with requisite accuracy.

Topaz. Section nearly normal to obtuse bisectrix.—Optic axial angle was found by first locating the principal ellipsoidal planes $\alpha\beta$ and $\alpha\gamma$ and then measuring the extinction angle of the section when α coincided with the microscope axis and after revolution of the section from that position through known angles about V_1 and V_2.

For the ellipsoidal planes the readings were:

	H_1	H_2	H_3	V_1	V_2
$\alpha\beta$ plane	180°	90°	236°·5	--	0°
$\alpha\gamma$ plane	"	"	326·	--	−17·5

For α in coincidence with the microscope axis, the readings were found from the projection to be: H_1, 185°, H_2, 90°, H_3, 326°, V_1, 30°, V_2, 22°·5. After the revolution about V_1 and V_2, the angles recorded were: H_1, 189°·5, H_2, 90°, H_3, 326°, V_1, 40°, V_2,−15°.

From these angles 2V was measured in projection and found to be 66°.

By direct observation of the optic axis, the same angle was also obtained. This method may in favorable instances give reliable results, but in general it cannot be considered an accurate method, owing to the undue influence in projection of slight deviations of the extinction angle on the value of the optic axial angle.

Second Section:

	H_1	H_2	H_3	V_1	V_2
$\alpha\beta$ plane	180°	90°	140°	--	+ ·5°
$\alpha\gamma$ plane	"	"	234	--	−32·5
Optic axis A_1	"	"	140	4°	+ ·5

The optic axis A_1 was determined by direct readings. After proper reduction to true angles the value 2V=63° was obtained from the projection plat.

Topaz. Section about perpendicular to the optic normal.—In this instance the principal elliposoidal planes were first determined and ellipsoidal axis β brought to coincide with the microscope axis and the extinction angle measured in that position. By trial that position of H_2 was found for which A_1 coincided with the principal plane of the lower nicol, and the optic axial angle thus ascertained by measuring the extinction angles of the section in different positions of V_1 and comparing the data of observation with those obtained by graphical methods from the projection plat on the assumption that A_1 did actually coincide with the principal plane of the lower nicol. In like manner, the section was revolved about V_2 and extinction angles measured until theory and observation furnished identical results.

The principal ellipsoidal planes of the section were determined by the readings:

	H_1	H_2	H_3	V_1	V_2
$\beta\gamma$ plane	180°	90°	333°	--	+16°
$\beta\alpha$ plane	"	"	241·5	--	− 1

For the different positions of H_2, the extinction angles for a given angle of revolution about V_1 were:

H_1	H_2	H_3	V_1	V_2
145°	123°	333°	17°	31°
145·5	122	"	"	"
144·5	124	"	"	"
144·3	123·5	"	"	"

On plotting these values in projection, it was found that 2V was about 64°–67°, but a more decisive result was not attainable. The method is not accurate and can only furnish very rough approximations.

In the second method, which involves revolution about an axis normal to that of the above, the values observed were:

H_1	H_2	H_3	V_1	V_2
148°·5	33°	332°	17°	31°
148 ·5	34	"	"	"
149	32	"	"	"

and from these angles, 2V was found to lie between 64 and 68°.

The determination cannot be termed satisfactory and this method, like the above, can furnish only rough approximations to the true values of 2V.

Summary.

(1) The optic axial angle of minerals in the thin section can be determined under the microscope in either convergent or parallel polarized light.

(*a*) In convergent polarized light, methods for the measurement of the optic axial angle are available for all sections in which at least one optic axis appears within the field of vision. Of these the method requiring the use of the Becke drawing table is of general application and furnishes results of a fair degree of accuracy—the usual probable errors being about ± 1° if both optic axes be visible, and ± 5° if only one optic axis be visible. More accurate and somewhat simpler in manipulation and of the same general application is the method involving the new double sorew micrometer ocular, described above. This ocular combined with the method of projection of Professor Wulff, is a general extension of the Mallard method, and, like the Becke method, utilizes the rule of Boit and Fresnel which defines the planes of vibration for any direction of wave propagation. With this ocular the probable errors of determination on sharp interference figures should not exceed 1° if both optic axes are visible, nor 3° if only one optic axis appears in the field.

(*b*) In parallel polarized light, the methods involving the Fedorow-Fuess universal stage are used and furnish satisfactory results, provided the position of one optic axis can be determined directly. If both optic axes are outside of the field of vision, the results obtained are usually unsatisfactory and inaccurate. Theoretically, it is possible to measure the optic axial angle of any biaxial transparent mineral on any section by means of the universal stage. If both optic axes appear within the field of vision, the error of determination should not exceed 1°, and if only one of the optic axes be visible, the accuracy may decrease to ± 5°. The exact location of a visible optic axis is assisted somewhat by use of the method of optical curves. Having once fixed the location of one optic axis, that of the second is determined by the method of extinction curves. If both optic axes lie entirely outside of the field, special methods must be resorted to, but in general without marked success, owing to the great difference in the value of 2V caused by a very slight deviation in the measured extinction angle.

The range of the field of vision of the universal stage is greater than that of any possible interference figure; the Fedorow universal stage methods are, therefore, applicable to a greater number of sections than the methods with convergent polarized light and may furnish results on sections otherwise useless for ordinary methods. Both experience and theory show that for all these methods the accuracy of the determination varies considerably with the section and mineral in question. The most accurate results can be obtained on sections for which both optic axes appear within the field of vision; less accurate but still satisfactory measurements can be made when only one optic axis appears, particularly when it is situated about midway from the center to the margin of the field.

For convergent polarized light, the general extension of the Mallard method by means of the new double screw micrometer ocular is the most satisfactory and accurate method available, but good results can be had by use of the Becke drawing table.

The readings in both methods require to be reduced to equivalent crystal angles and plotted in stereographic projection. The optical axial angle is then measured upon the plot by graphical methods.

(2) For the purpose of plotting and measuring observed and calculated angles, the stereographic projection is without doubt best adapted to optical work in general. The stereographic plot of Plate I is a photographic reproduction of the accurate drawing by Professor Wulff published in the Zeitschrift für Krystallographie.

(3) Since the interference figures are roughly orthographic projections of the phenomena in space, an accurate ortho-

graphic plat (fig. 1) has been added with the standard stereographic plat to serve as a base for future plotting of similar phenomena.

(4) Plate II is intended to serve as a graphical base by which to solve Mallard's formula, K sin E=D; also $n_1 \sin \alpha_1 = n_2 \sin \alpha_2$; also $\gamma' - \alpha' = (\gamma - \alpha) \sin \alpha_1 \sin \alpha_2$, α_1 and α_2 being the angles included between the given direction of wave propagation and the two optic axes respectively.

(5) The Mallard formula and method were tested by a new method and the agreement of the formula with fact for the special objective used and the particular precautions observed found to be remarkable. It was evident that for each microscopic objective similar tests should be made at intervals across the entire field in order to insure accuracy and certainty in the results obtained.

(6) A disk-shaped type of the Becke drawing table was constructed in the Geophysical Laboratory and found satisfactory in practice.

(7) An improvement was made in the Fedorow-Fuess universal stage, consisting in the addition of two hinged graduated circles on which to read the inclinations of the second vertical circle V_2, and found to be of service in several methods.

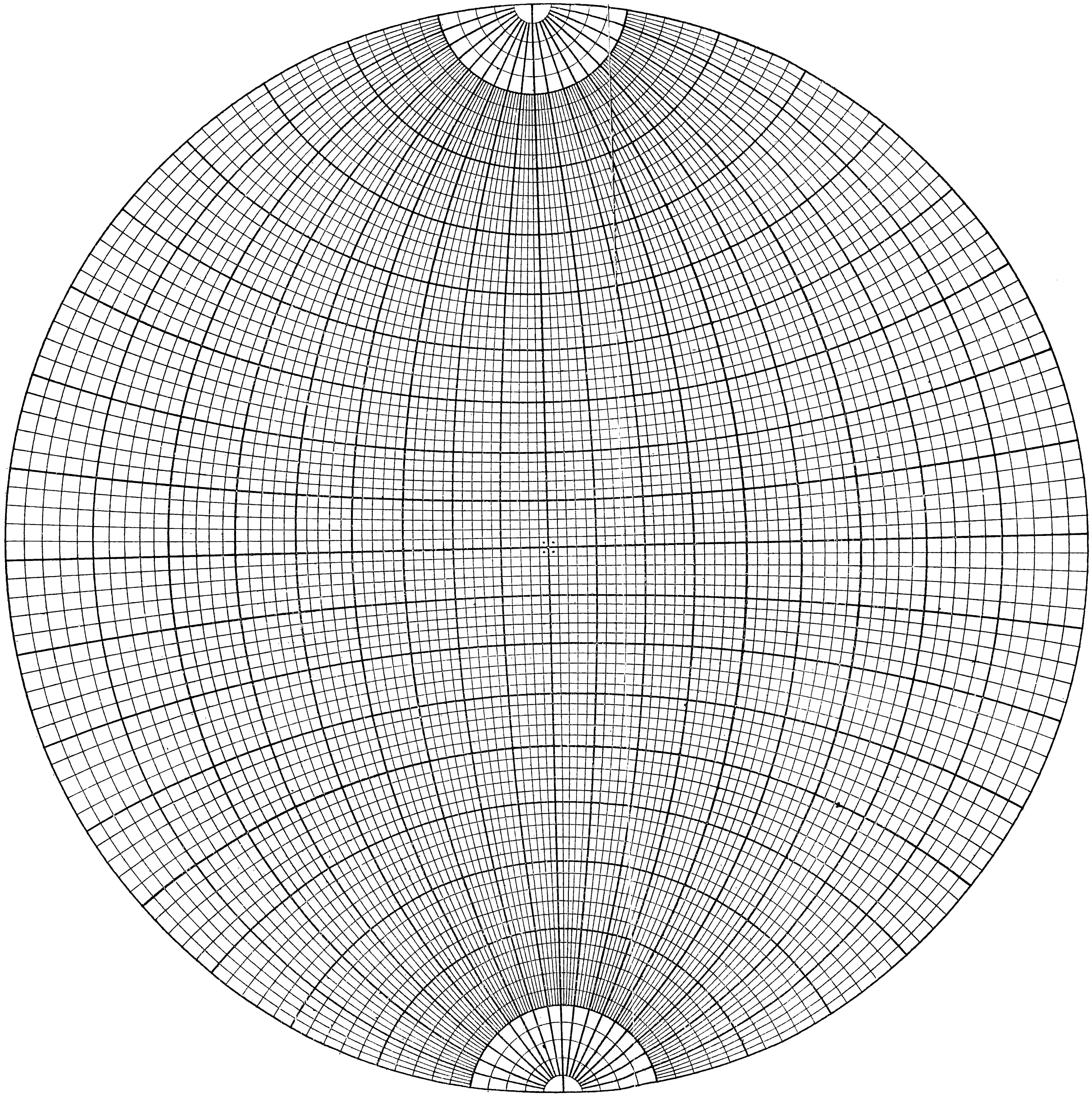
(8) A new form of condenser lens system which combines the advantages of the ten Siethoff qualitative adjustable condenser system with the exact movement of the universal stage, was also described and applied to the examination of minute mineral sections, especially of artificial preparations.

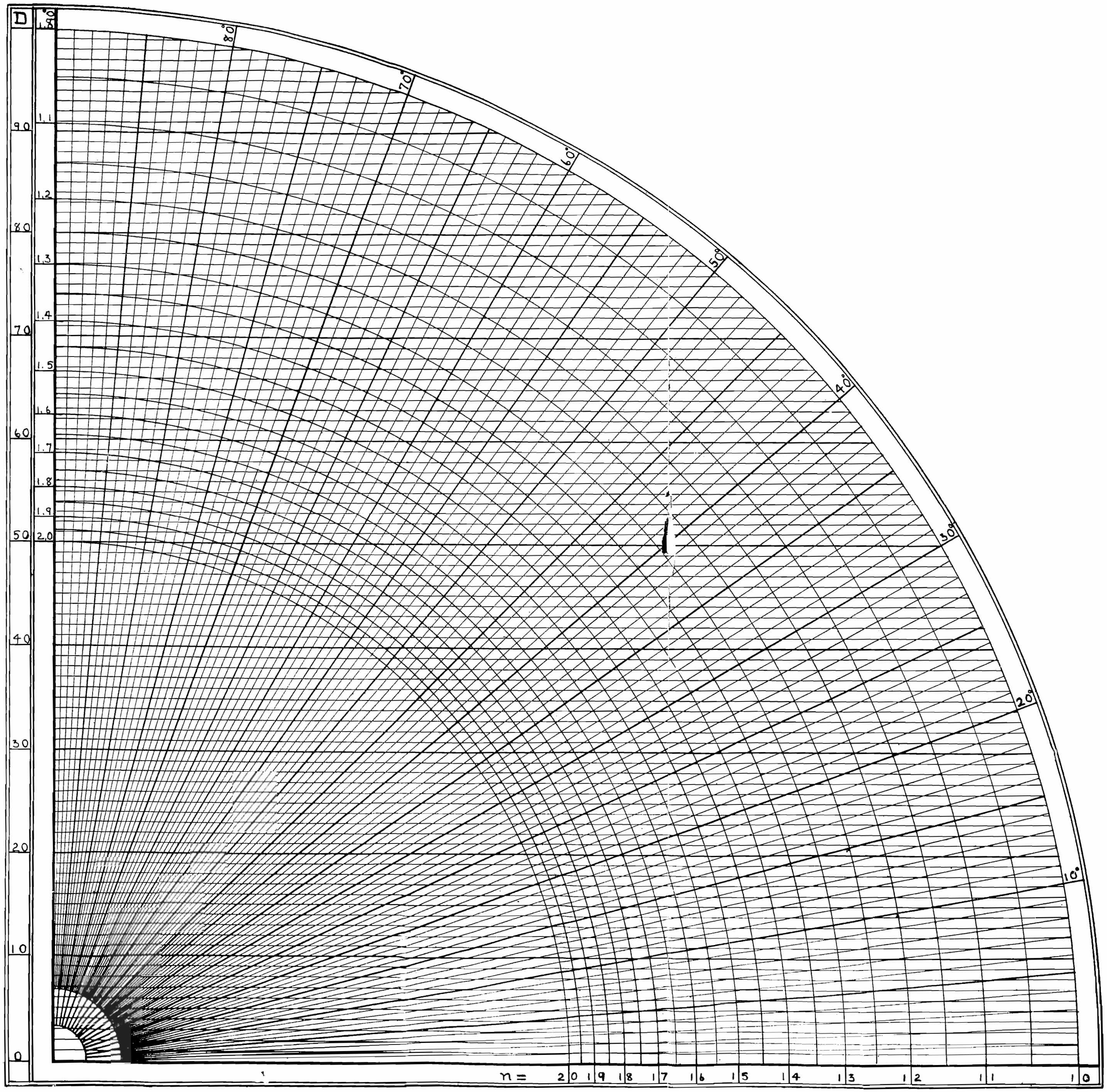
(9) A set of accurate drawings of the position of the dark axial bar of the interference figure in convergent polarized light for sections cut at various angles with one optic axis but always so that the optic axis is still visible, has been prepared, and the theoretically probable limits of error of determinations of the optic axial angle by the different methods and for the different sections established graphically.

(10) In the course of the investigation, several methods, based solely on extinction angles for different faces, were tried, but without exception they were discarded because of the difficulties in the measurement of the extinction angle and the undue influence of small differences in extinction angle on the value of the optic axial angle.

Geophysical Laboratory,
Carnegie Institution of Washington, May 15, 1907.

Plate I




D
0
10
20
30
40
50
60
70
80
90
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
10°
20°
30°
40°
50°
60°
70°
80°
90°
n = 2.0 1.9 1.8 1.7 1.6 1.5 1.4 1.3 1.2 1.1 1.0