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# TEMPERATURE AND HEAT FLOW IN A WELL NEAR COLORADO SPRINGS.

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ABSTRACT. Temperature measurements are reported for a well near Colorado Springs at an elevation of 6200 feet. This well is thought to penetrate the Basement rocks for perhaps as much as 2500 feet. A small flow of water, while complicating the interpretation of the temperatures, also affords a possibility of estimating roughly the conductivity of the rocks. The thermal gradient is determined to be 20° C/km; the mean conductivity is thought to lie between 0.005 and 0.007 (in cal, cm, sec and °C), giving a value for heat flow between $1.0 \cdot 10^{-6}$ and $1.4 \cdot 10^{-6}$ cal/cm$^2$/sec. This is shown to be consistent with measurements in other Colorado wells.

## INTRODUCTION.

In October, 1946, temperatures were measured in a well about 18 miles south of Colorado Springs, Colorado. These measurements are of interest for several reasons. In the first place, this well is unusually high, starting at 6200 feet above sea level; in the second place, the well was thought to penetrate the pre-Cambrian Basement for perhaps 2500 feet; in the third place, there has been a small flow of water since the completion of the well, and some interesting points arise in connection with the distribution of temperature in a flowing well.

These measurements have been used to determine the thermal gradient and thence the heat flow in a region where current theory would naturally suggest that the heat flow should be abnormally high, as a consequence of the high elevation and of the close proximity to the mountains of the Front Range. The purpose of the present paper is to report on the measurements and conclusions for this well; the more general problems, to which these measurements contribute a single detail, will be developed in subsequent papers. A brief preliminary account of the main considerations involved has already appeared[1]: it seems likely that the study of heat flow in elevated regions may ultimately furnish important clues to the fundamental question of the distribution of radioactive heat production.

## ACKNOWLEDGEMENTS.

Thanks are due to many individuals for the opportunity to perform these measurements. The existence of this well was brought to my attention by Prof. R. A. Daly, who learned of it from Prof. C. J. Roy, of Louisiana State University. The measuring equipment was kindly lent by Prof. T. S. Lovering of the University of Michigan, who had it on loan from the U. S. Geological Survey. Access to the well was arranged by Prof. D. B. Gould of Colorado College, who also provided transportation from Colorado Springs to the site of the well. I am indebted to Professor Gould and to Mr. Harry Oborne of Colorado Springs for valuable discussions of the geology of the region and for a copy of the log of the well, to various students at Colorado College, particularly Mr. Arthur Cosgrove, for assistance in setting up the equipment and making the measurements, and to Mr. Hart for permission to carry out the work on his property. Several colleagues have kindly read the manuscript and given valuable suggestions for its improvement.

## DESCRIPTION OF THE WELL.

This well, known as the L. V. Hart No. 1 Wildcat, of Colorado Petroleum, Incorporated, is located 800 feet east of the west line, 50 feet north of the south line of the southeast quarter of Section 17, T17S, R67W, El Paso County, Colorado, on a small, rather flat, structure known as the Red Creek Anticline. This region is described and mapped in the Colorado Springs Folio, No. 203, of the U. S. Geological Survey, by G. S. Findlay, from which Figure 1 has been traced. Recent work, slightly to the northwest, has been reported by

[1] Birch, Francis: 1947. Trans. Am. Geophys. Union, 28, pp. 792-797.

Glockzin and Roy.[2] The area is within the region used for summer field work by the Louisiana State University.

The central exposed portion of the anticline is identified as the Pennsylvanian Fountain formation which appears to rest locally against the Pikes Peak granite toward the northwest;

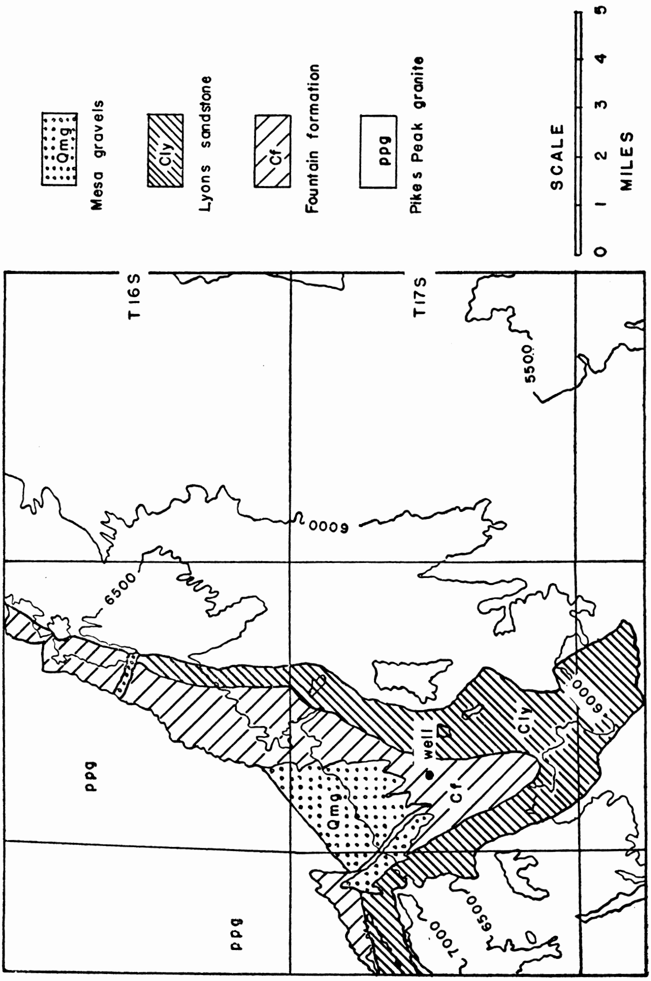


Figure 1. Topography and areal geology in the neighborhood of the Red Creek well.

the contact is said to be visible in stream cuts, though generally covered by the Quaternary gravels. The anticline plunges gently toward the south and east, while the ground

[2] Glockzin, A. R., and Roy, C. J.: 1945, Bull. Geol. Soc. Amer., **57**, 819-828.

surface rises slowly toward the north and west, about 300 feet in the first two miles and thereafter more rapidly; except for a few small hills, the surface elevation falls off slowly to the south and east.

Several other wells have been drilled not far from this site. About four miles to the southeast, in Pueblo County, Section 4, T18S, R67W, the Continental Oil Company drilled, and are said to have reached "granite" at 1784 feet. An old well, two miles to the southwest, was drilled to 2666 feet, without reaching "granite."

Drilling, with rotary tools, commenced November 18th, 1945, and was completed February 20th, 1946, at approximately 3500 feet, but a cement plug was set at 2970, with 5½-inch casing to 2872 feet. This casing was perforated at a number of depths: 1934-1939 feet; 2248-2256; 2475-2480; 2482-2488; 2496-2500; 2502-2507 and 2614-2622. "Additional water" was encountered as a result of the last perforation. Perforation was also accomplished between 925 and 950 feet for the purpose of providing water for the owner of the property. As indicated by the temperature measurements, however, most if not all of the water must at present enter at a depth of about 2300 feet.

A detailed log is available for the present well, but there appears to be some difference of opinion about its interpretation. Entries are given for every five to ten feet down to 3504, where drilling was abandoned. It seems hardly worthwhile to reproduce this entire log which covers eight long sheets of typewritten copy. From the surface to about 1000 feet, the cuttings are described as "coarse free granite wash," "red micaceous grit," with a few entries of "dense red limestone." About 750 feet of the first 1000 is characterized as "granite wash" of various degrees of coarseness and cementation. This is the Fountain formation, as usually understood, composed principally of arkosic sandstone of variable composition and grain size. At 1022 feet there is a change to "vitreous chert," and "dense red-brown dolomitic limestone," continuing to 1070. Artesian water broke through at 1065, but "may have been encountered above this depth." From 1070, there is another long stretch of "granite wash," "red micaceous grit," traces of chert and shale. Two feet of "fresh granite" were recovered by coring from 1323 to 1326 feet. Of this core Doctor

Roy writes[3]; "It is typical of the finer phases of the Pikes Peak batholith." Thereafter, we find "granite wash" and "micaceous grit" or "sandstone," fragments of "fresh granite," down to 2368. At this point the water from the upper part of the hole was shut off; and the character of the cuttings changed. From now on, we find "fresh granite fragments" or "fresh granite," alternating with "granite wash" and "grit." There are a few appearances of "fresh gabbro," of "biotite schist," and occasional mention of "oil stains." Of the 1300 feet between 1070 and 2370, more than 1200 feet are described as either "granite" or "granite wash." Of the 1136 feet between 2368 and 3504, approximately 400 are described as "fresh granite," 100 as "fresh gabbro" and the rest, mainly "granite wash" with "red grit" and "biotite schist" as the chief minor types. Another foot of core was obtained from 3500-3504, described as "fresh pink granite."

Other data are of interest in interpreting the record of this well. Only nine bits were used in drilling from the surface to 1050 feet, or an average of 117 feet per bit; 69 bits were used in drilling from 1050 to 2285, or an average of 18 feet per bit. An electrical resistivity log showed three distinct types of pattern, one very fine and irregular to 1022 feet, another, somewhat broader, between 1022 and 1070, and finally a very firm, long-period record from 1070 to the bottom, showing the highest values of resistivity.

The conclusion of the geologist on the well, Mrs. Fanny C. Edson, was that the lower part, below 2368, "could be a talus deposit of blocks and boulders formed adjacent to its source; the middle part, 2368 to 1022, could have been formed by torrential mountain streams with a very steep gradient . . ." The whole section is regarded as a "mechanically formed terrestrial molasse." On this view, the Basement rocks have not yet been penetrated. Other geologists familiar with the area feel confident that the Basement surface is at 1070, with a thin layer of the Manitou limestone underlying the Fountain arkose, which extends only to 1022. Doctor Roy writes[4] that examination of the cuttings "leaves no doubt that the Paleozoic stops at about 1070." Another possibility seems to be that the true Basement is encountered at 2368, where there

[3] Private communication.

[4] Private communication.

is a fairly clear change of rock type recorded in the log, and where, as we shall see later, the principal inflow of water now seems to arise. Since a contact between arkose and granite sometimes is deceptive even in surface exposures, it is probably not surprising that its position is not easily determinable by examination of well cuttings. More frequent coring might have proved helpful in the interpretation; it is noteworthy though not decisive, that at the only two levels where cores were sought, they yielded good-sized lengths of "fresh granite." The choice is between granite in place, with numerous dikes and inclusions which seem to be typical of the Pikes Peak granite, or a coarse conglomerate, predominantly granitic, which for the purpose of these measurements is practically indistinguishable from massive granite. Nevertheless, it is somewhat disappointing that what is generally regarded as the most reliable method of exploring underground, that is, sinking a well, should lead to an uncertain conclusion of this nature.

## MEASUREMENT OF TEMPERATURE.

The temperatures were measured with the portable equipment belonging to the United States Geological Survey, designed and used in many wells by Dr. C. E. van Orstrand. This equipment has been fully described[5]; it is essentially a hand-operated reel of steel wire on which thermometers may be lowered to and raised from any desired depth, as measured on a counter actuated by the motion of the wire. Several thermometers are placed in a steel tube which is sealed with pipe fittings to prevent entrance of water, attached to the wire and lowered to the desired depth; after a time sufficient to permit the thermometers to come to the temperature of the water, the thermometers are raised and the temperatures recorded. A time of seven minutes is said to be sufficient in the water, but in all cases longer times, usually approximately 30 minutes, were allowed.

The use of the maximum thermometer requires a means of cooling the thermometer below the lowest temperature to be measured. On this occasion, there was a small amount of snow on the ground which could be used for cooling the ther-

[5] Am. Pet. Inst. Prod. Bull. No. 205, 1930, Part II; Econ. Geol. 19, 1924, pp. 229-248.

mometers; the air temperature was also usually less than the water temperature, so that no special precautions were required. Under these conditions, when readings are made at successively greater depths, since each reading is higher than the preceding one it is not necessary to shake down the mercury after each reading. In some cases the mercury was shaken down, in others, the thermometer was simply read, replaced in the pressure tube, and lowered to the new (and greater) depth.

Four maximum thermometers were included with the equipment, graduated on the Fahrenheit scale and readable to 0.1° F. It was quickly discovered that one of these was unreliable,—the mercury did not remain at its maximum height. The other three were used together thereafter and gave remarkably consistent results, usually differing by no more than the uncertainty of reading. They were repeatedly compared with a standard mercury thermometer, and they proved extremely insensitive to mechanical vibration, the ride to and from Colorado Springs occasioning no change in the reading. It is of primary importance to prevent exposure of the thermometers to the water pressure, as was demonstrated by a few readings of other thermometers attached to the outside of the pressure-tight tube. At 1500 feet, the exposed thermometer read nearly 10° F higher than the protected thermometer, as a result of deformation of the glass envelope by the water pressure. Measurements in which glass thermometers are exposed to water pressure may be expected always to give excessively high gradients; the effect does not begin to be serious until the depth exceeds about 500 feet.

On October 11th, measurements were obtained at 500, 1000, 1500, 2000 and 2800 feet; these included the experiments with the exposed thermometers (Centigrade scale instruments) and a certain amount of fumbling with the unfamiliar equipment. On October 12th, with increasing expertness, measurements were made at 0, 200, 1200, 1400, 1600, 1800, 2300 and 2600 feet, in all cases using the three reliable thermometers. The recorded temperatures are shown in Table I.

By collecting the water issuing from a side connection near the top of the pipe, it was determined that the outflow was roughly 1 gallon per minute; it is possible that there was another outlet lower down from which some water escaped, and that this is a minimum value of flow at this time. It is of course impossible to say whether the flow has remained at this rate since the well was drilled, but there seems to be no evidence that it was ever vastly different.

TABLE I.

Measured temperatures in the Red Creek well.

| Depth feet | | Observed temperatures °F | Mean temperatures °F |
|---|---|---|---|
| 0 | | 61.7 (Standard Thermometer) | 61.7 |
| 200 | | 64.0, 64.0, 64.0 | 64.0 |
| 500 | First try | 57.8 (bad thermometer) | |
| | | 67.0 | |
| | | 68.0 (exposed thermometer) | |
| | Second try | 69.8 (exposed thermometer) | |
| | | 67.2, 67.1 | 67.1 |
| 1000 | | 72.5, 72.4, 76.5 (exposed) | 72.5 |
| 1200 | | 75.0, 74.8, 74.8 | 74.9 |
| 1400 | | 77.0, 77.2, 77.2 | 77.1 |
| 1500 | | 63.0 (bad thermometer) | |
| | | 78.4, 88.0 (exposed) | 78.4 |
| 1600 | | 79.0, 79.0, 78.9 | 79.0 |
| 1800 | | 80.7, 80.7, 80.6 | 80.7 |
| 2000 | | 82.0, 82.0 | 82.0 |
| 2300 | | 83.2, 83.2, 83.2 | 83.2 |
| 2600 | | 87.0, 87.0, 86.9 | 87.0 |
| 2800 | | 89.2, 89.2, 89.3 | 89.2 |

The mean temperatures are plotted against the depth in Fig. 2. The mean annual air temperature (29-year average) for Colorado Springs is 47.3° F. It is clear that the water temperature is higher than the rock temperatures near the surface; a study of Fig. 2 leads to the conclusion that the principal flow of water originates near 2300 feet, and that above this depth the water temperatures are always higher than the undisturbed rock temperatures at the same depth; below 2300 feet, the water is probably stagnant, and the measured temperatures are also the rock temperatures. Further quantitative support for this interpretation is given in a later section.

The data have also been analyzed to show the variation of gradient, in Table II. The gradient is given for 1400 to 1600 feet; the measurement at 1500, which depends upon a single thermometer, is not used. Except just above 2300 feet, the gradient is remarkably uniform in view of its determination by measurements at 200 foot intervals. The average from 2300 to 2800 is 21.9° C/km; the average from 2800 to the mean

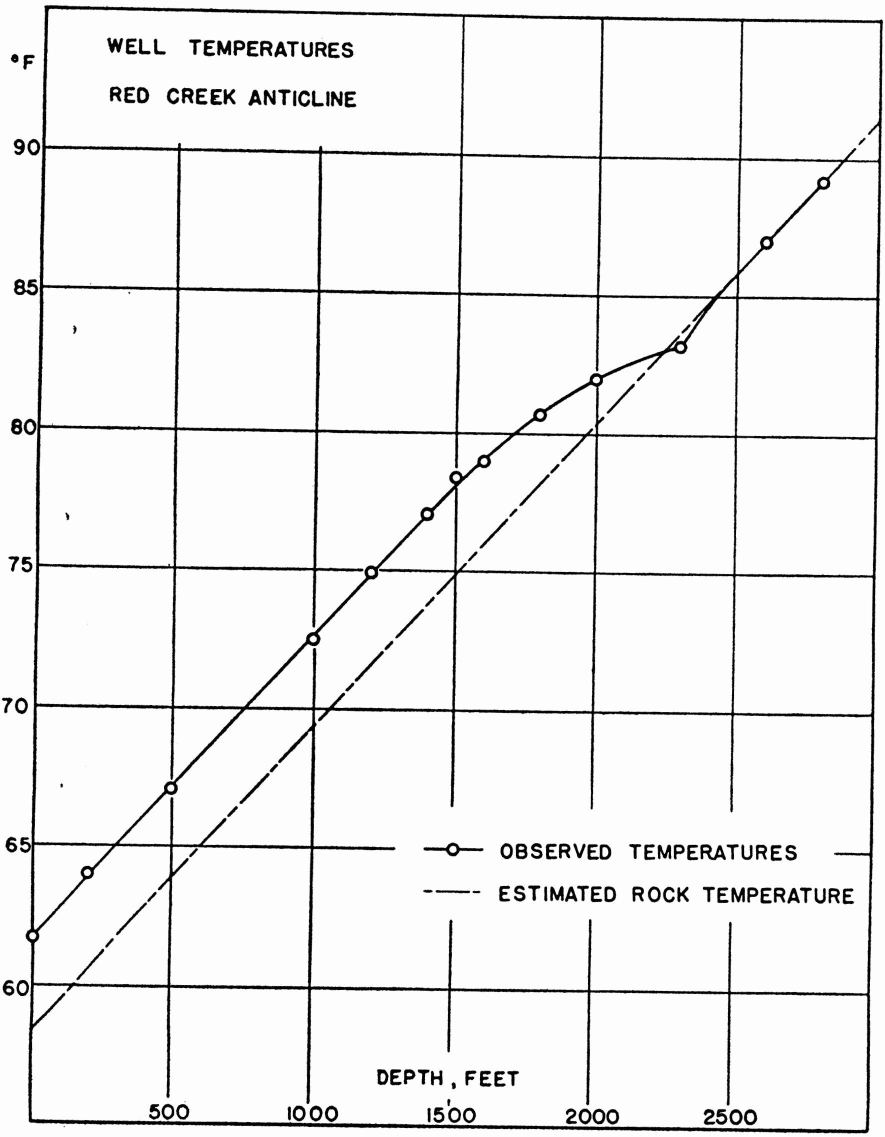


Figure 2. Observed and estimated temperatures in the Red Creek well.

annual air temperature is 27.2° C/km, certainly too high for the gradient in the rock.

The calculated values in the third column of Table II are obtained by passing a straight line through the two points at 2600 and 2800; they demonstrate that the temperatures above about 1600 feet are simply offset by a nearly constant amount from this straight line. It is possible that there is a small additional flow of water from some depth between 500

TABLE II.

Temperatures and Gradients in the Red Creek well.

| | Temperatures | | | | |
|---|---|---|---|---|---|
| Depth feet | Observed °F | Calculated °F | Obs.-Calc. °F | Mean Gradient °F/100′ | °C/km |
| 0 | 61.7 | 58.4 | 3.3 | | |
| | | | | 1.15 | 21 |
| 200 | 64.0 | 60.6 | 3.4 | | |
| | | | | 1.03 | 19 |
| 500 | 67.1 | 63.9 | 3.2 | | |
| | | | | 1.08 | 20 |
| 1000 | 72.5 | 69.4 | 3.1 | | |
| | | | | 1.20 | 22 |
| 1200 | 74.9 | 71.6 | 3.3 | | |
| | | | | 1.10 | 20 |
| 1400 | 77.1 | 73.8 | 3.3 | | |
| | | | | .95 | 17 |
| 1600 | 79.0 | 76.0 | 3.0 | | |
| | | | | .85 | 15 |
| 1800 | 80.7 | 78.2 | 2.5 | | |
| | | | | .65 | 12 |
| 2000 | 82.0 | 80.4 | 1.6 | | |
| | | | | .40 | 7 |
| 2300 | 83.2 | 83.7 | —0.5 | | |
| | | | | 1.27 | 23 |
| 2600 | 87.0 | 87.0 | 0 | | |
| | | | | 1.10 | 20 |
| 2800 | 89.2 | 89.2 | 0 | | |

and 1000 feet. We find an extrapolated surface rock temperature of 58.4° F and an offset of about 3.3° F. This gives a calculated temperature at 2300 feet 0.5° F higher than the measured temperature, a possible effect of the entrance of water near this level, but the gradient in the water above 1400 feet is very closely the same as below 2300. The best value is 1.1° F/100′ or 20° C/km. The difference between 58.4° F and the mean annual air temperature (at Colorado Springs) is 11° F, a high but not exceptional figure. Van Orstrand[6]

[6] Van Orstrand, C. E.: 1939, for example, see Chapter VI in "Internal Constitution of the Earth," McGraw-Hill, New York.

has found that in arid or semi-arid regions, the temperature just below the surface may be considerably higher than the mean air temperature. For four wells in Colorado, he found an excess of extrapolated ground temperature over mean air temperature ranging from 2.5° F to 8.5° F; differences of more than 12° F are given for a number of wells in southern California.

## THEORY OF TEMPERATURE DISTRIBUTION IN A FLOWING WELL.

Water has been flowing from this well since its completion about eight months before the temperatures were measured. The flow has probably varied during this period and the measured rate, about one gallon per minute, can be taken only as a rough indication of the average. The temperature-depth curve suggests that most of this water enters at a depth of about 2300 feet, very nearly at the rock temperature at that depth. Thereafter, as the water flows up the pipe, its temperature falls, though at a lower rate than the rock temperature, until, above about 1600 feet, the rate of fall of water temperature is observed to become sensibly uniform. In this upper region, the water has reached a distribution of temperature such that the loss of heat from a given linear section of pipe to the rock, at each level, is equal to the net excess of heat brought into this section by the flow of water. The gradient in the water is then equal to the gradient in the rock, the temperature in the water exceeding that of the rock at some distance from the well by the same amount at each level. The uniformity of gradient measured in the water between 1600 feet and the surface is thus a reflection of a uniform gradient in the undisturbed rock, which appears to continue nearly unchanged to the maximum depth.

If no heat were lost by the water after its entrance at rock temperature, the difference of 3.3° F between water and rock would be reached in a distance of 300 feet, corresponding to a gradient in the rock of 1.1° F/100′. The actual distance for reaching this difference of temperature is about 800 feet. With higher rates of flow, a greater distance would be required, and the water would reach the surface with a higher temperature, approaching the temperature of entrance. Thus in shallow wells, with even relatively small yields of water, the "steady" temperature difference might never be reached;

while with a flow of as much as 20 gallons per minute even the deepest well would not be deep enough. The "steady" temperature difference between the water and the rock at a distance continues to increase as the nearby rock warms up, but the change becomes exceedingly slow after a short time.

The "steady" temperature difference may be calculated approximately by the use of the known solution for a linear source of heat. Neglecting conditions near the surface of the ground and near the entrance level of the water, the problem is one of symmetry about an axis, and the flow of heat in the rock may be represented very closely by the ideal case of flow from a continuous linear source of heat. Within the pipe, the distribution of temperature in the water is, of course, very different from that of the linear source, the real temperature varying only enough across the section to provide the lateral flow of heat responsible for cooling the water and warming the rock. The details of the temperature distribution within the pipe are not necessary for an approximate account of the difference of temperature between the water and rock.

Suppose that heat is generated continuously along a line in an infinite, uniform medium, initially at constant temperature; then the rise of temperature at the time t, measured from the moment at which the production of heat began, and at a distance r from the line, is given by[7]

$$T = (H/4\pi K) \int_{z=r^2/4kt}^{z=\infty} z^{-1} e^{-z}\, dz = -(H/4\pi K)\, Ei(-r^2/4kt)$$

[7] See, for example, H. S. Carslaw, Conduction of Heat in Solids, Dover Publications, New York, 1945, p. 152. The solution given here is for an instantaneous line source; the required solution for a steady line source may be found by the method of p. 151. Thus, the rise of temperature T produced by an instantaneous line source of strength Q is given as $T = (Q/4\pi kt) \exp(-r^2/4kt)$. For a steady source producing H cal/sec per cm of length, commencing at $t = O$, the temperature rise at time t is

$$T = \frac{H}{4\pi K} \int_{t'=o}^{t'=t} \frac{e^{-r^2/4k(t-t')}}{t-t'}\, dt'$$

Substituting $z = r^2/4k(t - t')$, we find,

$$T = (H/4\pi K) \int_{z=r^2/4kt}^{z=\infty} z^{-1} e^{-z}\, dz, \quad \text{as given above.}$$

where k is the diffusivity and K the conductivity of the medium, and H the amount of heat lost per second, per centimeter length of the source. Here $-\mathrm{Ei}(-x)$ is a short notation for the integral, $\int_{z=x}^{z=\infty} z^{-1} e^{-z}\, dz$, a function for which tables exist.[8] For small values of the argument, we have the approximation, $-\mathrm{Ei}(-r^2/4kt) = \log_e (4kt/r^2\gamma)$, with $\gamma = 1.7810$.

To this solution for the rise of temperature produced by the line source, we may add a temperature, independent of time, increasing linearly with depth, which represents very nearly the temperature in the undisturbed rock. At points sufficiently far from the ground surface, the actual rock temperature will thus be the sum of the undisturbed rock temperature and the rise of temperature given above. The line source solution is divergent for points on the source itself, but it is finite for points a small distance away. We are interested in the temperature at the pipe, which we take to be very closely the temperature of the water. For a 5.5-inch pipe, the inside diameter will be about 14 cm, the outside about 15 cm. Taking $r = 7.5$ cm, $t = 2 \cdot 10^7$ sec (eight months) and $k = 0.01$ cm$^2$/sec, we have $r^2/4kt = 7 \cdot 10^{-5}$ and $4kt/r^2\gamma = 8000$. Since $\log_e 8000 = 8.99$, we find, nearly enough, $T_p = (9H/4\pi K)$ as the relation between the excess temperature of the water in the pipe, the conductivity K of the rock, and H, the rate of loss of heat from the pipe. The value assumed for diffusivity has been found to be reasonably representative for a large group of rocks, but a large error here has only a small effect, since it is the logarithm of this quantity which enters in the result. This statement is illustrated in Table III, where the effect of variation of the time t is shown. Evidently, the variation of either t or k affects the result in the same manner.

Now H is the difference between the heat brought into a section one centimeter long, per second, and the heat taken out, by the flow of water. If v is the linear velocity of flow, a the radius of the pipe, $\sigma$ the specific heat of the water and $\alpha$ the gradient of temperature along the pipe, then $H = \pi a^2 v \sigma \alpha$, and $T_p = (9\pi a^2 v\sigma\alpha/4\pi K)$.

In the present case, $\pi a^2 v$ is about 1 gallon/minute or 63 cm$^3$/sec, $\sigma = 1$ and $\alpha$ is 20° C/km or $20 \cdot 10^{-5}$° C/cm. With

[8] Jahnke, E., and Emde, F.: 1938, Tables of Functions, Teubner, Leipzig and Berlin.

these values, we find $T_p = 0.009/K$, where $T_p$ is in centigrade degrees. Thus we may use the observed offset of temperature to calculate the conductivity K. Taking $T_p = 3.3° F = 1.8° C$, we have $K = 0.005$. This is of the right order, but the various large uncertainties, especially as to the rate of flow, which enters directly in the result, should discourage emphasis of this particular figure. It is unlikely, however, that the conductivity of any important fraction of the formations through which this well passes exceeds 0.007. Measured values for good granites[9] range between about 0.005 and 0.008, and these are exceeded only by quartz-rich rocks such as quartzites, quartz schists and hard sandstones of nearly pure quartz, by dolomites and by ultra-basic rocks. None of these rocks of high conductivity is present in this well in appreciable amounts; the upper part of the hole is in arkosic sandstone or conglomerate whose conductivity is unlikely to be excessively high, and the lower part in "granite" and "granite wash," for which 0.007 seems to be a reasonable upper limit. Unpublished measurements show a range of values for "arkosic" sandstones between 0.004 and 0.007. Whether suitable sampling can ever be performed for the variable Fountain formation is questionable. The evidence of the temperature measurements is perhaps the best reason for concluding that the mean conductivity is relatively uniform from top to bottom, and that it probably falls between 0.005 and 0.007.

With these values for the conductivity and a thermal gradient of 20° C/km, we arrive at an estimate for the heat flow comprised between $1.0 \cdot 10^{-6}$ and $1.4 \cdot 10^{-6}$ cal/cm²/sec, the higher figure being very nearly the maximum allowable.

There are several other details of interest in the solution which we have used. Since $-Ei(-0.01) = 4.038$, the temperature rise at the pipe, $T_p$ reached nearly one-half of its value at the time of measurement for $r^2/4kt = 0.01$, or $t = 14 \cdot 10^4$ sec $= 39$ hours. Values of $T_p$ for other intervals of time are given in Table III. It is clear that, for a considerable time, the difference in temperature between water and rock changes very little if the flow of water continues at about its present rate.

[9] Birch, Francis, and Clark, Harry: 1940, Amer. Jour. Sci., **238**, p. 529 and 613. See also Section 17, Handbook of Physical Constants, Special Paper No. 36, Geological Society of America.

We may also inquire how far from the well the rock is appreciably heated. At the present time, the value of the integral is roughly 9. For what value of r does this reach 1 per cent of this value? From the tables, we find that $-Ei(-z)$ is equal to 0.09 for $z = 1.6 = r^2/4kt$, and hence that for $t = 2.10^7$, $r = 1140$ cm or 11.4 meters for a rise of 1 per cent, or in this case, 0.03° F. For the same time, the heating reaches 10 per cent of its value at the pipe at a radius of 4.9 meters. Appreciable heating of the rock by the well is thus confined to a radius of a few meters at the present time, but this radius increases, in proportion to the square root of the time, as long as the flow continues.

TABLE III.

Rate of variation of temperature rise.
(for $r = 7.5$ cm, $k = 0.01$ cm²/sec)

| Time seconds | | $-Ei(-r^2/4kt)$ | $T_p$ °F |
|---|---|---|---|
| $14.10^4$ | 39 hours | 4.038 | 1.5 |
| $1.10^7$ | 4 months | 8.29 | 3.1 |
| $2.10^7$ | 8 months | 8.99 | 3.3 |
| $3.15.10^7$ | 1 year | 9.43 | 3.5 |
| | 10 years | 11.74 | 4.3 |
| | 100 years | 14.0 | 5.1 |
| | 1000 years | 16.3 | 6.0 |

A somewhat better approximation is obtained by treating the well as a steady cylindrical surface source of heat, rather than as a line source. In this case,[10] the rise of temperature T at a distance r from the center of the cylinder of radius a, after an interval of steady heating t, is

$$T = (H/4\pi K) \int_{\tau = o}^{\tau = t} d(\log\tau) \exp\left[-(a^2 + r^2)/4k\tau\right] Jo(iar/2k\tau),$$

where Jo is the zero-order Bessel's function of the first kind. For r = a, at the surface of the pipe, this becomes,

$$T_p = (H/4\pi K) \int_{x = a^2/2kt}^{\infty} x^{-1} e^{-x} Jo(ix)\, dx$$

An approximate value for this integral may be obtained by a rough numerical method, by breaking the range of integration

[10] Carslaw, p. 153. Again the expression in the text refers to an instantaneous cylindrical surface source and must be integrated as on p. 150.

into intervals and using a mean value of Jo(ix) for each integral. The part remaining under the integral sign is then —Ei(—x), and may be found from the tables. In this way, a value 9.2 is obtained, as compared with 9.0 for the line source. The difference is not significant in this problem.

## DISCUSSION.

According to the interpretation advanced above, the gradient of temperature in the rock is very nearly uniform to 2800 feet, and equal to 20° C/km. This value may be derived either from the readings near the bottom of the well, or, in terms of the account given of the water temperature, from the linear portion of the curve for the upper part. We have also to consider whether this gradient is subject to important corrections for topography, changes of surface temperature or other effects. The topographical correction has been worked out by Jeffreys and Bullard.[11] It does not become appreciable unless steep slopes are within 2 or 3 km of the well. In the present instance, the slope is gentle and uniform within this radius; a steeper rise in one quadrant begins at the foot of the range, but this is more than 4 km distant and its effect appears to be negligible. Such corrections as would arise from this cause would in any case reduce the gradient, and thus also reduce the estimate of maximum heat flow.

The effects of changes of surface temperature are particularly elusive, though readily calculated for any specified conditions. Surface changes which took place as long ago as 1 million years are no longer of consequence, as a change of as much as 10° C at that time, which persisted to the present, would affect the gradient by not more than 1° C/km, a relatively minor correction. Changes within the last few thousands or tens of thousands of years, if persistent, might have effects appreciably larger near the surface. The fact that such large effects are rarely discernible with certainty suggests that alternations of surface temperature have probably occurred with a consequent reduction of the net effect.[12] The latest change will have the greatest proportional effect, but there is often uncertainty even as to the direction of this

[11] Jeffreys, H.: 1937-40, M. N. R. A. S. Geophys. Suppl. 4, 309; E. C. Bullard, ibid. 360.

[12] For a careful analysis of such a case, see W. O. Hotchkiss and L. R. Ingersoll, 1934, J. Geol. 42, p. 113.

change. The region of this study was not covered by the Pleistocene mountain glaciers, and studies of the general climatic variations do not afford quantitative values for the changes of temperature which must have occurred.[13]

There is available, however, a certain amount of data for other wells in this region. Van Orstrand has measured the temperatures in four wells just east of the Front Range, for which the data are given in Table IV.

Of these four temperature-depth curves, only the curve for the Longmont well has a shape that might reasonably be ascribed to a recent change of surface temperature. The other three suggest rather strongly changes of conductivity, at various depths and in all likelihood the changes of slope in the Longmont well should be attributed to the same cause. The very close resemblance of the curves for Longmont and Florence, the two most widely separated wells, is striking, except that changes of gradient which appear to be fairly sharp at Florence are more gradual at Longmont. All four of these wells are for the greater part of their depths in the Pierre shale; the changes of conductivity which would be necessary to afford a single value of heat flow for each well are of the same order as the differences among the numerous samples of shale from a California well studied by Benfield.[14] The Florence well is particularly interesting in showing two changes of slope, first from about 41° C/km to 35° C/km, then to 45° C/km or more. None of these points of change corresponds to especially notable features as indicated by the logs of these wells,[15] which reiterate, for the most part, such notations as "gray shale," "sandy gray shale," "soft shale" and so on, but changes of composition of the shale are nevertheless probably responsible for the changes of gradient. The evidence of these wells does not encourage a belief in recent climatic changes in this region which greatly affect the thermal gradients.

[13] Bryan, Kirk, and Ray, Louis L.: 1940, Smithsonian Misc. Coll. **99**, no. 2; Louis L. Ray, 1940, Bull. Geol. Soc. Amer. **51**, p. 1851-1918.

[14] Benfield, A. E.: 1947, Amer. Jour. Sci., **245**, p. 1.

[15] Logs of these or nearby wells are given in "Selected Well Logs of Colorado," by C. F. Barb, Quarterly of the Colorado School of Mines, Vol. 41, No. 1. Log 50 is the log of the Longmont well, log 109 of the Calhan well, log 127 of the Florence well. There is no log for the Fort Collins well, but logs 192 and 199 are for wells in adjacent sections.

## TABLE IV.

Temperatures and gradients in other Colorado wells.

| Depth feet | Longmont °F | Longmont °F/100′ | Longmont °C/km | Fort Collins °F | Fort Collins °F/100′ | Fort Collins °C/km | Calhan °F | Calhan °F/100′ | Calhan °C/km | Florence °F | Florence °F/100′ | Florence °C/km |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 47.6 | (air) | | | | | | | | | | |
| 100 | 58.1 | | | 52.7 | | | 53.0 | | | 58.2 | | |
| | | 1.52 | 27.9 | | 1.85 | 33.7 | | 1.68 | 30.5 | | 1.45 | 26.4 |
| 500 | 64.2 | | | 60.1 | | | 59.7 | | | 64.0 | | |
| | | 1.96 | 35.5 | | 1.84 | 33.5 | | 1.50 | 27.3 | | 2.30 | 41.9 |
| 1000 | 74.0 | | | 69.3 | | | 67.2 | | | 75.5 | | |
| | | 2.12 | 38.7 | | 1.82 | 33.2 | | 1.62 | 29.5 | | 2.26 | 41.2 |
| 1500 | 84.6 | | | 78.4 | | | 75.3 | | | 86.8 | | |
| | | 2.32 | 42.8 | | 1.66 | 30.2 | | 1.52 | 27.7 | | 1.88 | 34.2 |
| 2000 | 96.2 | | | 86.7 | | | 82.9 | | | 96.2 | | |
| | | 2.37 | 43.0 | | 2.02 | 36.8 | | 1.54 | 28.0 | | 1.96 | 35.7 |
| 2500 | | | | 96.8 | | | 90.6 | | | 106.0 | | |
| | | | | | 2.12 | 38.6 | | 2.02 | 36.8 | | 2.50 | 45.5 |
| 3000 | 119.9 | | | 107.4 | | | 100.7 | | | 118.5 | | |
| | | | | | | | | 1.90 | 34.6 | | 2.70 | 49.2 |
| 3500 | | 2.50 | 45.6 | | | | 110.2 | | | 132.0 | | |
| | | | | | | | | 2.20 | 40.1 | | | |
| 4000 | 144.9 | | | | | | 121.2 | | | | | |
| | | 2.62 | 48.0 | | | | | | | | | |
| 6500 | 210.3 | | | | | | | | | | | |

The measurements for the Longmont well have been published in C. E. Van Orstrand, Trans. Am. Geophys. Union, 1937, p. 1. Gradients for certain intervals in the other three wells are given in reference 6. The unpublished detailed observations for these three wells have been kindly supplied by the U. S. Geological Survey.

The values of heat flow obtainable by adopting reasonable values for the conductivity of the shale are in satisfactory agreement with the estimate for the Red Creek well. The conductivity of shale may run as high as 0.005, but this value would have to be associated with the lowest gradient in these wells, roughly 30° C/km; otherwise, keeping the heat flow fixed for a given well, we would be forced to take still higher conductivities for the sections of low gradient. With the highest gradients, nearly 50° C/km, we must associate the lowest values of conductivity, perhaps 0.002 to 0.003. Thus the heat flow is fairly certain to lie between $1.0 \cdot 10^{-6}$ and $1.5 \cdot 10^{-6}$ for all of these Colorado wells. It may be possible to narrow this estimate by measuring the conductivity of a number of samples of the Pierre shale, though it is clear that proper correlation with observed gradients is important. A sample of Pierre shale collected near Colorado Springs[16] from a fresh road cut showed a conductivity of 0.002; but the variation of the gradients demonstrate a variability of conductivity by a factor of nearly 2. Thus collection of samples and determination of temperatures in new wells probably offers the most hopeful method of improving the figures for heat flow.

For comparison, the values of heat flow for other regions are collected in Table V. The most extensive work, with the most favorable conditions, is that of Bullard[17] in South Africa. It is noteworthy that the gradient in the granite, penetrated for about 2000 feet at the Dubbeldevlei bore, is 22.3° C/km; Bullard found 0.0068 for the mean conductivity of 5 samples of this granite. Benfield[18] has given values for a number of bores in Great Britain; the figures for heat flow have been corrected for the removal of the last glaciation, unfortunately a large correction where the time is relatively short and the wells shallow. Benfield has also determined the conductivity of a large number of samples from a California well (Berry No. 1) in which temperatures had been measured by Van Orstrand, and obtained the heat flow shown in Table V. A value for the Big Lake No. 1-B well in West Texas is based on measured conductivities of 8 samples from 4 different

[16] For this sample, I am indebted to Professor Don B. Gould, of Colorado College.

[17] Bullard, E. C.: 1930, Proc. Roy. Soc. London, 173A, 474-502.

[18] Benfield, A. E.: ibid. p. 428-450; see also reference 14.

TABLE V.

Heat flow in various regions.

| Place | Surface Elevation—feet above sea-level | Rock | Heat flow in $10^{-6}$ cal/$cm^2$/sec |
|---|---|---|---|
| South Africa | | | |
| Witwatersrand | | | |
| Jacoba No. 3 | 4300 | Lava, porphyry | 0.95 |
| Doornhoutrivier | 4260 | Lava | 0.97 |
| Gerhardminnebron | 4787 | Quartzite | 1.28 |
| Doornkloof | 5448 | Quartzite | 1.20 |
| Reef-Nigel | 5120 | Lava, quartzite, dolerite | 1.03 |
| Carnarvon | | | |
| Dubbeldevlei | 4060 | Granite | 1.52 |
| Great Britain | | | |
| Balfour | | Sandstone, shale, clay | 1.16 |
| Holford | | Marl, rocksalt | 1.43 |
| Blythswood | | Sandstone, shale | 1.56 |
| South Balgray | | Sandstone, shale | 1.85 |
| United States | | | |
| California (Berry No. 1) | 680 | Shale | 1.29 |
| Colorado (Red Creek) | 6200 | Granite, arkose | 1.0 —1.4 |
| Texas (Big Lake 1-B) | 2800 | Limestone | 2.0 |
| Georgia | | | |
| (Griffin, LaGrange) | ~1000 | Gneiss | 1.4 |
| Michigan | | | |
| (Calumet and Hecla) | ~1200 | Trap | 1.0 |

cores.[19] An estimate may be made using the temperatures found by Hewett and Crickmay[20] in two shallow wells in the Carolina gneiss in Georgia. The measured gradients were 17.3° C/km and 21.9° C/km; with an estimated conductivity of 0.007, the mean heat flow for these two wells is $1.4 \cdot 10^{-6}$. The estimate for the Calumet and Hecla mine rests on an assumed conductivity of 0.005, combined with an equilibrium gradient of 19.6° C/km[21]; it is possible that the true conductivity is higher than 0.005. This review of other determinations, evidently of unequal reliability, suggests that the heat flow as found for the Red Creek well is not abnormally high.

In most recent discussions of the earth's heat, it is assumed that the upper or "granitic" layer possesses an average content of radioactive elements comparable with some mean value

[19] Birch, Francis, and Clark, Harry: 1945, AMER. JOUR. SCI., 243A, 69-74.

[20] Hewett, D. F., and Crickmay, G. W.: 1937, U. S. Geol. Survey Water-Supply Paper 819.

[21] See reference 12.

for the "granitic" rocks whose radioactivity has been determined in the laboratory. This assumption appears in the various papers of Joly, Holmes, Anderson, Jeffreys, Bullard, Adams, and others. It is also very widely supposed that the upper layer is thickened in mountainous regions and that present elevation is on the whole an indication of the amount of thickening. A consequence of these two ideas is that the flow of heat to the surface, at a sufficient interval of time after thickening, should be greater in mountainous regions, or more generally, at high elevations, than near sea level. The available observational data, admittedly inadequate, do not seem to bear out this expectation. Instead, the values of heat flow tend to cluster about $1.2 \cdot 10^{-6}$ to $1.3 \cdot 10^{-6}$, regardless of elevation. Further confirmation is evidently needed of this apparent absence of correlation between heat flow and surface elevation; studies of heat flow above 8000 feet would be of the greatest value. The effects upon geophysical theory of discarding either of the above assumptions would be far-reaching, and a more complete discussion is reserved for a future publication.

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