

RIDGE VOLUME DEPENDENCE ON SEAFLOOR GENERATION RATE AND INVERSION USING LONG TERM SEALEVEL CHANGE

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ABSTRACT. The basic quantitative distinction between global oceanic ridge volume and the global rate of seafloor generation is made fully explicit. From this, the question of inversion over time from the former quantity into the latter is then posed using a generalized expression to approximate global subduction zone distribution. Two numerical methods are described. Then, assuming the hypothesis that long-term (10^8 yr) eustatic sealevel change is due primarily to changing ridge volume, an inversion of a widely cited Phanerozoic sealevel curve (Vail) is also presented. The approach taken here is expected to be of direct importance for quantitative models of the carbonate-silicate cycle which seek to develop scenarios for atmospheric carbon dioxide variation over geologic time scales. Indeed, the testing of sealevel inversion, as performed here, may ultimately come from its degree of correspondence with past climate variation.

INTRODUCTION

A tectonic control on long term atmospheric carbon dioxide variation has often been proposed, but a quantitative model of the relevant processes was only recently formulated (Berner, Lasaga, and Garrels, 1983; Lasaga, Berner, and Garrels, 1985, henceforth BLAG I, II). The preliminary results not only support the idea but demonstrate impressively the predominating effect of global plate motion changes. Designed as a geochemical box-reservoir model, rate expressions for decarbonation are assumed proportional to the global rate of seafloor generation as this must equal the subduction rate by conservation of Earth surface area. Toward this end, the marine geomagnetic reversal record has been consulted and provides, among other things, the global area-age distribution of existing seafloor.¹ However, the more fundamental quantity, past seafloor generation rate — the quantity dominating sealevel, geothermal heat flow, and, preliminary results suggest, atmospheric carbon dioxide — actually equals this extant area-age distribution *plus* the area-age distribution of all seafloor lost to subduction zones. An inescapable uncertainty therefore enters any attempt to infer past seafloor generation rate from the present ocean floor. By contrast, this paper can be understood as an effort to bypass this fundamental problem by using a quantity that is always strongly coupled to young seafloor area — past oceanic ridge volume as inferred from sealevel. This usage of a critical quantity from the

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¹ The area-age distribution curve is defined as the differential area per unit age of all existing seafloor as a function of age.

past (sealevel), as opposed to one from the present (extant seafloor), marks the main advantage of this calculation.

In the so-called "corrected Southam and Hay" spreading rate history (BLAG I and II), a statistical approximation for the gradual loss of seafloor was assumed, and the area-age curve (from Southam and Hay, 1977) was "corrected" (that is, increased) accordingly. An identical approach was taken by Sprague and Pollack (1980) in a study of Cenozoic and Mesozoic heat flow in which they tested different loss rate formulations. Reflecting the strong fall-off of heat flow with age, they found a weak dependence on the loss rate model but a strong dependence on the seafloor generation rate. In these two examples, the loss rate model may be looked upon as a "survival probability" approximation. It will be adopted in this paper as well, but I believe it will be a higher order correction. Formally, it introduces a (weak) nonlinearity into the inverse problem for which a solution is not as readily obtained (see app. 1).

As with heat flow, the rapid fall-off of ridge slope with age results in ridge volumes being strongly dependent on crustal generation rate. And detailed efforts, determining poles of rotation, calculating velocities, and estimating boundaries, for the individual tectonic plates have been made (Pitman, 1978; Kominz, 1984). But here too, the limitations and difficulties with the reversal record manifest themselves. The broad categories of uncertainty, missing information (lost seafloor, the Cretaceous quiet zone), and dating errors are discussed authoritatively in Kominz (1984). Dating uncertainty appears in both the computation of the velocities and the ridge lengths as evidenced by the three time scales considered and the respective curves derived from them. Ridge volume, which is largely an integral convolution of the product of velocity and ridge length, shows a widening error bar toward the past with Late Cretaceous sealevel (80 Ma) estimated therefrom at 230 ± 100 m.² and ³

On the other hand, the so-called "first-order" sealevel curve presented in Vail, Mitchum, and Thompson (1977) extends over the Phanerozoic (fig. 1). As discussed in Vail, Hardenbol, and Todd, (1985), the long term changes for the Jurassic through Present were determined using geohistory analysis (Van Hinte, 1978).⁴ Additionally, the Late Cretaceous through Present segment was modified somewhat (see Vail, Mitchum, and Thompson, 1977) so as to parallel more closely the ridge volume calculations of Pitman (1978). The pre-Jurassic curve follows mostly from the

² The uncertainties in dating of anomalies may have a bearing on BLAG-type models, since it appears that time scales clear of constant spreading assumptions would yield dramatic velocity fluctuations (Rich and others, 1986). These authors report a correlation with marine diversity, the link being suggested as climate through the tectonics-carbon dioxide interaction.

³ Large fluctuations in plate velocities have traditionally been considered unlikely. A line of thought in support of fluctuations, prompted by the mechanical and seismic evidence of subducting slabs and suggesting a possible role for the 670-km mantle discontinuity, can be found in Gaffin (1986).

⁴ The "higher-order" sealevel curves represent the superposition of short term fluctuations from seismic stratigraphy onto the long term curve (see Vail, Hardenbol, and Todd, 1985). The short term fluctuations are not considered here because of widespread controversy regarding their origin and significance for eustasy (Thorne and Watts, 1984).

work of Sloss (1963) based on depositional sequences in the North American craton and is the most poorly constrained. From geohistory analysis, a “close agreement” is stated to exist with the sealevel estimates from ridge volume calculations (Vail, Hardenbol, and Todd, 1985). However, considering just the uncertainty bars on ridge volume estimates, the agreement cannot be, and may never be, precise. Therefore a quantitative

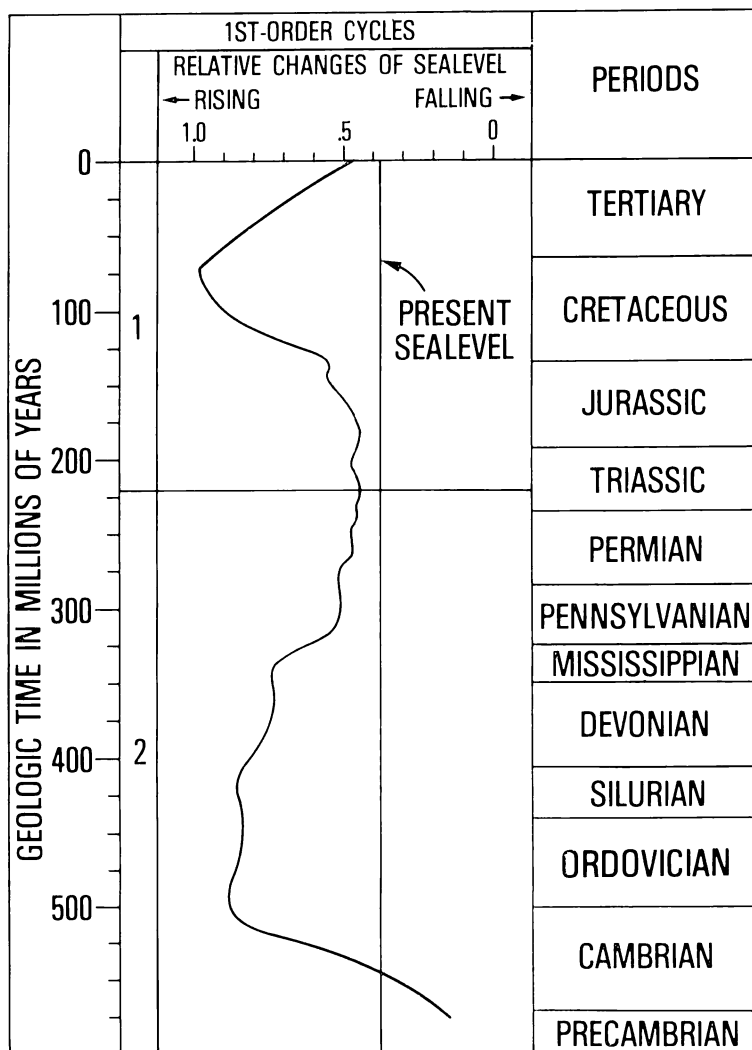


Fig. 1. Phanerozoic first-order eustatic sealevel changes as presented in Vail, Mit-chum, and Thompson (1977). Pre-Jurassic curve mostly from Sloss (1963). Jurassic through Cretaceous based on geohistory analysis as described in Vail, Hardenbol, and Todd, 1985 and Van Hinte, 1978. Cretaceous through Present (linear decrease) also from geohistory analysis but modified somewhat so as to parallel Pitman curve from 1978. Note that it was originally presented in “relative units”.

measure of the inaccuracy of the ridge volume-eustasy hypothesis is not available.

Qualitatively, nevertheless, the hypothesis is rather firmly established with numerous reports readily employing figure 1 as a proxy for ridge volume and hence tectonism. Fischer (1983) and Tissot (1979) have specifically discussed use of the curve in this fashion. A host of causal links, climatic and others, have been forged which will not be reviewed here (examples may be found in Wilkenson and Kevin Given, 1985; Sandberg, 1985; Shanmugan and Moila, 1982; MacKenzie and Pigott, 1981). In semi-quantitative support of the hypothesis and addressing more closely the actual features of the curve, Heller and Angevine (1985) argued that the Jurassic-Cretaceous transgression (periods not accessible to "forward" ridge volume estimates) probably represents the opening Atlantic as opposed to faster spreading. This distinction, however, essentially between ridge length and opening rate, will not be necessary when the discussion is generalized to "seafloor creation rate" as done at the outset in this paper.

Here it will be seen that I calculate the rates of seafloor generation directly from the volume changes as inferred from the sealevel curve. This means that, unlike other work, I do not use the data from today's area-age distribution curve; indeed, I do not use the ocean floor reversal record. And while the details of consumption by subduction zones are again unknown, the physical fact that volumes depend most strongly on young seafloor, which is likely to be the least consumed, becomes a key advantage. "Forward" calculations must carefully estimate past areas of young seafloor from what is now old, partially subducted, seafloor.

A more important source of error will probably be that associated with using sealevel as the volume proxy, mainly the periods of large scale glaciation. At least, I will attempt to represent this limitation as follows: intervals of glaciation, as listed in Caputo and Crowell (1985) will be denoted by a dotted line over the inversion results for those periods. Other mechanisms of eustasy (for example, continental area changes, thermal expansion) are one or two orders of magnitude smaller than ridge volume (Donovan and Jones, 1979). The assumption is made that such mechanisms are not a major and systematic component in the slow, large changes of sealevel shown in figure 1.

THE AGE-DEPTH EQUATION AND RIDGE GEOMETRY

Ridge volume and seafloor accretion, although temporally coupled, are not proportional. One intention of this paper is to make explicit the distinction between the two quantities, since it appears this has not been done as clearly as it should be. Furthermore, the discussion of inversion that follows would be impossible without having in hand the essential mathematical relationship.

The temporal coupling between volume and accretion exists via the age-depth relationship for cooling oceanic lithosphere, usually taken as:

$$d(t) = C_1 + C_2\sqrt{t} \quad (1)$$

with $C_1 = 2.5$ km (depth of crest below sealevel) and $C_2 = 0.35$ km/ $\sqrt{\text{age}}$ and $t \leq 70$ Ma (Parsons and Sclater, 1977). Older seafloor subsides more slowly for which empirical expressions are available (see, for example, Schroeder, 1984). Only the volumetric contribution for seafloor younger than 70 Ma will be considered in this paper, but it can be seen that the methods described are adaptable to older seafloor as well.

Perhaps the most important implication of (1) is that the velocity history of a spreading center is embedded in the ridge geometry so that by knowing that history past geometry may be reconstructed (see, for example, the illustrations in Pitman, 1978). Or, interestingly, by knowing the geometry, the history might be estimated: if a point x km from the ridge crest is situated at a depth d km below that crest (meaning its age may be taken as d^2/C_2^2) and the ridge slope there is d' , then the half-spreading rate, when x accreted, is given by:

$$v(x) = \frac{\delta x}{\delta t(x)} = \frac{\delta x}{\delta \left(\frac{d^2}{C_2^2} \right)} = \frac{C_2^2}{2dd'} \quad (2)$$

as follows from (1). An approximate version of the method suggested by (2) has been applied to Venusian altimetry data (Kaula and Phillips, 1981). However, it is less apparent that an analogous situation holds between volume change and seafloor generation rate possibly to be exploited in a similar manner over large expanses of time. Toward this end, consider the difference in elevation between seafloor age- t and seafloor age-70, denoted hereafter $g(t)$. According to (1) and neglecting older seafloor:

$$g(t) = \begin{bmatrix} C_2(\sqrt{70} - \sqrt{t}) & t \leq 70 \\ \approx 0 & t > 70 \end{bmatrix}. \quad (3)$$

Then, the volumetric contribution (per unit age) of t -age seafloor to the global ridge system is given by:

$$\Delta RV(t) = g(t) \alpha(t) \quad (4)$$

where $\alpha(t)$ is the differential area per unit age of all seafloor of age t , by definition, the area-age distribution curve. As discussed in the introduction, $\alpha(t)$ will be thought of as an original amount of seafloor generated per unit time t -yrs ago, diminished by a factor due to subduction. The seafloor generation rate will be denoted hereafter $A(t)$. The age dependent diminishing factor will be denoted by $P(t/t_{\max})$, where $t_{\max}$ is the maximum age of existing seafloor (presently 180). I write,

$$\alpha(t) = A(t) P\left(\frac{t}{t_{\max}}\right) \quad (5)$$

$P(t/t_{\max})$ evidently satisfies $P(0)=1$ and $P(1)=0$. Its intermediate shape, today, is discussed extensively in Parsons (1982), where the data suggest it has an approximately linear dependence on t . Furthermore, Parsons has interpreted the linearity as a "uniform" distribution of subduction

zones with age. However, in order to generalize the ensuing discussion, I will introduce a phenomenological exponent n and write:

$$P_n\left(\frac{t}{t_{\max}}\right) = 1 - \left[\frac{t}{t_{\max}}\right]^n \quad (6)$$

which always satisfies the limits, and $n=1$ is the linear dependence observed today.⁵ Now integrating (4) over age t , I obtain an expression for present ridge volume,

$$RV = \int_0^{t_{\max}} g(t) A(t) P_n\left(\frac{t}{t_{\max}}\right) dt. \quad (7)$$

Furthermore note here that $A(t)$ can change only in two ways: global ridge length changes and global spreading rate changes, and when discussing ocean basin volume there is no need to distinguish between the two cases.

Eq (7) can now be considered over geologic time, hereafter denoted T . In this case t (and therefore also $t_{\max}$) must be measured relative to T . To represent $A(t)$'s dependence on geologic time, one writes (without full clarity) $A(T, t)$. The maximum ocean floor age, $t_{\max}$, has certainly varied since with greater generation rates (for example, Atlantic rifting), maximum age is expected to drop relative to the present to conserve total seafloor area. Therefore, I will write $t_{\max}(T)$. As mentioned in footnote 5, the statistical model, P_n , may have varied in complicated ways, and a simplifying assumption will be made: its time dependence will be written as $P_n(t/t_{\max}(T))$. Eq (7) over geologic time, T , now becomes

$$RV(T) = \int_0^{t_{\max}(T)} g(t) A(T, t) P_n\left(\frac{t}{t_{\max}(T)}\right) dt. \quad (8)$$

However (8) still omits a very crucial fact: $A(T, t)$ is referring to seafloor generation rate at a particular geologic year. Since t is measured *relative* to T , that year can be none other than $T+t$. This fact must be incorporated and represents the following symmetry:

$$A(T \pm n, t \mp n) = A(T, t). \quad (9)$$

This is the same symmetry embodied by the more obvious statement: "... seafloor of age 20 M.A. in geologic year 150, was 19 in geologic year 151, was 21 in geologic year 149, and so on ...". Therefore (8) becomes more properly,

$$RV(T) = \int_0^{t_{\max}(T)} g(t) A(T+t) P_n\left(\frac{t}{t_{\max}(T)}\right) dt. \quad (10)$$

⁵ It is emphasized that P_n is only a statistical model meant to approximate the loss rate. The dynamics involved in its changes would be quite complicated even in idealized situations. For example, if a perfectly linear area-age distribution exists and seafloor spreading suddenly doubles everywhere, the linearity will be disturbed for a transient period even if all subduction zones and ridges remain fixed in their locations.

And conservation of total seafloor area is similarly written as:

$$C = \int_0^{t_{\max}(T)} A(T+t) P_n \left(\frac{t}{t_{\max}(T)} \right) dt \quad (11)$$

where C has the approximate value 360.10^6 km^2 (Kominz, 1984).

INVERSION

Accepting the hypothesis that $RV(T)$ may be determined from sealevel change over time, the question of inversion from these data to obtain, ultimately, $A(T)$ will now be considered. When a value for the exponent n is chosen (the model-dependent part of this calculation), the unknowns in the problem are $A(T)$ and $t_{\max}(T)$ which are non-linearly coupled to each other through (10) and (11). (Their present values, $A(0)$ and $t_{\max}(0)$, are known: $3.0 \text{ km}^2/\text{yr}$ and 180 Ma respectively.) I will in the following consider the case $n \rightarrow \infty$, which decouples $A(T)$ and $t_{\max}(T)$ and leads to a linear version of the problem *as long as the sealevel data do not require $t_{\max}(T) < 70 \text{ Ma}$ for any T* . In other words, the area of the seafloor included in ridge volume should not at any time T exceed the total (and fixed) area of the seafloor. This is a priori unknown. But if it is the case one can ignore (11), and, bearing in mind that

$$P_n \left(\frac{t}{t_{\max}} \right) \rightarrow \begin{bmatrix} 1 & t < t_{\max} \\ 0 & t = t_{\max} \end{bmatrix} \quad \text{as } n \rightarrow \infty$$

(10) simplifies to:

$$RV(T) = \int_0^{70} g(t) A(T+t) dt \quad (12)$$

The case being considered here, $n \rightarrow \infty$, corresponds to what may be called a “rectangular distribution” assuming no seafloor subduction until after age 70. Despite the simplification, it appears that this still includes a large array of ocean floor dynamics. So, for example, spreading rates on both sides of the ridge system are added to produce a total generation rate. Any ridge length increases (such as the Atlantic rifting) are also included. Ridge length decreases due to abandoned spreading centers, such as the East Pacific Mathematician Rise, are also included, since the latter are well known to subside according to the age-depth relation (Sclater, Anderson, and Bell, 1971). All spreading rate changes, either increases or decreases, again are implicitly included. All these processes simply serve to change the global crustal generation rate.

A NUMERICAL METHOD FOR THE LINEAR CASE, $n \rightarrow \infty$

I have inverted (12) by the following method which it will be useful to sketch. A numerical method for the non-linear case, when n is finite, is described in app. 1. Here, a family of $N+1$ fixed times in the geologic past, that span the interval of the sealevel curve, is chosen. These times, unlike T, t , and $t_{\max}$, are non-physical and can be selected entirely arbitrarily but are generally evenly and closely spaced. They will be denoted

by the symbol " θ_j " with j running from 1 to $N+1$. For the inversion shown in this paper, piecewise constant basis functions, u_j , were then defined as follows:

$$u_j(T) = \begin{cases} 1 & \text{if } \theta_{j+1} \geq T \geq \theta_j \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

Using these functions, an approximate representation for $A(T)$ can be written,

$$A(T) \approx \sum_{j=1}^N a_j u_j(T) \quad (14)$$

where a_j are unknown constant coefficients that are to be determined. Since this is defined for all times, the approximation for $A(T+t)$ is likewise,

$$A(T+t) \approx \sum_{j=1}^N a_j u_j(T+t) \quad (15)$$

The step proceeding from eq (14) to eq (15) makes clear the need to identify the actual year referred to in $A(T, t)$. On substitution of (15) into (12) I obtain:

$$RV(T_i) = \sum_{j=1}^N a_j \phi_{i,j} \quad (16)$$

where T has been replaced by T_i which is one of the times selected for the finite representation of the sealevel curve. The kernel, $\phi_{i,j}$, is:

$$\phi_{i,j} = \int_0^{70} g(t) u_j(T_i+t) dt \quad (17)$$

It is seen that (16) represents "i" linear equations in N unknowns, the coefficients a_j . The basis functions, $u_j(T_i+t)$, serve to restrict the region of integration, and only five possibilities arise that permit one to simplify (17). The five possibilities are given in the following:

Let $b_{ij} = \theta_{j+1} - T_i$ and $a_{ij} = \theta_j - T_i$.

(i)	if $b_{ij} \leq 0$	then	$\phi_{i,j} = 0.$
(ii)	if $a_{ij} \geq 70$	then	$\phi_{i,j} = 0.$
(iii)	if $b_{ij} > 70$ and $a_{ij} < 70$	then	$\phi_{i,j} = \int_{a_{ij}}^{70} g(t) dt.$
(iv)	if $a_{ij} < 0$ and $b_{ij} > 0$	then	$\phi_{i,j} = \int_0^{b_{ij}} g(t) dt.$
(v)	otherwise		$\phi_{i,j} = \int_{a_{ij}}^{b_{ij}} g(t) dt.$

(18)

Case (iv) is the most important because of the strong dependence on young seafloor. In cases (iii), (iv), and (v), the integrand $g(t)$ is readily evaluated (which is the purpose behind eq 18) in terms of the upper and lower limits:

$$\int_a^b g(t) dt = C_2 \left[70^{1/2}(b-a) - \left(\frac{2}{3}\right) b^{3/2} + \left(\frac{2}{3}\right) a^{3/2} \right] \quad (19)$$

The physical origin of (18) can be shown transparently using a picture (fig. 2). Seafloor area younger than 70 Ma of age is represented by the large solid rectangle. This area is not conserved over time. The ridge crest is on the top edge. Newly generated seafloor is of age T ($t=0$) and moves, as it ages, downward through the ridge system, after which it is subducted at an age presumed greater than $T+70$ ($t=70$). The elemental seafloor

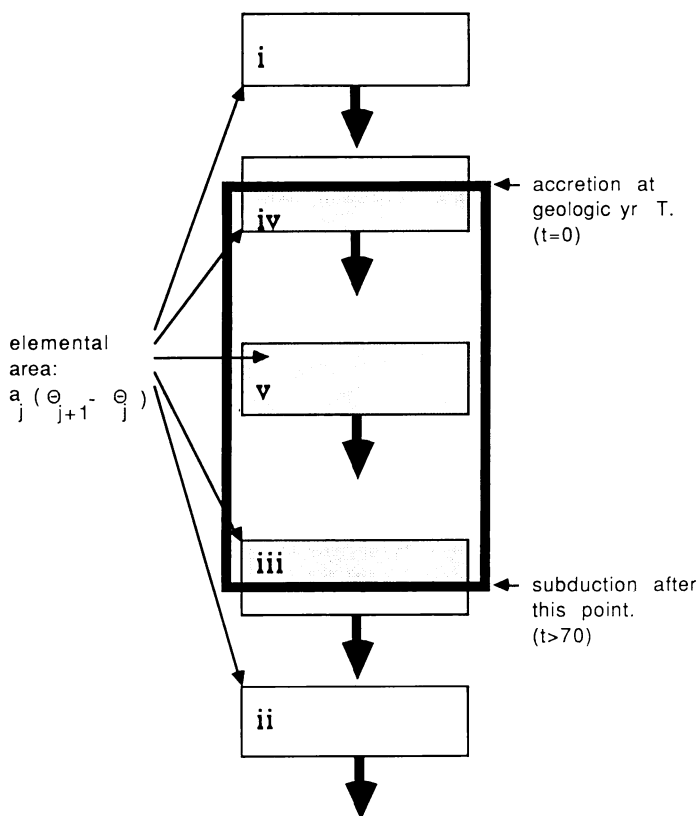


Fig. 2. The five possible configurations of the elemental inversion area: $a_j \times (\theta_{j+1} - \theta_j)$, relative to an existing ridge volume at ridge time "T". This version (linear) assumes no subduction loss until after seafloor age 70. This assumption was found not to violate conservation of total ocean floor area when the sealevel curve is scaled at 230 m (Cretaceous highstand). A violation might occur if sealevel is scaled above 300 m in which case the method (non-linear data fitting) discussed in app. 1 must be applied in order to obtain a viable solution.

area, " $a_j \times (\theta_{j+1} - \theta_j)$ " is represented by the smaller rectangles. Topologically, it can exist in any one of five configurations relative to the existing ridge volume system. These are depicted by roman numerals (i) through (v) and correspond to the five cases described numerically above. It is only the shaded areas of the elemental seafloor area that contribute to ridge volume.

If M (generally equidistant) points of the sealevel curve are selected, then (16) represents an $(M \times N)$ linear system that can be solved for the unknown accretion rates a_j . These rates should be considered acceptable only if they do not violate (10), the total area constraint. The known generation rate a_1 ($=3.0 \text{ km}^2/\text{yr}$) can be formally included in the matrix equation if desired (see Gaffin, 1986) but will instead be used to calibrate the sealevel proxy for $RV(T)$ as follows.

SEALEVEL PROXY FOR $RV(T)$

A cross section of the ocean basin geometry used for inversion is shown in figure 3. The same dimensions are used in Kominz (1984). Present air-sea interface is taken as 360.10^6 km^2 . The hypsometric increase in the interface area is taken as 170.10^6 km^2 per kilometer of sealevel rise. I have chosen to relate $RV(T)$ to a new variable, $TV(T)$, ("total volume") which is defined as the integral over depth of oceanic surface area from sealevel at time T down to the 70 Ma isobath ($\approx 5.4 \text{ km}$ below present sealevel *plus* isostatic changes). It is in calculating this volume integral that one makes the correction for isostatic readjustment. Specifically, an observed change in sealevel of $\Delta h(T)$ at the surface induces an isostatic response of $(\rho_w/(\rho_m - \rho_w)\Delta h(T))$ at the base of the water column. The quantity $\Delta h(T)$ is precisely that supplied by the sealevel curve. Finally, the proxy for ridge volume is then simply:

$$RV(T) = TV(T) - WV \quad (20)$$

ocean basin model used for inversion

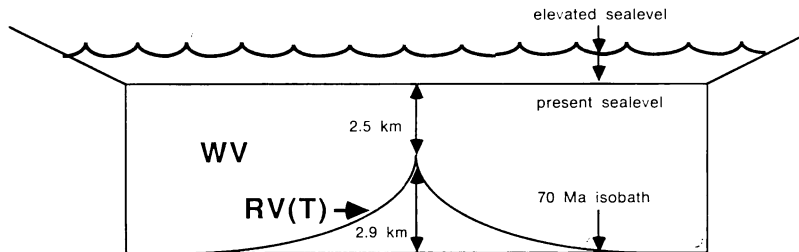


Fig. 3. Cross section of the ocean basin geometry used in this inversion. It includes a global ridge volume to a depth of 70 Ma and assumes a constant water volume over the long term. The necessary data are sealevel relative to the present. Figure 1 is used and scaled at 230 m for the Cretaceous highstand (Kominz, 1984). Two corrections are included: (i) isostatic adjustment and (ii) continental hypsometry.

where WV is (assumed constant) water volume. For the inversion presented, the system was calibrated by “shooting” the solution repetitively with trial values for WV until a_1 was found equal to its present value, $3.0 \text{ km}^2/\text{yr}$.

INVERSION OF THE PHANEROZOIC SEALEVEL CURVE AND KNOWN ICE AGES

Figure 4 top shows the long term sealevel curve (scaled at 230 m for the Cretaceous highstand) after it had been digitized into 10 Ma intervals from year 0 to year 550. This means the curve is represented by 56 points denoted “T_j” in eq (16). To define the seafloor generation rates, a family of times of 10 Ma duration was used from year 0 to year 550, and an additional interval of 70 Ma duration was employed between year 550 and 620. Using previously defined notation,

$$\{\theta_j\} = \{0, 10, 20, \dots, 540, 550, 620\} \quad j \text{ runs from 1 to 57.}$$

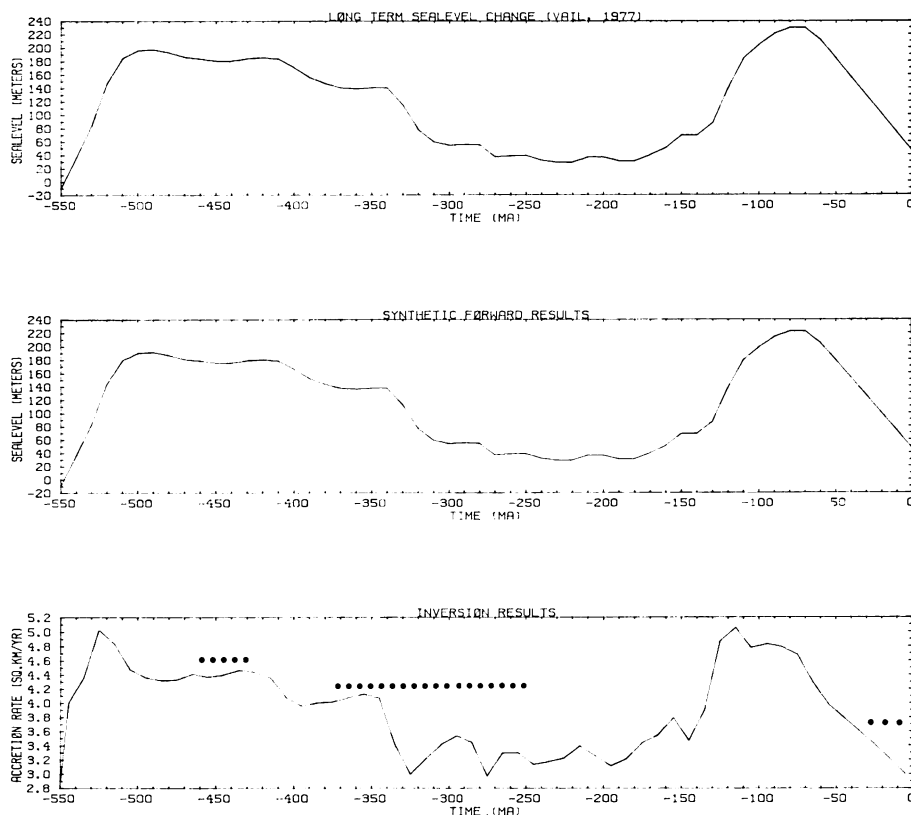


Fig. 4. Bottom panel: the seafloor generation rate curve consistent with the sealevel curve shown in the top panel and assumptions made in the text. The top panel is a “digitized” reproduction of figure 1. Middle panel is the “synthetic forward calculation” showing the “correctness” of the inversion results. Intervals of glaciation as described in Caputo and Crowell (1985) are denoted by a dotted line over the inversion results to symbolize this uncertainty.

These 57 times define 56 unknown accretion rates according to (14) so eq (16) is a linear matrix equation, where the matrix is (56×56) . The individual entries are $\phi_{i,j}$ which is defined in (17) and made more explicit in (18). Standard linear software was used to solve for the unknown accretion rates. Water volume, "WV", was varied by trial and error, until an accretion rate of $3.0 \text{ km}^2/\text{yr}$ was returned for the unknown a_1 . This occurred for $WV = 1.75 \cdot 10^9 \text{ km}^3$. This value is somewhat higher than actual water volume in the ocean (see Donovan and Jones, 1979), but the difference simply reflects the idealized geometry and is not significant since it is the *change* in sealevel that drives the inversion. The solution is exact and was checked to see if it violated surface area conservation (11); at no time is (11) violated. Therefore the solution is acceptable within the limits of this simplified model and is presented in figure 4 (bottom). A synthetic "forward" calculation using these accretion rates and eq (16) in the forward direction is shown in figure 4 (middle). The long term sealevel curve is successfully reproduced by the method.

The Paleozoic ice age record is discussed in Caputo and Crowell (1985). Unequivocal glaciations are listed therein as (I) from Caradocian (458 Ma) to Llandoveryan (428 Ma) and (II) from Famennian (367 Ma) to Tatarian (250 Ma). No further ice sheets are known until the Oligocene (30 Ma). To convey the unquantified effect on water volume in the oceans during these times the inversion results are dotted over those periods. It is quite apparent that the Ordovician and Late Cenozoic ice ages are relatively short and not significant for this inversion over the Phanerozoic time scale. Although the Permo-Carboniferous glaciation is indeed extensive in time, it nevertheless appears that one can argue that something additional to ice was involved in that sealevel regression since it was preceded by non-glacial periods of high sealevel and followed by non-glacial periods of low sealevel. Here I have assumed it was ridge volume, but other mechanisms such as continental collision certainly may have been involved. However, massive ridge volume decrease during the Permo-Carboniferous, with its corresponding effect on atmospheric carbon dioxide, is suggestively in line with the large scale glaciation that occurred during this time.

THE PHASE LAG BETWEEN SEAFLOOR GENERATION CHANGES AND SEALEVEL CHANGES

It should be stressed that the timing and magnitude of the accretion rate curve cannot be simply read off from the sealevel curve. Slopes are evidently quite different. In order to emphasize these differences an alternative view of the long term results for the Jurassic-Present is shown in figure 5. This is a "phase" plot of sealevel versus accretion rate, where the data are plotted using their corresponding geologic times. So, for example, at present (the 0 datum) accretion rate is 3.0 and sealevel is 50 m (an ice "correction" used by Pitman (1978) and Kominz (1984) and adopted in the Vail curve). The existence of a delay between accretion rate and sealevel is brought out by the overall phase "orbit". For example, peak

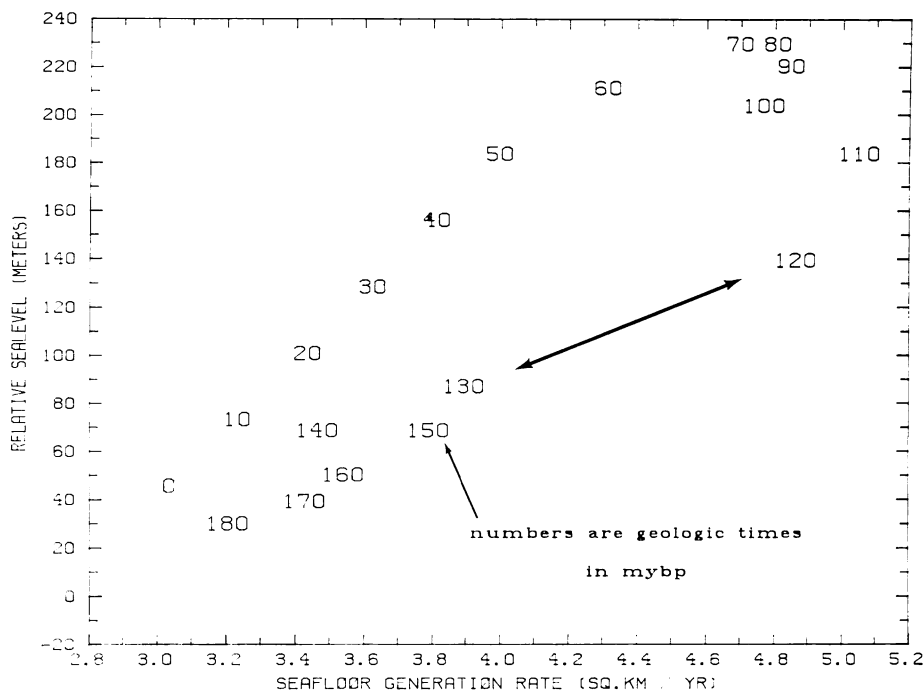


Fig. 5. An alternative view of the inversion results shown in figure 4 for the Jurassic through Present. This is a "phase plot" of accretion rate versus sealevel primarily meant to demonstrate that simple proportionality between the two does not exist. Such a graph brings out other features like the anomalously large accretion rate change between 130 and 120 Ma. Also see footnote 6.

accretion occurs near year 110, whereas peak sealevel occurs near year 80 Ma. Similarly one sees that sealevel at times 30 and 120 are almost the same, but the corresponding accretion rates are significantly different (3.6 versus 4.8 km² per yr).⁶

Additionally, figure 5 brings out clearly the anomalous increase in accretion rate between years 130 and 120 of about 1 km² per yr. For a 50,000 km length ridge system (typical for the Tertiary) this would translate into a mean opening rate increase of 2 cm per yr at every location and a likewise increase at subduction zones with corresponding effects on volcanic activity and atmospheric carbon dioxide.

ATMOSPHERIC CARBON DIOXIDE VARIATION

The quantity obtained in figure 4 (bottom) is directly applicable to paleoclimate modeling of atmospheric carbon dioxide. In BLAG notation:

$$f_{sr}(T) = \frac{A(T)}{A(O)} \quad \text{and for reference, } f_{sr}(80) = 1.6. \quad (21)$$

⁶The author has detected an identical phase lag between sealevel and magnetic reversal frequency. This important clue will be reported on in a future publication.

For purposes of illustration, a land area fraction, consistent with the geometry shown in figure 3, can be defined as:

$$f_a(T) = \frac{140 \cdot 10^6 - 170 \cdot 10^6 \cdot \Delta h(T)}{140 \cdot 10^6} \quad \text{from which } f_a(80) = 0.72 \quad (22)$$

where $140 \cdot 10^6$ km² is taken as present land area, and $\Delta h(T)$ is sealevel change in kilometers from figure 1.

An approximate method for evaluating the first order changes in CO₂ is described in BLAG I (p. 674) and follows from equilibrium for atmospheric carbon dioxide:

$$\text{CO}_2(T) = \frac{2(k_{MD} - k_{WD})D + (k_{MC} - k_{WC})C - 2F_{wSi} + k_{prep} M_{Ca} M_{HCO_3}^2}{k_{prep} K_{eq}} \quad (23)$$

Holding the carbonate and silicate reservoirs fixed at their present values, using the quantities calculated in (21) and (22), and integrating the rate expressions for Mg⁺⁺, Ca⁺⁺, and HCO₃⁻, I obtain for the Late Cretaceous:

$$\text{CO}_2(80) \approx 2.5 \cdot 10^{18} \text{ moles} .$$

This falls roughly halfway between the values obtained from the "Pitman" and the "Southam and Hay" spreading rate histories. Because of the linearity of the inversion, $f_{sr}(T)$ will scale with the calibration for the sealevel curve, so that, for example, lower sealevel estimates for the Cretaceous would result in lower carbon dioxide estimates for that time. Additionally, stronger feedbacks from the climate system onto weathering would have the same effect (Budyko and Ronov, 1979).

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APPENDIX I

Inversion for the case of finite n

When n is finite, eqs (10) and (11) are classified as a "nonlinear data fitting problem" (Stoer and Bulirsch, 1980). As such it is generally solvable only by means of an iteration procedure from an initial guess, for example, the Gauss-Newton algorithm (Stoer and Bulirsch, 1980). As in the text one may approximate A(T) and tmax(T) by a discrete set of unknowns in the following way: (1) select two families of fixed times $\{\theta_j\}$ and $\{\tau_i\}$, (2) choose a set of basis functions and express A(T) and tmax(T) in terms of these functions and a vector of unknown multiplier coefficients:

$$X^t = (a_1, \dots, a_N, tm_1, \dots, tm_N) . \quad (A1)$$

A function $F(X)$ can be defined that is zero for the solution of (10) and (11). This function is:

$$f_k(X) = \begin{cases} \int_0^{\text{tmax}(T_k)} g(t) A(T_k + t) P_n(t, \text{tmax}(T_k)) dt - TV(T_k) + WV & k=1, \dots, N \\ \int_0^{\text{tmax}(T_{k-N})} A(T_{k-N} + t) P_n(t, \text{tmax}(T_{k-N})) dt - C & k=N+1, \dots, 2N \end{cases} \quad (A2)$$

Each function is defined by the data time T_k at which it is evaluated. Furthermore since,

$$\frac{\partial A}{\partial a_i}(T+t) = u_i(T+t, \theta) \quad (A3)$$

$$\frac{\partial \text{tmax}}{\partial t m_i}(T) = u_i(T, \tau) \quad (A4)$$

$$\frac{\partial}{\partial x} \int_a^{b(x)} f(x, s) ds = \int_a^{b(x)} \frac{\partial}{\partial x} f(x, s) ds + f(b, s) \frac{\partial b}{\partial x} \quad (A5)$$

the Jacobian for the problem is:

for $i=1, \dots, N$:

$$\frac{\partial f_k}{\partial x_i}(X) = \begin{cases} \int_0^{\text{tmax}(T_k)} g(t) u_i(T_k + t, \theta) P_n(t, \text{tmax}(T_k)) dt & k=1, \dots, N \\ \int_0^{\text{tmax}(T_{k-N})} u_i(T_{k-N} + t, \theta) P_n(t, \text{tmax}(T_{k-N})) dt & k=N+1, \dots, 2N \end{cases}$$

for $i=N+1, \dots, 2N$:

$$\frac{\partial f_k}{\partial x_i}(X) = \begin{cases} \frac{[u_i(T_k, \tau)]}{[\text{tmax}(T_k)^{n+1}]} \int_0^{\text{tmax}(T_k)} n t^n g(t) A(T_k + t) dt & k=1, \dots, N \\ \frac{[u_i(T_{k-N}, \tau)]}{[\text{tmax}(T_{k-N})^{n+1}]} \int_0^{\text{tmax}(T_{k-N})} n t^n A(T_{k-N} + t) dt & k=N+1, \dots, 2N \end{cases} \quad (A6)$$

where $u_i(x, y)$ are basis functions with the second argument distinguishing between the $\{\theta\}$ and $\{\tau\}$ family for $A(T)$ and $\text{tmax}(T)$ respectively. All integrals can be evaluated in closed form which is important in regard to efficiency. As discussed in the text, the case $n=1$ is the "triangular" distribution observed today, and the case $n \gg 1$ should reduce to the solution shown in this paper. Indeed it can be seen that rows $N+1$ through $2N$ of the Jacobian, representing the non-linear area constraint, are zero for this case. The results of studying this system will be presented in a future publication.

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