[American Journal of Science, Vol. 282, June, 1982, P. 808-855]

ON THE MECHANICS OF IGNEOUS DIAPIRISM, STOPING, AND ZONE MELTING

BRUCE DAVID MARSH

Department of Earth and Planetary Sciences, Johns Hopkins University, Baltimore, Maryland 21218

"All of them are offered rather to emphasize once more the difficulty and importance of the problem than to suggest that a complete solution has been found" (R. A. Daly, 1903).

ABSTRACT. *The velocity of ascent of a hot spherical diapir moving through wall rock whose viscosity is extremely sensitive to temperature is estimated by bounding the drag on the body. The drag is modelled as a variable viscosity planar shear flow (upper bound) and as a fluid sphere enclosed in a spherical volume of another fluid whose thickness is arbitrary (lower bound). In each model the thickness of the zone of variable viscosity is related to the thickness of the thermal boundary layer about the body. The drag is mostly controlled by the thickness of the thermal aureole and is less sensitive to viscosity than in Stoke's Law. A typical velocity for a body of radius 3 km is about 10^{-9} m/s if the viscosity of the wall rock at the contact is about 10^{18} p. We show that it is essentially the value of the viscosity at the contact that controls the drag. For this type of diapirism the roof rock must be heated at least to its solidus to allow a reasonable ascent velocity. But since the body of magma contains a limited amount of heat, a first body can only ascend about half way through the lithosphere. A second body coursing the same path within a few hundred thousand years of the first can ascend to within about 20 km of the surface. These results suggest that the heat transfer from the body is at small values (that is, 1-10) of the Peclét number (Pe). A new relationship between Pe and the Nusselt number in this intermediate range of Pe is suggested ($Nu = 1 + 1/2\ Pe^{1/2}$). If the body becomes slender it must process less roof rock, and this possibility is examined through the heat transfer and dynamics of drag. The heat transfer is hardly affected, but the drag models suggest that the body will instead become slightly oblate. Altogether, unless bodies of magma beneath island arcs are unusually large (that is, radius $> \cong 10$ km), more than a single body is needed to penetrate the entire lithosphere. Once a path is forced through the lithosphere, a third drag model shows that bodies about half as large as the earlier ones can ascend about twenty-five times faster. The earliest bodies probably travel at about 150mm/yr, and the later ones at about 4 m/yr.*

Ascent by stoping is essentially undetectable by thermal and chemical contamination, if the blocks are larger than about 3 m (radius) in a basaltic magma and about 30 m in a granitic magma. The process of marginal shattering to produce blocks can be effected by thermal stresses in the wall rock. These stresses are caused by heating of the wall rock, and they are surprisingly large, as originally suggested by Daly. In peridotitic wall rock they amount to about 4 kb per 100 degrees of heating. These stresses are much larger than those usually considered for magmatic inflation and deflation. Because the probable block size could not be estimated, the *ascent velocity of a stoping magma remains essentially unknown. The congestion of the body by stoped blocks probably limits the ascent distance to a few body heights.*

The mechanics of ascent by zone melting are a blend of those of diapirism and stoping, and it is severely limited again by the finite thermal energy of the magma itself.

INTRODUCTION

That magma ascends buoyantly under the action of gravity has seldom been questioned, but the means by which it actually moves remain a mystery. The shapes of intrusive bodies brought forth in the past century the familiar question of emplacement: What happened to the rock initially occupying the space now held by the pluton? This is the essence of the "room problem," once the heart of the granitization controversy itself.

Forcible intrusion, whereby wall rock is pushed aside, and Daly's stoping, whereby blocks of spalled roof rock sink through the magma, are still considered the principal means of intrusion. It seems to have been Grout (1880-1958; for example, 1932 p. 200) who suggested that at depth magma may rise "by rock flowage in the roof and walls." This is forcible intrusion far from the Earth's surface. In the following, the general mechanics of Grout's mechanism are investigated by estimating the ascent velocity of a viscous globe of magma moving buoyantly through wall rock whose viscosity is extremely sensitive to temperature. Daly's stoping mechanism is examined by placing limits on the amount and size of material that can pass through a magma without thoroughly contaminating it chemically and thermally. Ascent by zone melting is also briefly examined.

Doming of the Earth's surface, sometimes associated with forcible emplacement, can only occur near the surface, for the magma has only a limited buoyancy with which to lift overburden. At depth, therefore, diapirism, which was classically known as forcible intrusion, stoping, and zone melting are the possible means of magma ascent. Dike formation and propagation are not treated in this work (but see Shaw, 1980).

A wealth of detailed geological observations are available on the emplacement of plutons, but little theoretical framework exists within which to interpret these facts. The present investigations intend to increase our knowledge of the physical characteristics of magma ascent. Once the major characteristics of each method of ascent (that is, dikes, diapirs, et cetera) are well understood, geological reasoning may then decide the many questions about magma ascent. And although these investigations are aimed at understanding magma ascent beneath island arcs (for example, Marsh, 1979a), they are generally applicable.

MECHANICS OF GROUT'S DIAPIRISM

Field evidence.—There is ample evidence from the internal and external structure of plutonic rocks that they have been emplaced in a broadly viscous fashion (Balk, 1937; Grout, 1945; Buddington, 1959). In particular, Buddington's extensive review of field work regarding the em-

TABLE 1
Symbols

Roman

A — exponential factor of viscosity variation, area of body, and slope of solidus.
a — radius of body.
a_1 — radius of thermal anomaly.
b — radius to outer edge of thermal boundary layer.
Cp — specific heat.
D_i — drag on concentric spheres ($i = s$) and cylinder ($i = cyl$).
d — thickness of shear zone.
d' — thickness of thermal boundary layer.
E — Young's modulus of elasticity.
e — d'/a, also ellipticity of body.
f — d/d', fraction of thermal boundary layer containing shear zone.
g — gravitational acceleration.
H — latent heat of fusion and crystallization.
h — length scale of heat transfer.
i — subscript for wall rock ($i = 1$) and magma ($i = 2$).
J — heat transfer parameter ($A \bullet Nu \bullet K/hV'_2$).
K — thermal diffusivity.
K_c — thermal conductivity.
K_i — drag factor for wall effect of sphere ($i = s$) and cylinder ($i = cyl$).
L — length of ascent path, also Zo.
l — μ_1/μ_2, also length of heated zone of thermal stress.
N — number of stoped blocks.
n — numerical exponent.
P — pressure.
Q_{cd} — heat flux from conduction.
Q_{cv} — heat flux from convection.
Q_t — total heat flux ($Q_{cd} + Q_{cv}$).
R — volume ratio (roof rock/magma), also aspect ratio of body.
r — radial coordinate.
T — average temperature of magma.
T_m — temperature of wall rock.
T_o — initial temperature of magma.
T_s — temperature of solidus.
t — time; t_o, total ascent time.
U — tangential velocity in boundary layer.
V — velocity of body.
V_o — velocity in shear zone.
V'_2 — volume of body of magma.
W — velocity of convection in magma in zone melting.
w — wave number of thermal anomaly for thermal stress.
X — volume fraction of melt or crystals.
y — Z/L.
Z — depth coordinate.

Dimensionless parameters

Nu — Nusselt number (Q_t/Q_{cd}).
Pe — Peclét number (Va/K).
Pr — Prandtl number ($\mu/\rho K$).
Ra — Rayleigh number ($\rho \alpha g \Delta T a^3/\mu K$).
Re — Reynolds number ($\rho Va/\mu$).

Greek

α — coefficient of thermal expansion (volumetric).
$\Delta\rho$ — density difference.
η — a/b.
μ_i — viscosity.
μ^*_i — effective viscosity of wall rock at contact with magma.
ν — Poisson's ratio.
ρ_i — density.
σ_{rr} — radial component of thermal stress.
$\sigma_{\theta\theta}$ — tangential component of thermal stress.
τ — shear stress.
τ_{ij} — stress tensor.
ϕ — tangential coordinate of boundary layer.

placement of granite as a function of depth showed that plutons emplaced near the surface are mostly discordant. At increasingly greater depths emplacement tends to be conformable, the pluton and the country rock have deformed sympathetically. Buddington suggests that whereas emplacement is discordant in cool country rock, it is hot country rock that makes emplacement conformable.

In mapping some plutons of the Sierra Nevada, Moore (1963) found that "both country rock and intrusive rock behaved plastically." He found the pluton contacts to be straight or gently curved over long distances with minor fracturing. The existence of long (26 km), thin (~ 100 m) unbreached septa or screens of wall rock between adjacent plutons indicates gentle wall rock deformation during emplacement. The plutons in general are conformable with each other and the wall rock, each seems to have gradually deformed to make way for the other. At greater depths, where things are much hotter, Grout's idea of viscous deformation during diapirism should be even more closely correct.

General dynamics.—The velocity of magma ascending through viscous wall rock is almost entirely controlled by the viscosity distribution in the wall rock. For slow flows (that is, vanishingly small Reynolds number, Re), as in the present consideration, the ascent velocity is hardly affected by the shape of the body. The drag on a circular disk at small values of Re, for example, lessens only by a factor of two-thirds when it moves edge-on compared to when it moves face-first into the flow. For the present discussion, then, it is not important to stipulate the exact shape of the body, although for simplicity a spherical shape is generally assumed.

It is also clear from the well known extension of Stokes's result for a solid sphere to a viscous sphere by Hadamard (1911 and Rybczynski (1911) that the drag is hardly affected by the viscosity of the magma itself. For the drag lessens only by a factor of two-thirds for a fluid sphere of zero viscosity over that of a body of infinite viscosity (that is, a solid).

These conclusions, that the body shape and viscosity are unimportant, express the fact that for Stokes-flow the scale of the velocity field about the body is insensitive to the exact properties of the body itself. But the velocity field is exceedingly sensitive to the viscosity distribution in the wall rock.

In a medium of constant viscosity, the velocity field extends, for a spherical body, for example, out to about ten body radii. The usual thin deformation envelope about plutons clearly indicates that the wall rock viscosity was not spatially uniform, or not Newtonian, but perhaps plastic. Grout (1945) recognized this and suggested: "It may even be possible that some of the overlying rocks become so reduced in viscosity in a relatively narrow zone that they move aside and down along the sides, permitting the rise of a large mass chiefly by the adjustments in a mobile contact zone." Magma much hotter than its wall rock moves with a thin thermal boundary layer about it. Across the boundary layer or thermal aureole the viscosity changes from some reduced level nearest the magma to its unperturbed value at the outer edge of the boundary layer. The velocity

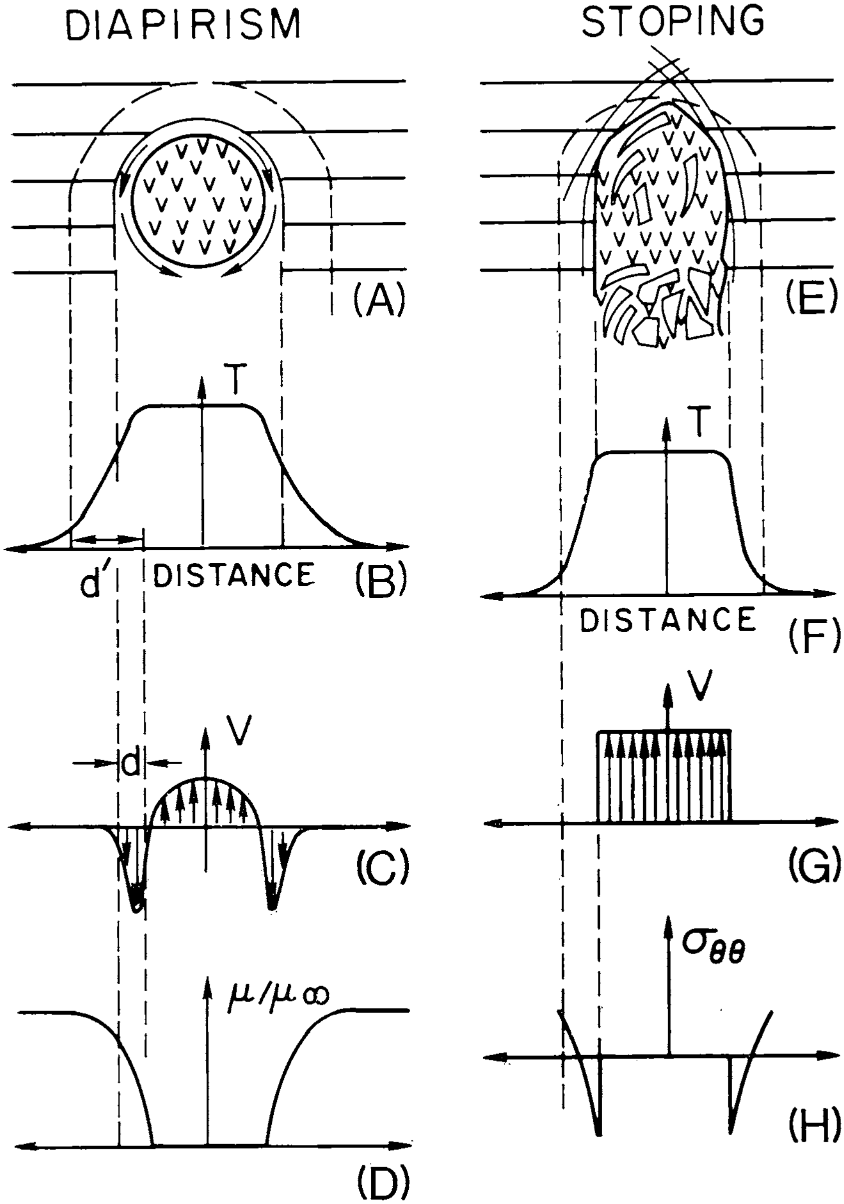


Fig. 1. Schematics of magma ascent by diapirism and stoping. In diapirism, a hot spherical, fluid mass of magma moves by softening a thin region of wall rock nearest the body. The body travels with a thermal halo of thickness d′ (see A and B) that is much thicker than the softened region of thickness d. The velocity distribution (C) is strongly downward nearest the body, this is allowed by the strong change in the viscosity (D) of the wall rock there upon being heated by the magma. In stoping there is also a thermal halo (F), but the magma moves by dislodging rock from its roof and walls, perhaps by thermal shattering (thin lines arching over the body). The average velocity (G) is upward in the magma, the wall rock is still, and the return flow is by sinking blocks. The principal tangential thermal stress (H) is compressional (negative) within the thermal halo and tensional (positive) beyond the halo.

field, and therefore the drag suffered by the body, is strongly affected by the variation of viscosity in the wall rock.

To clarify the importance of a spatially (actually thermally) variable wall rock viscosity, consider a simple model of the drag on the body. The drag on any body is the product of the local shear stress or pressure, depending on the body shape, and the surface area of the body. For a spherical, solid body of radius a moving at a velocity V in a fluid of uniform viscosity μ, the drag D_s, as found by Stokes (1851), is $6\pi a \mu V$. Since the shear stress is determined by the velocity gradient at the surface of the body, the drag can be rewritten as $(4\pi a^2)$ $(\mu 3V/2a)$, where the first quantity is the surface area and $3V/2a$ is an effective average velocity gradient over the body surface. Here the velocity field is taken to diminish completely over a distance a outside the body. This is not exactly so, but it illustrates that, with all else being equal, if the velocity field is confined to a region much thinner than a, the drag will be increased significantly. This is closely related to the wall effect so important for spheres settling in cylinders as in Millikan's oil drop experiment. If the drag is, on the other hand, to be held constant while the shear zone is thinned, the viscosity must be lessened accordingly, presumably by heating the fluid.

This example brings out the essential factors influencing the drag suffered by an ascending body of magma. If the wall rock viscosity is uniform near the body, the body will deform the rock out to great distances, and, considering the great viscosity of the lithosphere, the ascent velocity will be very small. So small in fact that, unless the body is unusually large, heat transfer calculations indicate solidification long before it approaches the surface (Marsh, 1978; Marsh and Kantha, 1978). To ascend successfully, the wall rock viscosity, and hence the drag, must be lessened near the margins of the body as suggested by Grout.

Drag with variable viscosity.—To appreciate how seriously the softened or heated wall rock nearest the magma affects the drag suffered by the magma, consider the following simple analysis. Since the boundary layer is thin compared to the size of the body, flow near the body, but in the wall rock, can be taken to be similar to flow between two parallel plates (for example, as in Couette flow). Furthermore, since, as discussed already, the constitution of the magma is relatively unimportant to the drag, take the body to be solid, but very hot compared to its wall rock. (Here and elsewhere we ignore the effects of volume changes due to thermal expansion and melting, which are generally of little importance.)

The thickness of the shear zone, dictated by the thickness of the thermal boundary layer, is taken to be some arbitrary thickness d. In this steady state, slow, viscous flow, the flow is driven by the upward motion of the solid mass of magma moving at some arbitrary velocity Vo (see fig. 1). Beginning with Euler's equation of momentum, we are left with:

$$\frac{d\tau_{ij}}{dx_j} = 0 \qquad (1)$$

or, for unindirectional flow

$$\tau_{ij} = \text{CONSTANT} \equiv C\,. \qquad (2)$$

This applies to any continuum. For a Newtonian fluid,

$$\tau = \mu \frac{dV}{dy} \equiv C\,; \tag{3}$$

where X_1 has been taken to be in the direction along (vertical) the magma-wall rock interface, positive downward, and $X_2 = y$ is normal to the interface, being positive outward. Since no restriction has been placed on the spatial variation of viscosity, let it vary strongly with y. To gain the drag ($\tau \times$ area), C must be found.

The solution of (3) is:

$$V = C \int \frac{dy}{\mu(y)} + C_1\,, \tag{4}$$

where C_1 is a constant of integration. Assuming, for the moment, that the integral can be evaluated once $\mu(y)$ is given, applying the boundary conditions, $V(d) = 0$ and $V(0) = -V_o$, gives two equations from which C_1 can be eliminated yielding:

$$C = V_o \Big/ \int_0^d \frac{dy}{\mu(y)} \tag{5}$$

This can be simplified further by noticing that the integral to be evaluated is $\overline{(1/\mu(y))}d$, where the barred quantity represents the mean value of $(1/\mu(y))$. Generally speaking, for a function $\mu(y)$ which varies extremely from, say, μ_1 at the interface ($y = 0$) to μ_0 at $y = d$, the viscosity of the unheated wall rock (that is, $\mu_1 << \mu_o$), the mean of the reciprocal of this function $(1/\mu(y))$, is very nearly μ_1. Then,

$$C \equiv \tau \sim \frac{V_o}{d}\,\mu_1\,, \tag{6}$$

where it is emphasized that μ_1 is the *smallest* value of the viscosity in the shear zone. This somewhat surprising result evidently explains why a hot knife or ball so easily penetrates cold butter and paraffin. This result can actually be deduced directly from (3) by noting that, since the shear stress is constant throughout the shear zone, it can be evaluated anywhere, and thus at the magma-wall rock contact it depends on μ_1.

The proportionality in (6) can be removed by choosing an explicit function for the variation of viscosity. Taking $\mu(y) = \mu_1 \exp(Ay/d)$, for example, the integral in the denominator of (5) becomes $(d/A\mu_1)$, and $\tau = V_oA\mu_1/d$. The value of A depends on the strength of the variation in viscosity; for a variation of 5 and 15 orders of magnitude, A is, respectively, 11.5 and 34.5. So, at worst, A lies in the range 10 to 35.

Assuming that (6) describes the mean drag (per unit area) everywhere on the spherical body (this will be discussed further), by equating the total drag to the buoyancy the apparent ascent velocity is found:

$$V_o = 1/3\,\frac{\Delta\rho g a d}{\mu_1 A} \tag{7}$$

where $\Delta\rho$ is density contrast, g is gravity, and the other symbols are as before; A is included not to imply acceptance of the function suggested above but as a reminder of its necessity. This law has the form of Stokes's Law, and when the viscosity is constant everywhere $A = 1$, and, as shown earlier, $d \cong a$, it differs only by a factor of two thirds from the exact result.

Physically, this simple result (7) is inaccurate, because the flow around the sphere is actually caused by a pressure gradient between the front and the back of the body produced by the buoyancy, and the body itself is very fluid. Nevertheless, if the pressure term in (1) is retained, the drag is scarcely different from that of (7). Judging from Stokes's result, this is not surprising because everywhere on the body the drag is the same be it due to pressure, at the leading edge, or shear stress at the mid-section. (A comprehensive treatment when the drag is dominated by pressure at the leading edge is given by Morris, ms.) Knowing either the maximum shear stress or dynamic pressure against the direction of motion gives the total drag. But, since the velocity of the wall rock in the narrow layer must be much faster than the ascent velocity of the body itself, for conservation of mass the ascent velocity (V) relates to the drag-governing velocity (V_o) : $V_o = Va/2d$. The ascent velocity of the magma is then given by:

$$V = 2/3 \frac{\Delta\rho g d^2}{A\mu_1} \qquad (8)$$

which is again similar to Stokes's result when $d = a$ and $A = 1$; differing from the fluid sphere result of Hadamard and Rybcznski by a factor of 3 (that is, when the magma is much less viscous than the wall rock.) Although this result is apparently independent of the body size, it depends implicitly on the body radius through d.

It is clear from the preceding that the wall-rock viscosity nearest the magma and the width of the softened layer (d) control the ascent velocity. d will be some fraction (f) of the thickness of the thermal aureole (d′) about the body. This thickness (d′) depends on the ascent velocity, the nature of the velocity field nearest the body, and the thermal properties of wall rock. The thermal aureole thickness is inversely proportional to the ascent velocity. And, as will be shown in a succeeding section (*Heat transfer*), a representative measure of it is given by

$$d' = a(1 + 0.5Pe^{1/2})^{-1} \qquad (9)$$

where Pe, a pure number, is the so-called Peclét number ($\equiv Va/K$, K is the thermal diffusivity of the wall rock), a measure of heat transfer by convection to that by conduction.

The thickness of the softened zone (that is, $d \equiv fd'$) is determined by the temperature of the magma relative to the solidus temperature of the wall rock and the melting characteristics (that is, heat of fusion and melt production with temperature increase) of wall rock, which controls the velocity distribution. So thickness d depends on the viscosity field, and, strictly speaking, d cannot be determined without knowledge of

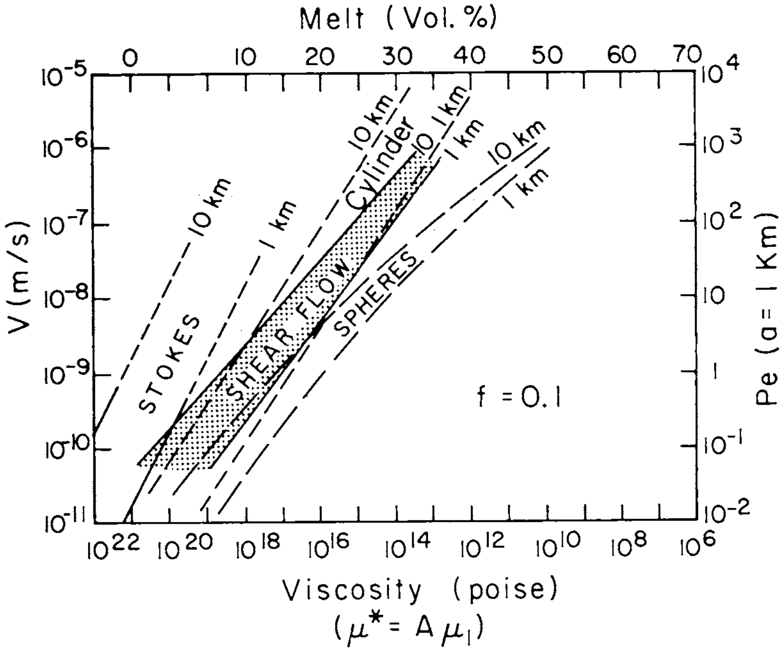


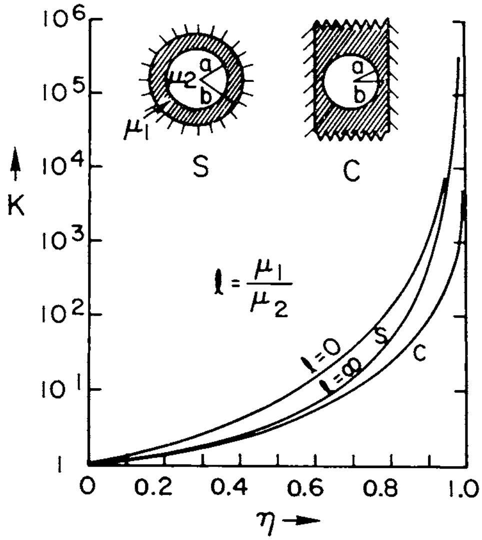
Fig. 2. (A) The ascent velocity (left axis) of a hot diapir as a function of the effective viscosity at the contact. The stippled region marks the results for the drag modeled as a shear flow. The region labeled SPHERES is when the drag is modeled by a sphere enclosed in a viscous sphere of fluid. The region labeled Cylinder is for a fluid sphere rising in a cylinder of viscous fluid. For reference, the region to the left labeled STOKES marks the results of the well-known law due to Stokes; these hold only for isoviscous flows, whereas the others are for variable viscosity. For each result, a curve is given for a body of radius 1 and 10 km. The factor f (= 0.1) signifies that the momentum boundary layer (the softened zone) is one-tenth as thick as the thermal halo. Across the top are *estimates* of the degree of melting of peridotite necessary to produce the effective viscosity along the base. The right axis gives the dimensionless ascent velocity or Peclét number associated with the ascent velocities for a sphere of radius 1 km. Because wall-rock melting probably seldom becomes greater than about 10 to 20 percent, the ascent is unlikely to be faster than about 10^{-8}m/s, and thus Pe < 10.

these other parameters. But virtually nothing is known about the viscosity of rock as a function of melt content. For the present the fraction (f) will be introduced as a parameter, but its relationship to melting of the wall rock remains to be found.

With (9) and the definition $d \equiv fd'$, the ascent velocity (8) becomes:

$$V = 2/3 \frac{\Delta\rho g a^2}{A\mu_1} f^2 (1 + 0.5 Pe^{1/2})^{-2} \tag{10}$$

This result is transcendental in velocity (recall, Pe = Va/K), but the ascent velocity as a function of the effective contact viscosity $\mu^*(= A\mu_1)$ can be found by choosing a velocity, calculating Pe, and then solving for μ^*. For $\Delta\rho = 0.5 \times 10^3$ kg/m^3, g = 10 m/sec^2, a = 1 km and 10 km, and f = 0.1, the ascent velocity is shown in figure 2. It is evident there that



(B) The K- or drag-factors that control the drag as a function of wall-proximity as determined by η (= a/b). When $\eta = 0$, the outer or container wall is infinitely far away, the body resides in an infinite body of fluid, and K = 1. S marks the curves for a sphere of radius a embedded in a spherical container of radius b of viscous fluid. Curves are given both for a solid body ($l = 0$) and an inviscid body ($l = \infty$). C marks the curve for an inviscid sphere of radius a within an infinitely long cylinder of radius b containing a viscous fluid. Notice that the viscosity of the body has relatively little effect on the drag. In the models developed herein $0.9 \leqslant \eta \leqslant 1.0$, so the asymptotic values of K when $\eta \cong 0(1)$ are mostly of interest.

at the larger velocities body size is relatively unimportant in determining the ascent velocity, quite unlike Stokes's result.

This result has two limits: For large values of Pe (>>1), $(1 + 1/2\ Pe^{1/2})^{-2} \cong 4\ Pe^{-1}$, which allows the explicit result:

$$V \cong \left[\frac{16}{3} \frac{\Delta\rho g a K f^2}{A\mu_1} \right]^{1/2} ;\ \text{(large Pe)} \tag{11}$$

And when Pe << 1:

$$V \cong \frac{2}{3} \frac{\Delta\rho g a^2}{A\mu_1} f^2;\ \text{(small Pe)} \tag{12}$$

In obtaining (10), d has been taken to be a constant, but in fact the boundary layer grows thicker with distance around the body, and this thickening might seriously affect the drag. That this is not a serious effect can be seen by employing Levich's (1962, p. 407) result for the variation of d′ with θ. When his formulae are transformed to the present formulation, the drag integrated over the spherical surface differs from that given above by a factor of about two-thirds. This assumes, however, that the shear stress is of the same order as the pressure in controlling the drag. This is so for Stokes's original problem, but it may not be so here, for the drag, especially at large Pe when d′ is thin, may be largely dependent on the retarding pressure at the leading edge (Morris, ms). The above drag model is thus more than likely an upper bound on the ascent velocity. To set a lower bound on the ascent velocity, a more complete and restrictive drag model can be constructed.

Consider the drag suffered by a spherical body of magma encased in a larger concentric sphere of fluid wall rock; so to speak, the drag on a yolk within an egg. The fluid separating the two spheres is taken to represent the anomalously hot wall rock of the thermal boundary layer. Consideration of the drag when the inner sphere is slowly oscillating seems to have been instrumental in bringing Stokes in 1845 to discover the Navier-Stokes equations themselves and finally in 1851 to the well-known law discussed earlier. The problem here follows that of Stokes's original (1851) problem except now there are boundary conditions to be met at the radius (r = b) of the outer shell in addition to the usual ones at the edge of the inner sphere (r = a). The solution to this problem of the drag on a fluid inner sphere (radius a and viscosity μ_2) contained within a fluid sphere (radius b, viscosity μ^*_1) was found independently here and by Haberman and Sayre (1958; see also Happel and Brenner, 1973). The drag suffered by the fluid inner sphere is:

$$D_s = 6\pi\mu_1 aV\left(\frac{1+2/3l}{1+l}\right)K_s \tag{13}$$

where $\eta = a/b$, $l = \mu^*_1/\mu_2$, and

$$K_s = \frac{1-\left(\frac{1-l}{1+2/3l}\right)\eta^5}{1-\left(\frac{9}{4}\,\frac{1+2/3l}{1+l}\right)\eta+\left(\frac{5}{2}\,\frac{1}{1+l}\right)\eta^3-\left(\frac{9}{4}\,\frac{1-2/3l}{1+l}\right)\eta^5+\left(\frac{1-l}{1+l}\right)\eta^6}$$

When $\eta = 0$, (13) reduces to the Hadamard-Rybcznski result (Ks = 1), and when $\eta = 1$, the drag becomes infinite, for the two spheres coincide.

For the magma contained within wall rock $\mu^*_1 >> \mu_2$, $l >> 1$, and the ascent velocity is found to be:

$$V = 1/3\,\frac{\Delta\rho g a^2}{\mu^*_1}\left(\frac{1-3/2\eta+3/2\eta^5-\eta^6}{1+3/2\eta^5}\right) \tag{14}$$

where μ^*_1 is taken to be the effective viscosity ($=A\mu_1$) at the magma-wall rock contact, and η can be related to d through (9): $\eta \equiv a/(a+d) = a/(a+fd')$.

For most values of Pe, it is easily shown that $\eta \cong 1$ and thus the last term in (14) can be simplified by letting $\eta = 1$ in the denominator and by expanding the numerator in a Taylor series about $\eta = 1$. Retaining the leading and largest term in this expansion gives a result accurate to within 10 percent or less of (14) itself for $\eta > 0.7$:

$$V = 2/3\,\frac{\Delta\rho g a^2}{\mu^*_1}(1-\eta)^3 \tag{15}$$

And from the earlier results:

$$1 - \eta = f/(1 + f + 1/2Pe^{1/2}) \tag{16}$$

(15) then becomes:

$$V = 2/3 \frac{\Delta\rho g a^2}{\mu^*_1} \left[\frac{f}{1+f+1/2Pe^{1/2}}\right]^3 \tag{17}$$

The relationship between V and μ^*_1 for a = 1,10 km, f = 0.1, $\Delta\rho = 0.5 \times 10^3 kg/m^3$, and $g = 10\ m/sec^2$ is shown by figure 2. As was anticipated in developing this more complete drag model, these velocities, for any given viscosity, are about ten times smaller than those of (10). And it is evident that (17) depends more weakly on the body size than does (10). This is so for these models because ascent is essentially controlled by the thickness of the thermal aureole (that is, thermal boundary layer) which depends only weakly on the body size.

Like (10), (17) also has two special limiting forms: for large values of Pe (that is, >> 1)

$$V \cong \frac{16}{3} \frac{\Delta\rho g a^{1/2}}{\mu^*_1} K^{3/2} f^3 \text{ ; (large Pe)} \tag{18}$$

and for very small Pe (that is, << 1)

$$V \cong \frac{2}{3} \frac{\Delta\rho g a^2}{\mu^*_1} f^3 \text{ . (small Pe)} \tag{19}$$

In the limit of large Pe, it is clear from (18) and also from (11) that the ascent velocity depends weakly on the body radius as $a^{1/5}$ and $a^{1/2}$, respectively. This is a fortunate result, for within the Earth the sizes of ascending diapirs are largely unknown. The parameter f in these equations is not sensitive in controlling the ascent velocity, for it must be in the range of, say, 0.1 to 0.5.

Within the Earth itself the magma is probably not contained within a spherical envelope nor by a concentric shear zone but more likely by a cylindrical envelope with a hemispherical cap. The actual drag or ascent velocity will probably lie between the curves given by (10) and (17), and henceforth these results (fig. 2) are taken as upper and lower bounds on the actual ascent velocity. It is, nevertheless, clear from figure 2 that, even though body size is relatively unimportant, the viscosity of the wall rock must be known to estimate with any certainty an ascent velocity.

Recalling that the viscosity of peridotite just above its solidus is probably about that of the low velocity zone of the upper mantle (that is, $\cong 10^{20}$p; Cathles, 1973), it is clear from figure 2 that, for any reasonable ascent velocity (for example, $\cong 10^{-6}$ cm/s), the wall rock must be partially molten. The change in viscosity with melt fraction (X) can be estimated roughly by assuming it to vary in a linear logarithmic fashion from $\mu_o(10^{20}p)$ at the solidus (X = 0) to $\mu_m(\sim 10p)$ at the liquidus (X = 1):

$$\mu_1 = \mu_o(\mu_m/\mu_o)^X \tag{19A}$$

Using this function and the indicated numerical values the percentage of melt (upper abscissa) is given as a function of wall-rock viscosity (lower abscissa) in figure 2. Because the peridotite solidus and liquidus are separated by about 500° to 700°C (for example, Ito and Kennedy, 1967; Mysen and Kushiro, 1977), it is unlikely that more than about 20 percent melting can ever be caused by the magma. And this limits the ascent velocity to less than about 5×10^{-9} m/sec.

It is clear from this discussion that the change in wall-rock viscosity upon melting is the single most important parameter in controlling diapirism. If the viscosity near the solidus decreases faster than this linear logarithmic approximation, for any given amount of melting the magma can ascend considerably faster than this estimate.

Drag for repeated ascent.—If the pathway forced by the initial body of magma is coursed by another body before it cools off, the drag suffered by this later body depends primarily on the proximity of the unsoftened wall rock, for the roof rock has already been processed. This drag can be likened to that experienced by, say, a sphere rising within a cylinder of fluid.

The drag suffered by such a spherical fluid body (radius a and viscosity μ_2) moving along the axis of an infinitely-long cylinder (radius b) containing a fluid of viscosity μ_1 has been studied by Haberman and Sayre (1958; see also Happel and Brenner, 1973). Their complete solution for the wall effect (K_{cyl}) as a function of $\eta(= a/b)$ is given only numerically, but they also give the analytic approximate solution:

$$K_{cyl} = \frac{1 - 0.75857\left(\frac{1-l}{1+2/3l}\right)\eta^5}{1-2.1050\left(\frac{1+2/3l}{1+l}\right)\eta + 2.0865\left(\frac{1}{1+l}\right)\eta^3 - 1.7068\left(\frac{1-2/3l}{1+l}\right)\eta^5 + 0.72603\left(\frac{1-l}{1+l}\right)\eta^6} \tag{19B}$$

where $l = \mu^*_1/\mu_2$. This expression matches the complete solution very well up to values of η of about 0.6 beyond which there is increasing disagreement. As $\eta \to 1$, the denominator should go to zero (that is, $K_{cyl} \to \infty$), but it instead tends to a rather small finite value ($\sim 8.87 \times 10^{-3}$, for l large). Since the present interest is for η near unity, the denominator has been made to go zero by arbitrarily adjusting the last constant from 0.72603 to 0.7345. Except when $\eta \cong 1$, this is a harmless adjustment.

As was similarly done for the sphere within a sphere, letting $\eta = 1$ in the numerator, expanding the denominator in a Taylor series about $\eta = 1$, and with l large (that is, $\mu_1 >> \mu_2$), it is found that

$$K_{cyl}^{-1} \cong 5.76 \times 10^{-2}(1 - \eta) \tag{19C}$$

where only the largest term in the series has been kept. The drag (D_c) on the fluid sphere is then given by

$$D_c = 6\pi\mu^*_1 aV\left(\frac{1+2/3l}{1+l}\right)K_{cyl}, \tag{19D}$$

which for l large, $D_c = 4\pi\mu^*_1 aVK_{cyl}$. The ascent velocity is then given by

$$V_{cyl} \cong \frac{1}{52}\frac{a^2\Delta\rho g}{\mu^*_1}(1-\eta)\,. \qquad (19E)$$

And using the earlier formulation (16) for $(1-\eta)$,

$$V_{cyl} \cong \frac{1}{52}\frac{a^2\Delta\rho g}{\mu^*_1}\left(\frac{f}{1+f+1/2Pe^{1/2}}\right) \qquad (19F)$$

where it is understood, as before, that for a spatially varying wall rock viscosity $\mu^*_1 = A\mu_1$ (see the remarks near 6).

Once again, for a = 1,10 km, $\Delta\rho = 0.5 \times 10^3$ kg/m^3, g = 10 m/sec^2, f = 0.1, and K = 10^{-6} m^2/sec, the relationship between ascent velocity and effective wall-rock viscosity is shown by figure 2. This formulation predicts an ascent velocity quite similar to that of eq (10) (that is, the sphere with a shear zone), and the second ascent is not significantly faster than the initial ascent, except at the largest velocities. This model has assumed, however, that the second body is the same size as the first one, and that its ascent is governed by its own thermal boundary layer or aureole. If the first body is, say, of a radius of 10 km and the second body is of any smaller radius the effect of the wall is less, and the body might ascend considerably faster. Starting with an initial chimney whose radius is, say, a_1 the ascent velocity (V_2) for smaller bodies of radius a_2 relative to the initial velocity V_1 is found, from (19D, for V_2) and (19E, for V_1), to be $V_2/V_1 = 347\,(a_2/a_1)^2K_{cyl}^{-1} \cong 347\eta^2K_{cyl}^{-1}$, since $a_2/a_1 \cong a/b$ and for most ascents $(1-\eta_1)$ in (19E) is about 0.05. This calculation shows $V_2/V_1 \cong 25$ for $\eta = 0.5$ to 0.6, and it decreases to about 10 as η increases to 0.8 and decreases to 0.2 from these values. For reasons of drag, *it is most advantageous for succeeding bodies to be about half the size of the initial body: they can ascend about twenty-five times faster.*

Thermal requirements.—To soften the roof rock enough to allow the magma to ascend at a reasonable speed, the roof rock must be heated at least to its solidus. The entire column of roof rock originally above the body must be so affected, and this heat comes from the magma. Because the volume of magma is generally much smaller than that of roof rock and the magma has only a limited amount of energy expendable for processing roof rock, the ascent distance is limited by the energy necessary to bring the column of roof rock to its solidus.

The magma itself gives up heat through cooling, crystallization, loss of potential energy, and exsolution of volatiles. Potential energy losses produce heat by viscous dissipation primarily in the wall rock. Exsolution of gas heats magma, as does crystallization, but this effect is impossible to evaluate due to our lack of understanding of the volatile content of magma at depth.

Regardless of the magma type, magma loses about 250°C during solidification, an additional 300°C (equivalent) from latent heat, and an equivalent of about 150°C in potential energy in ascending, say, 100 km. In terms of temperature, the magma has about 700 degrees per unit volume to yield in processing roof rock. A quantitative measure of the magmatic heat (enthalpy + potential energy + heat of crystallization) going to heat the roof rock (enthalpy + latent heat) from its ambient temperature (T_m) to its solidus (T_s) is given by the following equation.

$$\frac{dT}{dt} - \frac{\rho_1-\rho_2}{\rho_2 Cp_2} gV - \frac{H_2}{Cp_2}\frac{dX_2}{dt} = -\left(\frac{\pi a^2}{V'_2}\right)\left(\frac{\rho_1 Cp_1}{\rho_2 Cp_2}\right) V \left(T_s - T_m + \frac{H_1 X_1}{Cp_1}\right) \qquad (20)$$

The subscripts 1 and 2 refer, respectively, to the wall rock and magma, T is the mean temperature of the magma, V′ is volume, V is ascent velocity, ρ is density, Cp is specific heat, g is gravitational acceleration, H is latent heat or (for wall rock) heat of fusion, X is the fraction of crystallization or fusion, and a is the equatorial radius of the magma and also of the column of roof rock. This equation describes how the magma cools with time as it moves upward. The left side describes the magma's loss of enthalpy (first term), potential energy (second term), and heat gained by crystallization (third term); the right hand side describes the heat needed to bring the approaching wall rock to the desired temperature; the possibility of heat loss by fusion of some constant fraction of the wall rock is described by the last term on the right. Loss of heat to conduction, which does not help the magma proceed, is ignored for the present.

Recalling that dt = dZ/V and integrating (20) from Z = 0 to Z and T_o, the initial magma temperature, to T gives after some rearrangement:

$$\int_0^z \left(T_s - T_m + \frac{H_1 X_1}{C_{p_1}}\right) dz = -\left(T - T_o - \frac{\Delta\rho g z}{\rho_2 C_{p_2}} - \frac{H_2 X_2}{C_{p_2}}\right) \Big/ \frac{\pi a^2}{V'_2} C^*_p \qquad (21)$$

where $C^*_p \equiv \rho_1 C_{p_1}/\rho_2 C_{p_2}$ and $\Delta\rho = \rho_1 - \rho_2$. Convenient forms for T_s and T_m are: $T_s = T_2(1-AZ/T_2L)$ and $T_m = T_3\ (1-(Z/L)^n)$; A is the difference (T_2-T_1) of the solidus temperature at the start of ascent and that at the surface, T_3 is the temperature of the wall rock at Z = 0, L is the distance to the surface, and n is an integer that best describes the shape of the local geotherm. (Although T_o, T_2, T_3 could often be equal, letting them be unequals in the derivation allows for flexibility in evaluating a variety of starting conditions; for example, a magma starting ascent at a super solidus temperature.) Furthermore, let the body shape be that of a prolate spheroid of equatorial radius a and volume $V'_2 = (4/3)\pi a^2 b$ (that is, cigar-like).

Upon substitution, this last equation becomes:

$$\frac{1}{n+1}\left(\frac{Z}{L}\right)^{n+1} - \frac{1}{2}\frac{A}{T_o}\left(\frac{Z}{L}\right)^{2} + \frac{H_1X_1}{T_oC_{p_1}}\left(\frac{Z}{L}\right)$$
$$- \frac{\Delta\rho g\,L}{\rho_2 C_{p_2} T_o}\left(\frac{4}{3}\right)\left(\frac{b}{L}\right)\left(\frac{Z}{L}\right) = \frac{4}{3}\,\frac{T_o - T + H_2X_2/C_{p_2}}{T_oC_p{*}}\left(\frac{b}{L}\right) \qquad (22)$$

For any set of physical properties and T_s, T_m, T_o, and final temperature T, this equation gives the distance (Z/L) to which a body of size (b/L) can ascend before losing all of its heat to the roof rock.

Beginning near the base of the lithosphere $L \cong 125$ km, $T_o \cong 1300°C$, $T \cong 1000°C$ (final temp), $T_2 = T_3 = 1250°C$, $\Delta\rho = 0.5 \times 10^3$ kg/m³, $g = 10$ m/sec², $H_2 \cong 0.42$ J/kg, $X_2 = 1$, $Cp_2 = Cp_1 = 1.25 \times 10^{-3}$ J/kg deg, $\rho_1 = 3.4 \times 10^3$ kg/m³, $\rho_2 = 2.9 \times 10^3$ kg/m³, and with $n = 2$, and assuming there is no fusion of wall rock for this first calculation ($X_1 = 0$), the above equation becomes:

$$\frac{b}{L} = \frac{y^3 - 0.29y^2}{1.95 + 0.53y} \qquad (23)$$

where $y = Z/L$. This result is shown in figure 3. If instead of combining the heat of crystallization with the original enthalpy, this latent heat is released linearly over the ascent path, like potential energy, the denominator of the right hand side of (23) becomes $0.92 + 1.56\,y$. The results are pretty much the same.

Because the body and the column of roof rock have the same radius, the ascent distance is only dependent on the height of the body (b). For a given volume of magma, however, b determines the shape or aspect ratio of the body itself. The results of figure 3 show that for an initial ascent ($n = 2$) the height b is unimportant until $Z/L \cong 0.5$, beyond which the body must be increasingly slender, for a given volume, to sustain ascension.

There may be a limit on how tall a body can become. For once the pressure difference between the wall rock and the magma becomes greater than the strength of the casing of wall rock, the wall rock may flow in and sever the magma. (This possibility is uncertain and warrents further investigation.) Wall rock possessing a strength of at most, say, 1 kb can support a density difference of 0.5×10^3kg/m³ over a distance of about 20 km or $b \leqslant 10$ km, and this length is marked on figure 3. For $n = 2$, this limits the ascent distance to $Z/L \leqslant 0.7$. The body can at most only rise 70 percent of the distance to the surface before stopping due to solidification.

In the above calculation, $n = 2$ was chosen as a good representation of the geotherm for an initial ascent. If a second body, after a sufficiently short time, follows the same path as the first body, it will be insulated by the heat lost by the first body, and its cooling rate will be less. The second body can ascend more quickly until it reaches the first body, and, since it still holds much of its original heat, it can pass the first body and ascend

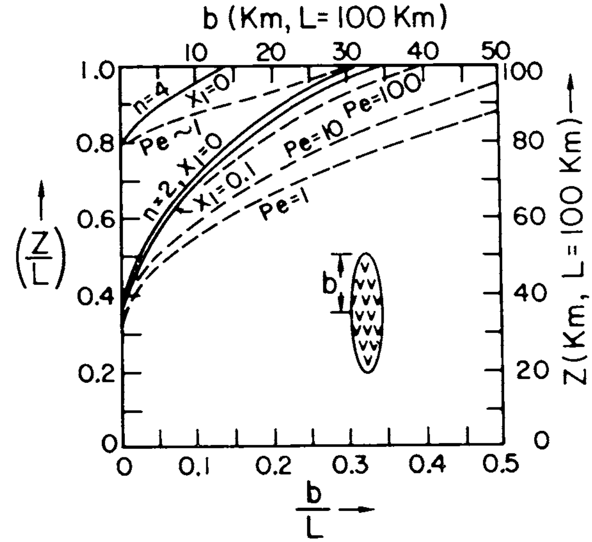
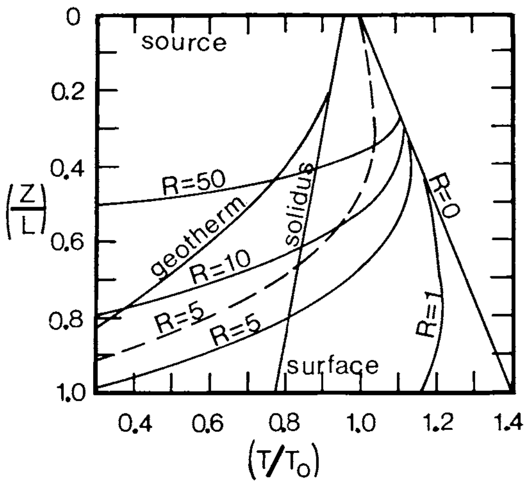


Fig. 3 (A) The half-height b that a body of magma in the shape of a prolate spheroid must take on if it is to ascend a distance L to the surface by heating its column of roof rock to its solidus. (It does not necessarily take on this shape.) The left axis (Z/L) marks the fraction of the distance ascended, and the lower axis marks the body half-height normalized by L. For an ascent distance of 100 km and the thermal parameters given in the text, the body half-height is given by the upper axis, and the distance by the right axis. The lower set of curves (two solid, three dashed) are for an initial ascent (that is, n = 2, see text) at various speeds and degrees of melting of the wall rock, as indicated by the values of Pe and X_1, respectively, against each curve. The top solid curve of this group is for no loss of heat laterally (that is, fast ascent) and no melting of the wall rock. The next solid curve below is for no lateral heat loss but with 10 percent melting of the wall rock. The three dashed curves are for no melting but for a successively slower ascent velocity. The uppermost two curves (n = 4) represent a second ascent, and they are both for no melting: the solid curves is for no lateral heat loss (that is, faster ascent), and the dashed curve is for the slowest ascent.

some distance beyond. (The actual details of how one body passes another are ignored.) To estimate the distance to which the second body can ascend, (22) is solved where n is some larger integer; the perturbed geotherm is rather flat at depth and very steep near the surface. A curve for n = 4 is also shown by figure 3, and the result indicates, assuming again a limit on b of 10 km, that this second body of magma can ascend to within 5 percent of the surface.

The effect of melting wall rock during ascent can also be estimated from (22). For a small amount, say, 10 percent ($X_1 = 0.1$) of partial melting where the heat of fusion for peridotite near its solidus is taken to be about that of diopside, its solidus mineral, ($H_1 = 0.4$ J/kg) the resulting Z − b curve is only slightly different from the case when $X_1 = 0$, as is shown by figure 3.

Since the body actually carries a thermal halo about itself, the amount of heat lost to the roof and wall rock may be significantly greater than that so far estimated. Heat lost laterally does little to expedite ascension. Through this thermal boundary layer of thickness d′, the temperature



(B) The dimensionless, average temperature (horizontal axis) of a body of magma as it ascends through wall rock having the indicated solidus and initial temperature. The magma moves under the assumption that the roof rock must be heated to its solidus. Against each curve is a value of R that effectively represents the ratio of the volume of roof rock to magma. For R = 0, the temperature increases because of heat gained by the magma from latent heat of crystallization and loss of potential energy. For small values of R, say, 1 to 5, if the magma is to arrive at the surface still partially molten, the magma must reduce the volume of roof rock necessary to process by reducing its cross section. The dashed curve for R = 5 shows the result when the latent heat of crystallization is canceled by an equal fusion of roof rock.

drops from, say, T_s to T_m, the solidus to background wall rock temperature. The rock within d′ has, to a fair approximation, its temperature raised by $(T_s-T_m)/2$.

The volume of wall to be heated is determined by the size of d′, which is itself inversely proportional to the ascent velocity or the Peclet number (Pe = Va/K). As will be shown in a succeeding section, for Pe ~ 1, $d' \cong a$, and for large values of Pe ($\geqslant \sim 10$) $d' \cong a/(0.46\ Pe^{1/2})$. The energy necessary to heat this additional volume $2\pi a d'\,(1+d'/2a)Z$ of wall rock by, say, $(T_s - T_m)/2$ degrees is:

$$\pi a^2 \rho_1 Cp_1 V(e + e^2/2)(T_s - T_m)$$

where $e \equiv d'/a$.

Upon adding this term to the right hand side of (20), with due regard to its sign (−), this part of (20) becomes:

$$-\left(\frac{\pi a^2}{V'_2}\right)\left(\frac{\rho_1 Cp_1}{\rho_2 Cp_2}\right)\ V \left[\left(1 + e + \frac{e^2}{2}\right)(T_s - T_m) - \frac{H_1 X_1}{Cp_1}\right] \quad (24)$$

When the ascent is fast e ~ 0, the earlier result (22) is recovered, while for the slowest ascent e ~ 1, and the wall and roof rock must be heated an equivalent of 2.5 times the amount (T_s-T_m). These effects are also shown in figure 3. For the slowest ascent (Pe ~ 1), the maximum initial ascent distance is about 0.55L, and at velocities faster than about Pe = 100 heat loss laterally to the wall rock is relatively unimportant.

The energy balance (20) can also be used directly to monitor the average temperature (T) of a body of fixed shape. Rewriting (22)

$$\frac{T}{T_o} = 1 - R\left[\frac{T_2}{T_o}\left(1 - \frac{A}{2T_2}y^2\right) - \frac{T_3}{T_o}\left(1 - \frac{y^{n+1}}{n+1}\right) + \frac{H_1X_1}{Cp_1T_o}y\right] + \frac{\Delta\rho g L}{\rho_2 Cp_2 T_o}y + \frac{H_2X_2}{Cp_2T_o}y \qquad (24A)$$

where $y = Z/L$, the heat of crystallization (last term) is linearly distributed over Z, and $R = \pi a^2 L Cp^*/V'_2$. This latter parameter (R) essentially measures the volume of roof rock that must be processed relative to the volume of the body of magma itself, for $Cp^* \cong 1$. Using the constants stated already near (23)

$$\frac{T}{T_o} = 1 - 0.96\, R\left[\frac{y^3}{3} - 0.096y^2\right] + 0.39y \qquad (24B)$$

and a series of curves calculated from this are shown by figure 3. At small values of R the heating of the magma above its initial temperature reflects the heat gained by loss of potential energy and through crystallization. This is an ideal example, for a part of this energy must also be dissipated within the wall rock. But this also conserves the magma's energy. It is clear from these curves that, at least for this particular geotherm, solidification takes place at depth unless R is less than about four. That is, the magma can only process about four times its own volume of roof rock. If this magma is to arrive at the surface, its cross-sectional area must accordingly be made smaller. These results also suggest that it is unlikely that a single body can reach the surface without becoming vertically greatly extended.

During the time of decay of the initial thermal anomaly, a second body must follow if it is to receive the benefit of a preheated passageway. This time can be estimated from a description of the decay with time of the axial temperature of a cylindrical thermal anomaly within an infinite medium (for example, Crank, 1956, p. 28)

$$\frac{T - T_m}{T_o - T_m} = 1 - \exp(-a_1^2/4Kt)$$

where a_1 is the radius of the initial thermal anomaly, and the other symbols are as before. The time necessary for the initial temperature contrast to decay by, say, 50 percent is found to be

$$t = 1.14 \times 10^4 a_1^2 \qquad \text{(yrs)}$$

where a_1 is now in units of km and a value of 10^{-6} m²/sec has been taken for K. For $a_1 = 2$ km, t = 44,000 yrs; for $a_1 = 5$ km, t = 285,000 yrs; for $a_1 = 8$ km, t = 730,000 yrs. These are minimum estimates, because if there is any melt present upon solidification it will release heat and prolong cooling. For, say, 10 to 20 percent melt, an equivalent of about 30 to 60 degrees of additional temperature must be lost; this should increase the cooling times by about 15 percent. All things considered, to benefit from and preserve the initial thermal anomaly bodies must ascend on the order of once every 100 to 500 thousand years.

In the results shown as figure 3, it has been assumed that the body may, within reason, take on the shape of a prolate spheroid (that is, cigar-like) to preserve its heat and enable it to process a longer, but thinner, column of roof rock. This of course does not mean that this is actually the shape of the body; these results merely show how far a body might travel were it to possess such a shape. It is thus of some interest to know how this shape affects the heat (and mass) transfer from the magma.

Heat transfer.—The heat transfer from a body of arbitrary shape and size moving through either a viscous fluid or a solid, perhaps as in an elastic crack, is described by (Marsh, 1978, eq 8)

$$T = Je^{-Jt} \int e^{Jt} T_m(t) dt \tag{25}$$

where T is the mean temperature of the magma, T_m is the mantle or wall rock temperature far from the body, and t is time. The parameter $J (\equiv A \cdot Nu \cdot K/hV'_2)$ describes the actual transfer of heat in terms of the ratio of surface area to volume (A/V'_2), the Nusselt number (Nu), the characteristic length scale of the body (h) for conductive heat loss, and the thermal diffusivity K. The physical foundations of this equation and some solutions of it were given by Marsh (1978) and Marsh and Kantha (1978). Although these authors treated mainly a spherical body, their conclusion that for a successful ascent $Jt_o < 1$, where t_o is the total ascent time, is general and is not restricted to a particular geometry or mode of transport; the choice of Nu does, however, depend on the mode of ascent.

The Nusselt number, a pure number, measures the total heat transfer from the body relative to that by conduction. In its simplest definition, it measures the ratio of the conductive to convective length scales; it measures the thermal boundary layer thickness. It can easily be shown by scaling arguments (for example, Eckert and Drake, 1972) that the Nusselt number in the present context is solely a function of the Peclét number $(Pe \equiv Vb/K)$ and can be represented by

$$Nu = C_1 + C_2 Pe^n \tag{26}$$

where C_1 and C_2 are constants of order unity, and n is a constant that depends on the nature of the velocity field. It is important to realize that Nu is defined to include all the complicated thermo-mechanical effects of heat transfer from the body. As far as the form of (26) is concerned, the exact body shape is relatively unimportant as long as it is *roughly* spherical, as we shall assume in the following discussion.

In flows of constant viscosity, n depends on the scale of the velocity field relative to that of the thermal field; this is measured by the Prandtl number, $Pr \equiv \mu/\rho K$. Under conditions of constant viscosity, Pr within the Earth is practically infinite, and only the character of the velocity field nearest the body is at all important in determining the rate of heat transfer. At these large values of Pr, Nu depends on the magnitude of Pe: when Pe is near unity (that is, convection ≅ conduction), $C_1 = 1$, $C_2 = 1/2$, and $n = 1$, for either a solid or fluid sphere (higher order terms arise for the solid body; O'Brian, 1963; Acrivos and Taylor, 1962). When Pe is large (that is, $> \cong 10$), $C_1 = 0$, $C_2 = 0.46$, and $n = 1/2$ for a fluid sphere, and $C_1 = 0$, $C_2 = 0.625$, and $n = 1/3$ for a solid sphere (Levich, 1962). This result has been improved by Acrivos and Goddard (1965) who found that for a solid sphere $C_1 = 0.461$ and $C_2 = 0.6245$. The result for the liquid sphere is yet to be improved.

Before considering what C_1, C_2, and n might be in flows of variable viscosity, it is useful to realize how the exact value of n comes about in problems involving flows of large Pe number and constant viscosity. If the components of fluid velocity near the surface of the sphere (either liquid or solid) are given as $V \cong -b_1 y \sin\phi$ and $U \cong b_2 y^g \cos\phi$ in, respectively, the r and ϕ directions, the equation for temperature in the boundary layer form is (b_1 and b_2 are constants):

$$\mathrm{Pe}\left[b_2 y^g \cos\phi \frac{\partial T}{\partial y} - b_1 y^f \sin\phi \frac{\partial T}{\partial \phi} \right] = \frac{\partial^2 T}{\partial y^2} \qquad (26A)$$

where $y = r - a$, a is the sphere radius, and T is dimensionless $(=(T - T_m)/(T_0 - T_m))$. To eliminate Pe, which enables a similarity solution to be found, experience suggests the substitution $y = X\mathrm{Pe}^{-1/P}$, whereupon it is found that not only must $g = f + 1$ but $P = g + 1 = f + 2$, and by definition $n = 1/P$. It is thus enough to know the dependence of either V or U on y, the boundary layer variable (for they are related through the continuity relation), to find the value of n, and this can be done by inspection. In a fluid of constant viscosity, for example, for a solid sphere $V \cong -(3/2)V_0 y \sin\phi$, $f = 1$, and $n = 1/3$, and for a fluid sphere $V \cong -V_0 \sin\phi$, $f = 0$, and $n = 1/2$.

In flows of variable viscosity, values of C_1, C_2, and n as a function of Pe are unknown. But if the size of Pe* (* = variable viscosity) can be estimated These values should be similar to those in flows of constant viscosity for Pe = Pe*. By definition, Pe is the product of the Reynolds number (Re) and the Prandtl number (Pr, a measure of the ratio of the velocity boundary layer thickness to that of the thermal field), or Pe ≡ RePr and Pe* = Re*Pr*. For diapirism, Re and Re* are very nearly the same, but Pr >> Pr*. This is so because for constant viscosity the thickness of the thermal boundary layer is much less than that of the velocity field, whereas for variable viscosity the thickness of the velocity field is much less than the thermal boundary layer thickness. This reasoning implies that Pe* << Pe, because Pr* << Pr. (More on this is forthcoming from Morris, ms.)

In sum, to choose the appropriate constants for (26) an estimate of Pe must be made. In the curves shown already for the ascent velocity determined by the drag model (fig. 2), it is clear that to move at a reasonable speed the wall rock must be heated at least to its solidus. Because the magmatic temperature is probably never much above the solidus of the wall rock, except possibly near the surface or when penetrating continental crust, the initial penetration velocity is unlikely to be greater than about 10^{-9} to 10^{-8} m/s, and this is essentially true regardless of the size of the body. The magnitude of the associated Pe, which determines the formulation of Nu, is between 1 and 100 for a body with a radius of between 1 and 10 km. Hence Pe* is probably not greater than about unity. Since the exact nature of the heat transfer for the variable viscosity flow is unknown (but see Morris, ms), we will assume in the following that Pe and Pe* surely lie in the range of 0 to 100.

This range of Pe spans the region between small and large values of Pe, where the relation of Pe to Nu is analytically unknown. Extrapolating the small Pe formulation to larger values of Pe and extrapolating the large Pe formulation to small values of Pe show a large disagreement between the two formulations in the region of Pe near 1 as shown in figure 4. Since both results seem well established in their own domains of validity — but experimental verification does not seem abundant — to make the results match, we have (ad hoc) added 1 to Levich's liquid sphere result and 0.5 to the improved results for a solid sphere (fig. 4). (A need for the solid sphere results will arise later.) Levich (1960, p. 87) himself suggests such an interpolation, and this action is not without some experimental justification.

In a study of heat transfer within and from drops of liquid at low Reynolds number, Head and Hellums (1966) measured Nu at a value of

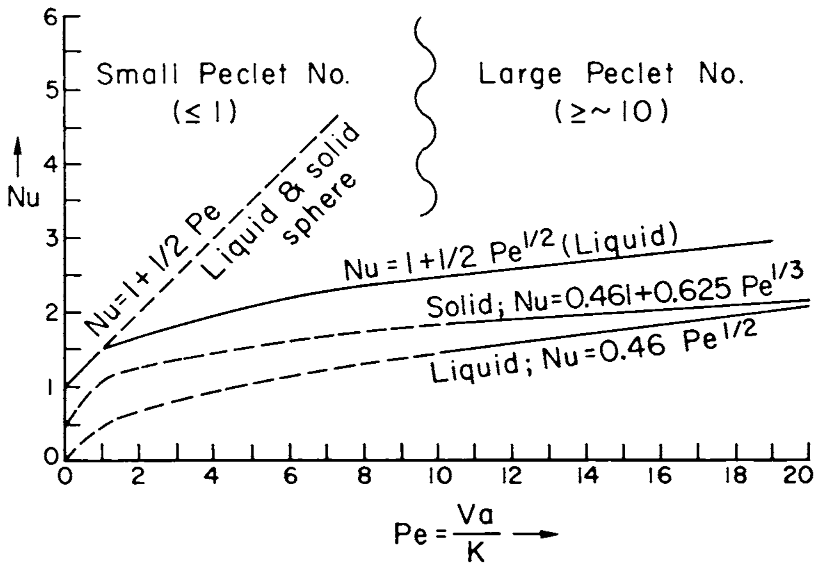


Fig. 4. Analytical relations between Nusselt number (Nu, left axis) and Peclét number (Pe, lower axis) for liquid and solid spheres moving in an isoviscous Newtonian fluid. Analytic results exist at values of Pe less than unity and greater than about ten (solid lines). To cause the extrapolations of these results to agree in the intermediate range of Pe, the function describing the long solid line is suggested.

Pe of 1850. Using Levich's original result (that is, $Nu = 0.46\ Pe^{1/2}$), the value of Pe would have to be about 2100 to match the measured value of Nu. Using the adjusted formula (that is, $Nu = 1 + 0.46\ Pe^{1/2}$) the expected value of Pe for the measured value of Nu is about 1900; a significantly closer agreement even at these large values of Pe.

In physical terms, for a given temperature difference the overall heat transfer from the body is measured by Nu which is determined by the thickness of the diffusion layer (d′). Adjusting the Nu-Pe relation makes d′ smaller and Nu correspondingly larger for any given value of Pe. The effective diffusion layer thickness (d′) which controls cooling is:

$$d'/a = Nu^{-1} = (1 + 0.46\ Pe^{1/2})^{-1} \tag{27}$$

This result is similar to that for the thickness (d) of the mobile part of the thermal boundary layer, which controls the drag or the transfer of momentum, as already shown by eq (9).

Solutions to (25) are given by Marsh and Kantha (1978) as a function of Jt_o and with the adjusted Nu-Pe relation

$$Jt_o = A\left(\frac{Z_o}{V'_2}\left(\frac{K}{Va}\right) + 0.46\frac{Z_o}{V'_2}\left(\frac{K}{Va}\right)^{1/2}\right) \tag{28}$$

where Z_o is the distance ascended in a time t_o, and all other symbols are as before. To ascend to the surface $Jt_o = 1$; to ascend 80 percent of the way to the surface, $Jt_o = 3$; and Jt_o increases accordingly for proportionally shorter ascent distances (see fig. 3 of Marsh and Kantha, 1978).

For a body of any given volume, it is clear from (28) that the larger its surface area (A), the faster it must ascend to arrive at any level with the same temperature as, say, an equivalent sphere of magma. The surface area (A) of a prolate ellipsoid (b, semi-major axis, and a, semi-minor axis) relative to that of a sphere (A_s) of equal volume is given by:

$$\frac{A}{A_s} = \frac{1}{2}\left[\left(\frac{a}{b}\right)^{4/3} + \left(\frac{b}{a}\right)^{1/3}\left(\frac{\sin^{-1}e}{e}\right)\right] \tag{29}$$

where e is the ellipticity ($= (b^2 - a^2)^{1/2}/b$). For a sphere e = 0, and for a needle e approaches 1. The last quantity on the right, for e up to about 0.7, which already is beyond our bounds for a magma shape, is always between 1 and 1.1 (e = 0 to 0.7, respectively). This quantity therefore can be dropped with little loss of generality.

If b is assumed, as mentioned already, to be at most about 10 km, for an equivalent spherical volume of magma having a radius of from 1 to 10 km, the quantity b/a varies, respectively, from about 33 to 1. And the quantity A/A_s in (29) varies, respectively, from 1.61 to 1.0. That is, in the worst case of a magma 20 km tall and 660 m in diameter the increased cooling due to an increase in surface area over that of a sphere is by a factor of 1.61. Considering that a propagating dike (an oblate ellipsoid) of aspect ratio R must travel $R^{4/3}$ times faster than an equivalent sphere to maintain the same temperature (Marsh, 1978), this is not much of an effect.

Body shape.—From figure 4 it is clear that heat transfer from the body is, over a large range of Pe, governed by a Nusselt number of, say, about 3. This external Nu must be balanced by an internal Nu that depends on the rate of convection as measured by the Rayleigh number (Ra). Since Ra varies with the cube of the characteristic length (that is, radius), in order to balance the internal and external Nu the body may be forced to assume a special shape. The relation of Nu to Ra ($=\rho\alpha g\Delta Ta^3/\mu K$; α is the coefficient of thermal expansion, other symbols as before) is in general of the form (for example, Shimazu, 1959; Kreith, 1973, p. 393; Shaw, 1974):

$$Nu = C_3\, Ra^{1/m} \tag{30}$$

C_3 varies from about 0.5 to 0.1 depending on the geometry, and m varies between 3 and 4 for laminar flow and between 2 and 3 for turbulent flow.

For $C_3 = 0.1$ and $m = 3.5$, a value of $Nu = 3$ demands a Ra of about 1.5×10^5 which for a conservative choice of Ra parameters ($\alpha = 5 \times 10^{-5}$ deg^{-1}, $\Delta T = 1°C$, $\mu = 10^5 p$, and $K = 10^{-6}$ m^2 sec^{-1}) implies a length scale of only about 10 m. This is not much of a constraint, for any body thicker than 20 m will satisfy it.

Lighthill (1953; Ostrach, 1964) has found for convection in vertical tube-like bodies closed at one end and with a horizontal temperature gradient that the fluid will stagnate near the closed end if the tube is too long (L) relative to its radius (a). Stagnation occurs when the product (a/L) Ra is less than 311 and the associated value of Nu is 0.364. In terms of the aspect ratio (b/a) of the prolate ellipsoid mentioned above, this implies that $b/a > Ra/622$, and since $Ra \sim 10^5$, $b/a > \sim 200$. That this is much larger than the aspect ratios considered already, where the minimum was about 30, implies that even the most needle-shaped magmas will probably not experience stagnation. But in dikes where the aspect ratio may reasonably exceed 1000 stagnation is sure to occur.

In sum, the internal dynamics of the magma probably have little influence on the shape of the body itself. The external heat transfer and dynamics must therefore control the shape.

A spherical body of liquid obeying the Hadamard-Rybczynski form of Stokes's law remains spherical no matter how large it is, because the normal component of the stress inside and outside the body differs always by the same constant (Batchelor, 1967, p. 238). For the hot body considered here, the normal stress may diminish along the body from front to back: along the outer surface of the body the shear stress is balanced by a pressure gradient driving the fluid wall rock from the front to the back of the body. Dimensionally this pressure (P_s) at any point of the surface is given by,

$$P_s \sim \frac{\mu VL}{d^2} \tag{31}$$

where the curvature of the surface has been neglected, L is the length of the surface, and the other quantities are as defined already; μ and V are evaluated at the surface. Since the thickness of the softened zone (d) increases with distance along the body — and V and μ also probably decrease somewhat — the pressure on the surface may decrease downward along the body. The body will attempt to equalize this pressure on all parts of the surface by expanding in direct proportion to the increase in d. Thus the body may mushroom and take on the shape of a tear drop. But this adjustment can only be a minor one, because this pressure gradient is essential for the rise of the body itself.

These qualitative considerations suggest that an actual magma may tend toward the shape of an oblate ellipsoid (that is, pill-like) rather than that of a prolate ellipsoid (cigar-like). This action increases the body's cross-sectional area, forcing it to process a larger volume of roof rock, and thereby allowing the whole body to ascend a distance shorter than if it had any more oblate shape. The exact amount of flattening that would occur is difficult to estimate accurately, for it may depend on the speed of ascent. For large values of Pe, the momentum boundary layer stays thin over the upper half of the body which retards flattening, and for small values of Pe ($\cong 0$), the Stokes case is approached where the body is spherical. Overall, the body may tend to be spherical.

A SIMPLE MODEL OF ASCENT

The preceding results imply that for the magma to move fast enough to penetrate a significant part of the lithosphere the wall rock must be slightly molten. This melt may strongly reduce the drag suffered by the body. If upon melting, however, the wall rock viscosity changes extremely, as when, say, the wall rock is a single pure phase with a unique melting point, the ascent velocity will be dictated solely by the ability of the magma to supply the requisite heat for melting. The body moves only as fast as the on-coming roof rock can be heated above its solidus (T_s) to T_1. Heating the roof rock only to its solidus will, it is assumed, produce no motion. This convective heat flux (Q_{cv}) must be supplied by the body in the amount:

$$Q_{cv} \equiv A_1 \rho_1 Cp_1 V[T_1 - T_s + X_1 H_1/Cp_1] \tag{32}$$

where A_1 is the cross-sectional area of the roof rock moving toward the body of magma, the other symbols are as defined already. By definition: $Nu \equiv Q_t/Q_{cd} \equiv (Q_{cv} + Q_{cd})/Q_{cd}$, where Q_t is the total heat flux from the body and Q_{cd} is the conductive heat flux from the magma when it is at rest and the wall rock temperature is just below its solidus temperature. For a body of arbitrary shape, $Q_{cd} \equiv A_2 K_c (T_s - T_m)/a$, where A_2 is the effective surface area of the body which is involved in balancing Q_{cv} of (32). From these definitions, Q_{cv} can be replaced in (32) by Q_{cd} (Nu − 1). Hence upon some rearrangement (32) can be put in the form:

$$\frac{Va}{K} = \frac{A_2}{A_1}\left[\frac{T_s - T_m}{T_1 - T_s + X_1 H_1/Cp_1}\right](Nu - 1) \tag{33}$$

And in keeping with the definitions of Nu and Q_{cd},

$$Nu \equiv \frac{T - T_m}{T_s - T_m}\left(\frac{a}{d}\right). \tag{34}$$

As T, the magma's average temperature, reduces to T_s the body comes to rest, d approaches a, and Nu = 1; the heat transfer is solely by conduction. Combining (34) and (33) gives the desired result:

$$\frac{Va}{K} = \frac{A_2}{A_1}\left[\frac{T_s-T_m}{T_1-T_s+X_1H_1/Cp_1}\right]\left[\frac{T-T_m}{T_s-T_m}\left(\frac{a}{d}\right)-1\right] \quad (35)$$

When heating does not go much beyond the solidus ($T_1 \cong T_s$), where $A_1 \cong A_2$, and for sufficiently small values of Pe (that is, near zero) d ≅ a, a simplified version of (35) results

$$\frac{Va}{K} \cong \left[\frac{T_s-T_m}{X_1H_1/Cp_1}\right]\left[\frac{T-T_m}{T_s-T_m}-1\right] \quad (36)$$

The last result can be checked easily by simple experiment: A one inch bronze ball containing a thermister at its center is heated by a one-quarter inch copper tube connected to a soldering iron. The copper tube is inserted into the bronze ball through a hole drilled into it. This hole also contains the thermister, the leads of which run up the copper tube and out through a hole in its wall to be connected to a resistance bridge. The temperature in the ball is regulated by controlling the voltage to the soldering iron. In an experiment, the hot ball is placed on a block of paraffin initially at room temperature (T_m), and its steady state velocity of penetration is measured as a function of the steady state central temperature (T). Some results of this setup are shown as figure 5.

For this paraffin with $T_s = 50°C$, $H_1 = 0.224$ J/kg, $Cp_1 = 2.1 \times 10^{-3}$ J/kg deg (Perry and Chilton, 1973), and $T_m = 22°C$, the first parameter in (36) is about 0.26. The experimental slope is around 0.5. This is an

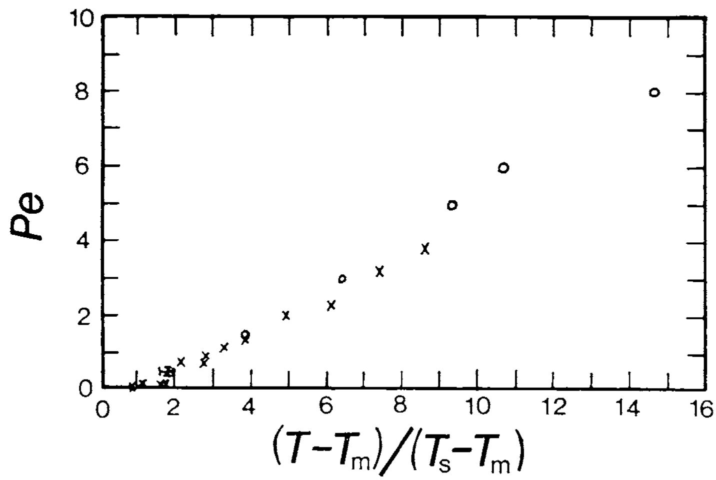


Fig. 5. Dimensionless penetration velocity (Va/K, left axis) of a hot metal sphere in paraffin as a function of the sphere's central temperature (T) relative to the melting temperature of paraffin (Tm). The crosses are from the present experiments and the circles are the constant-flux experiments of Hardee and Sullivan (1974).

expected difference because as Pe increases so does (a/d) and generally for $1 \leqslant Pe \leqslant 10$, $1.5 \leqslant (a/d) \leqslant 2.6$. So the slope of (36) might be expected to vary from about 0.40 to 0.68, which agrees well enough with observation.

If these results are applied directly to the Earth, for 10 percent melting of peridotitic wall rock containing diopside ($H_1 = 0.358$ J/kg, $Cp_1 = 1.17 \times 10^{-3}$ J/kg deg) as its solidus phase the slope of (36) is about 6.5. And in the lithosphere for averaged values of T_s ($\cong$ 1000°C), T_m ($\cong$ 800°C), and T ($\cong$ 1200°C) the ascent velocity is given by $Va/K \cong 6.5$. In continental crust where the solidus phase is, say, quartz ($H_1 = 0.136$ J/kg, $Cp_1 = 1.046 \times 10^{-3}$ J/kg deg), the ascent velocity of a granitic diapir might be about $Va/K \cong 15.4$, about twice as fast as for peridotitic wall rock.

The results of this section essentially ignore the dynamics of ascent and estimate the ascent velocity when melting of the wall rock is the rate controlling step. That (36) fits the experimental data as well as it apparently does, upon closer inspection, may be somewhat fortuitous. The quantity a/d in (35) is actually, as indicated by (27), also a function of Pe, so (35) is nonlinear. Combining (34) with (27) and substituting this result into (35) for d/a shows that Pe is equivalent to a constant (that is, $\sim [(1/2)\ (A_2/A_1)\ (T_s-T_m)\ (T_1-T_s + X_1H_1/Cp_1)^2]$). This probably only implies that for (33) and (34) to be consistent with (27) this definition of Pe must be made, but this definition is not consistent with experiment. Therefore in the definition of Nu (that is, eq 34) the experimental results imply that $d \cong a$, and Nu depends solely on the size of the super-conductive temperature gradient. Although this reasoning, and the somewhat unconventional definition of Nu, is somewhat circular, such reasoning is necessary to explain the experimental results. And this does serve to outline the important factors in such a model.

The main idea of this section was expressed in a report by Hardee and Sullivan (1974). Although their analysis is quite different from the present results, their experimental results using a constant flux source in paraffin are also consistent with (35); these results are also shown in figure 5. These workers also found that the effect of increased buoyancy on the migration velocity is small. When the effective buoyancy is increased by a factor of twenty-five the observed value of Pe changed by about two at small values of Nu (~3), and Pe changed hardly at all at larger values of Nu. This emphasizes the fact that under these conditions the velocity is controlled almost completely by heat transfer and not by the dynamics of drag. But because mantle wall rock never becomes completely molten, it is unlikely in the Earth that heat transfer completely controls ascent.

MECHANICS OF DALY'S STOPING

In magmatic stoping, blocks of disengaged roof rock sink through the magma. The magma essentially mines its way upward. Stoping was first invoked to understand intrusion by J. G. Goodchild (1892, 1894) and A. C. Lawson (1896), but it was R. A. Daly (1871-1957; for example, 1903) in his study of Mount Ascutney, Vt., who rediscovered and developed the idea to its present state of appreciation. Today there is no doubt that some plutons have moved by stoping. Undeformed wall rock and large xenoliths of roof rock are testimony to its occurrence (Pitcher, 1979). But the extent to which it prevails is still questioned. And the objections to stoping remain the ones initially proposed by H. Cloos, Harker, Iddings, Lindgren, and other prominent geologists of the day. Namely: (1) Can the xenoliths sink? (2) Why are so few xenoliths actually seen in the field? (3) If stoping occurs during the whole ascent, how does the magma remain hot?

Daly (1933, p. 280) answered these objections: (1) The xenoliths are denser. (2) The stoped xenoliths are assimilated into the magma. (3) "[Stoping] is responsible merely for the completion of the process," relating ". . . to the mode of intrusion . . . . through the last few thousand feet of uprise" (1914, p. 195). It is clear today that the crustal assimilation and caldera collapse associated with large silicic volcanic centers is exactly Daly's idea of stoping. It might also be important in deep magma transfer.

Contamination.—Daly clearly saw the implications, and therefore the tests, of the stoping theory: "In most cases nearly all of the sunken blocks *must* be melted or dissolved or both" (1914, p. 216). This poses the question: "First, what is the actual, though small, proportion of foreign material that can be dissolved by primary magma? Second, what is the absolute volume of a magmatic body which is capable of such a degree of contamination?" (1933, p. 301). Since nearly fully molten basaltic lavas are commonly found in island arcs lying both on oceanic and continental crust, it is clear that these magmas have hardly been thermally contaminated by blocks of cold crustal rock. And the well known general lack of correlation between the composition of a basaltic lava and crustal material implies that chemical contamination is not large. But many isotopic studies show that there must be some contamination, albeit small.

The amount of contamination depends critically on the size of the block and its sinking velocity, namely, the ratio of block surface area to magma volume and the residence time of any block in the magma. The size of the block determines its surface area and also essentially its sinking velocity. At one extreme, large blocks may spall singly from the roof and drop quickly through the magma, the block surface area relative to its volume is small, and its residence time is short. The magma might be hardly contaminated. At the other extreme, small pieces of roof rock may spall in a continuous shower and thoroughly contaminate the magma in a very short time. Block size is controlled by what Daly called marginal shattering, caused by thermal stresses set up in the wall rock by the approaching hot magma. In addition, the stress distribution attending magmatic expansion, perhaps upon degassing, may also contribute to marginal shattering. All in all, the exact mechanics of marginal shattering is not understood, and the block sizes are thus unknown. But something more on this account will be discussed later.

Nevertheless, a simple, but highly revealing, model can be constructed as follows. The magma is assumed to move continuously upward and contain at all times a set fraction of blocks of an arbitrary size. Each block cools or chemically contaminates the magma in direct proportion to its surface area and the difference between its temperature or composition and that of the magma. For slowly settling (that is, small Reynolds number) spherical blocks (the exact shape is unimportant) the flux of heat from the magma into N blocks is (Levich, 1962, eq 14.19; see Marsh and Kantha, 1978, for the general method)

$$Q = 7.98\ NK^{2/3}V^{1/3}a_1^{4/3}\rho_1 Cp_1(T - T_m(t)) \qquad (37)$$

where a_1 is the block radius, T is the mean temperature of the magma, $T_m(t)$ is the mean block temperature, which varies with distance upward or ascent time (t), and the other symbols are as before.

The loss of temperature by, say, a spherical body of magma of radius a is, by conservation of energy:

$$\frac{4}{3}\pi a^3 \rho_2 Cp_2 \frac{dT}{dt} = -7.98\ NK^{2/3}V^{1/3}a_1^{4/3}\rho_1 Cp_1(T - T_m(t))\,. \qquad (38)$$

To begin with, if the mean temperature of all the blocks is a constant during ascent, say, T_m, (that is, the mean wall-rock temperature for the whole ascent) (38) can be immediately integrated to yield:

$$\frac{T-T_m}{T_o-T_m} = \mathrm{EXP}\left[\ -7.98\,\frac{NK^{2/3}V^{1/3}a_1^{4/3}}{V'_2}\,t\ \right] \qquad (39)$$

where V'_2 is the original volume of magma, T_o is the initial magmatic temperature, and $\rho_1 Cp_1 \cong \rho_2 Cp_2$ has been assumed. (A general method of solution for various $T_m(t)$ is given by Marsh and Kantha, 1978.) If the magma ascends a distance L at velocity V, t = L/V, and if the total number of blocks is written as some fraction (f) of the ratio of the total volume of magma to the volume of a single block, (39) can be written as

$$\frac{T-T_m}{T_o-T_m} = \mathrm{EXP}\left[\ -1.91\,f\left(\frac{Va_1}{K}\right)^{-2/3}\frac{L}{a_1}\right] \qquad (40)$$

An exactly analogous equation can be written for the change in concentration (C) of any chemical component by substituting the appropriate parameters: concentration (that is, chemical potential) on the left hand side and on the right replacing the thermal diffusivity (K) by the chemical diffusivity (D) appropriate for the component C.

If the magma is to ascend a distance L and undergo no more than, say, 10 percent contamination, the left side of (40) must be 0.9 or greater, and the ensuing block settling velocity must be

$$V = 77.5(fL)^{3/2}\,K\,a_1^{-5/2} \qquad (41)$$

or greater. For magma half filled with blocks at all times (f = 0.5), the constant in brackets is 8.7×10^{12} for L = 100 km and 2.74×10^{10} for L = 10 km; $K = 10^{-6}$ m²/s. These equations are shown as lines in figure 6. If

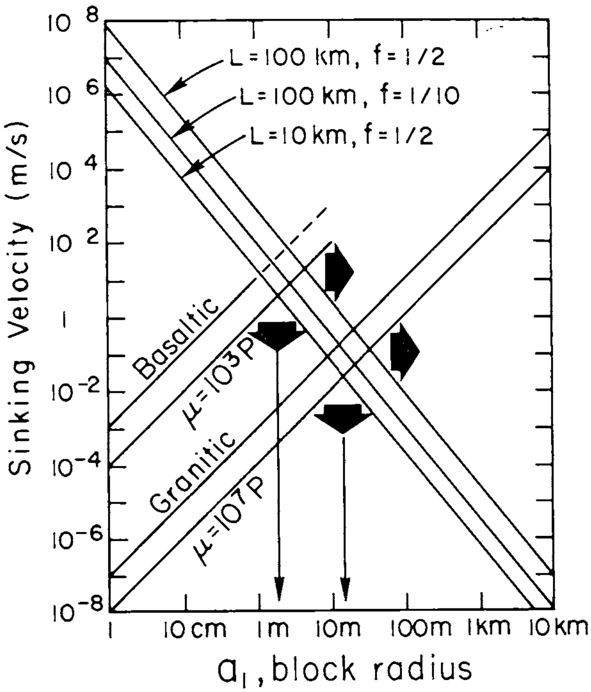


Fig. 6. The sinking velocity (residence time) and minimum radius that spherical stoped blocks of concentration f must take on if the host magma is to be thermally contaminated by no more than 10 percent in ascending a distance L (lines sloping from left to right). The lines of positive slope give the sinking velocity as regulated by the viscosity of the magma and the concentration of blocks; the two lines for each magma type indicate the uncertainty in this calculation of sinking velocity. To prevent significant contamination the stoped blocks must be larger than about 2 m (radius) in a basalt and about 20 m in a granitic magma.

the block can settle faster than that indicated by (41), the magma will not become seriously contaminated. The actual settling velocity of a block can, for all but the largest, be calculated from Stokes's law (assuming $\Delta\rho \cong 50$ kg/m^3), and this velocity is also shown by figure 6 for a magma viscosity of 10^3p (basalt) and 10^7p (granite). The uncertainty shown includes the increase in drag (decreased velocity) due to the presence of other blocks (f = 1/2) as estimated from the results of figure 2B. Blocks with radii larger than a few meters in a basaltic magma and larger than a few tens of meters in a granitic magma settle fast enough to avoid serious thermal contamination.

In a companion study now in preparation for publication, a more general result has been found showing that each of the cooling curves presented by Marsh and Kantha (1978, fig. 3) can be produced by stoping blocks of a radius

$$a_1 = \left[1.91 \left(\frac{9}{2} \frac{K\mu}{g\Delta\rho} \right)^{2/3} \frac{L}{A} \right]^{1/3} \tag{41A}$$

where μ is the magma's viscosity (block-free), g is gravity, $\Delta\rho$ is the density contrast between block and magma, A is the value of Jt_0 from those cooling curves, and the other symbols are as before. For A = 1, L = 100 km, $\Delta\rho = 50$ kg/m^3, and $\mu = 10^3$ poise (basalt), the size of the block must be about 2 m to cause the magma to cool along this curve ($Jt_0 = 1$). For a granitic magma with $\mu = 10^7$ p, the size of the block must be about 20 m.

A surprising feature of (41A) is that the size of the block does not depend on the concentration of blocks in the magma, as long as they can fall freely and do not choke the body. Briefly, this is so because the increase in cooling with the increase in concentration of blocks is exactly offset by the increase in velocity of the magma past the blocks with increasing fraction of blocks. Overall the block size is similar to that given by figure 6.

Because typical chemical diffusivities are a factor of one thousand or more smaller than the thermal diffusivity, chemical contamination is much less severe. Blocks can be almost a factor of ten smaller. Because of this difference, it seems clear that *if a magma arrives at the surface thermally uncontaminated, it is probably chemically uncontaminated.* The chemical contamination commonly detected through isotopic measurements, especially Nd, therefore also implies thermal contamination.

Thermal contamination can be prevented if the stoped blocks are preheated by the approaching magma; namely, stoping occurs within the thermal halo about the body. The heat of the thermal halo has indeed already been given up by the magma, but the rate of heat transfer is much less than if cold blocks continually pass through the magma. And at great depth where the wall-rock temperature is close to that of the magma itself, thermal contamination is minimized. Similarly, stoping of wall rock compositionally identical to the magma will produce no chemical contamination. Also if magma is chilled against the stoped blocks it will insulate the magma against both types of contamination.

Marginal shattering.—The above calculations say nothing about the actual rate of ascent, but only how large the stoped blocks must be to prevent contamination. The rate of rise rests with the magma's ability to break blocks from the roof. Daly called this process marginal shattering. And he suggested that it is caused by thermal stresses developed in the wall rock due to sudden and uneven heating by the advancing magma. The magnitude of this stress, as estimated by Daly (1903), is about 10 kb (see correction, Daly, 1914, p. 201) for an anomalous temperature of 1000°C. Since this stress is well beyond the strength of rock (~ 1 kb), it is useful to investigate this effect.

Since the magma travels with a thermal halo about it, the thermal stresses can be approximated as those developed in a spherical shell of outer radius b containing the magma of radius a and having the magma mean temperature (T) at its contact with the magma (r = a) and the normal, undisturbed wall rock temperature (T_m) at its outer margin (r = b). If for simplicity the temperature distribution is taken to be symmetrical, in a spherical coordinate system (r, θ, ϕ) the only nonzero principal stresses are σ_{rr} and $\sigma_{\theta\theta} = \sigma_{\phi\phi}$. These stresses can be calculated from (Timoshenko and Goodier, 1970):

$$\sigma_{rr} = \frac{2E\alpha}{1-\nu}\left[\frac{1-(a/r)^3}{1-(a/b)^3}\frac{1}{b^3}\int_a^b T(r)r^2dr - \frac{1}{r^3}\int_a^b T(r)r^2dr\right] \quad (42)$$

$$\sigma_{\theta\theta} = \frac{E\alpha}{1-\nu}\left[\frac{2+(a/r)^3}{1-(a/b)^3}\frac{1}{b^3}\int_a^b T(r)r^2dr + \frac{1}{r^3}\int_a^b T(r)r^2dr - T(r)\right] \quad (43)$$

where E is Young's modulus, α is the coefficient of thermal expansion, and ν is Poisson's ratio.

Letting the variation of temperature be

$$T(r) = ((T - T_m)/(b/a - 1))\,(b/r - 1)\,, \quad (44)$$

the stress distribution from (42) and (43) becomes:

$$\sigma_{rr} = \left(\frac{E\alpha(T-T_m)}{1-\nu}\right)\frac{\eta}{1-\eta^3}\left[\eta + 1 - b/r(1+\eta+\eta^2) + \eta^2(b/r)^3\right] \quad (45)$$

$$\sigma_{\theta\theta} = \left(\frac{E\alpha(T-T_m)}{1-\nu}\right)\frac{\eta}{1-\eta^3}\left[\eta + 1 - \frac{b}{2r}(1+\eta+\eta^2) - \eta^2\left(\frac{b}{r}\right)^3\right] \quad (46)$$

where as before $\eta = a/b$, and the radial stress (45) has been made to be zero at $r = a,b$; this condition can be relaxed to satisfy ambient hydrostatic conditions or even nonhydrostatic pressure due to expansion of the whole body itself by employing the principle of superposition of stresses.

The tangential stress is negative (compressive) at the margin of the magma ($r = a$), and it becomes positive (tensional) at the outer border of the thermal aureole ($r = b$). The radial stress is compressive throughout the thermal aureole, and it has a maximum when $d\sigma_{rr}/dr = 0$, or where

$$\frac{r}{b} = \sqrt{3}\,\eta\,(\eta^2 + \eta + 1)^{-1/2}. \quad (47)$$

The difference in the tangential stress at the inner and outer borders of the thermal aureole is:

$$\sigma_{\theta\theta}{}^b - \sigma_{\theta\theta}{}^a = \frac{E\alpha(T-T_m)}{1-\nu} \quad (48)$$

And it is this coefficient that controls the magnitude of the thermal stress in any of the above equations. For a peridotitic wall rock, with $E \simeq 1.5 \times 10^{11} N/m^2$, $\alpha = 2 \times 10^{-5} deg^{-1}$, and $\nu = 0.25$ (Birch, 1966); (48) amounts to 40 bars/deg; this comes to 4 kb for each hundred degrees of temperature difference between the magma and its wall rock. (Very near the surface where the wall rock may contain free water, an equally large stress can be produced by heating this water.)

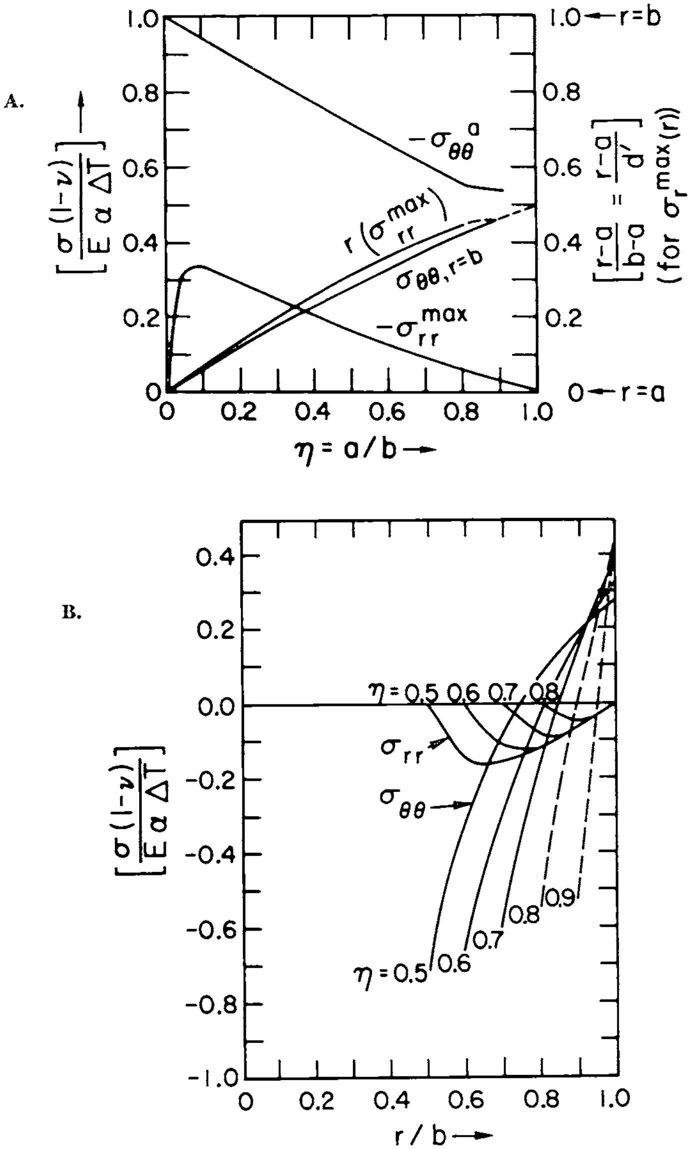


Fig. 7 (A) Principle stresses (dimensionless, left axis) in the tangential ($\sigma_{\theta\theta}$) and radial (σ_{rr}) directions about a heated spherical cavity of radius a in a boundless medium. The medium is heated symmetrically and diminishingly to a distance b beyond the cavity, and the stresses are given as a function of the thickness (η = a/b, lower axis) of this heated region. The uppermost curve ($-\sigma_{\theta\theta}{}^{a}$) is the negative of the stress at the inner contact with the magma (r = a). The lower two curves are the tangential stress at the outer limit (r = b) of the thermal aureole and the negative of the maximum radial stress. The remaining curve gives the position within the thermal aureole (right axis), where the radial stress is maximum as a function of the thickness of the aureole itself.

(B) The variation in principal stress (dimensionless, left axis) as a function of distance through the thermal aureole (lower axis). Each curve is for a constant aureole thickness. For the tangential stress, the stress difference between the inner and outer margins of the aureole is always one. The largest radial stress occurs with the thickest aureole.

The variation of tangential stress at r = a,b and also of the maximum radial stress is shown as a function of η in figure 7A; also shown is the variation of the position of the maximum in radial stress with changes in η. The actual spatial variation of stress as a function of r/b is shown as figure 7B. The earlier results on diapirism showed generally that for the slowest ascent (by any means) $\eta \cong 0.5$ and that generally $0.5 \leqslant \eta < 1$. With this in mind, it is clear that typical dimensionless tangential stresses (that is, $\sigma \equiv \sigma_{ii}\ (1-\nu)/E\alpha\Delta T$) at the margins of the aureole are: $\sigma_{\theta\theta}{}^{a} \cong -0.6$ and $\sigma_{\theta\theta}{}^{b} \cong 0.4$. This amounts to, respectively, a compressive stress of 2.4 kb/100°C nearest the magma and a tension of 1.6 kb/100°C at the outer edge of the thermal aureole. The maximum radial stress of $\sigma \cong 0.1$ amounts to 400 bars/100°C. And this maximum lies about 40 percent of the way across the thermal aureole (that is, $r - a \cong 0.4d'$). The deviatoric stress (that is, $(\sigma_{\theta\theta} - \sigma_{rr})/2$) is largest near the inner and outer margins of the aureole.

It is clear from these results that even for the most reasonable temperature gradients the thermal stresses will easily exceed the strength of the wall rock. The rock nearest the magma will reduce these stresses by deforming plastically (for example, Nadai, 1963, p. 404; Ashby and Verral, 1977). Since the yield strength is sensitive to temperature, it increases away from the magma whereas the thermal stress decreases until the neutral surface is encountered. Beyond the neutral surface the rock will yield under tension at stress levels lower than when under compression (for example, Jaeger, 1962). The central portion of the aureole may contain relatively undeformed rock. But since the thermal aureole advances with the magma, all the rock within the aureole will have been deformed or weakened. Since the tangential stress is large within the inner, plastically-deforming portion of the aureole, the slip surfaces may roughly resemble the stress pattern itself, perhaps resembling the well-known logarithmic spirals. And because unprocessed wall rock is not continuous around the bottom of the magma, large curved sheets of wall rock may slough into the magma and slide around the body, allowing advancement. Owing to its similarity to exfoliation during weathering, this process might be called *infoliation stoping.*

The rate of infoliation stoping determines the ascent rate, and the ascent rate governs the thermal aureole thickness, which governs the block size. Other than the meager size restrictions already placed by the limits of sinking velocity and thermal contamination, an estimation of the rate of stoping presently seems inaccessible. The large tangential stresses are themselves independent of the thermal aureole thickness; they depend only on the temperature difference before and after heating. The radial stress is a function of aureole thickness, but it is generally not large. For sensible values of η, it hardly affects the deviatoric stress.

Anderson's stresses.—Stresses are also set up about the magma because of any pressure difference between the magma and the prevailing hydrostatic pressure of the undeformed wall rock. The well known work of Anderson (1936) shows that, if the magmatic pressure exceeds the hydrostatic pressure, the probable system of fractures dip inward toward the spherical body of magma. But an inward dip prevents large blocks from dropping into the magma. If, on the other hand, the hydrostatic pressure is larger, the fractures dip outward if the body itself dips outward at less than 60° and the depth of the center of the body is less than twice the body radius (Roberts, 1970). The versatile, three dimensional solutions of Koide and Bhattacharji (1975) show the fracture pattern in an infinite medium to be considerably affected by the aspect ratio of the body itself. None of this work considers thermally-induced stresses.

During the late stages of crystallization of magma the significant decrease in volume may produce a pressure deficiency that would produce outward-dipping fractures and promote caldera collapse, stoping, or so-called bell jar intrusion (Roberts, 1970). But more generally when the magma is moving at depth either as a viscous diapir or by stoping, because of the magma's buoyancy, an overpressure occurs at the roof of the magma, and an underpressure occurs at the base of the body. The pressure is that of a dipole of equal and opposite sign. As was first shown by Lamé, the pressure about an internally-pressurized spherical vault decreases as r^{-3}, just as do the thermal stresses (Timoshenko and Goodier, 1970, p. 395). But the relative magnitude of the thermal stress to that of bouyancy is $(E\alpha\Delta T)/((1-\nu)2\Delta\rho ga)$, which for all sensible body sizes is $\geqslant 10$. The stress due to buoyancy is much less important than that from heating of the wall rock in promoting stoping, and it may be safely ignored.

Quarrying with fire.—Although the calculated thermal stresses are unusually large, just as Daly suggested, there is evidence suggesting that these results may in fact be realistic (Daly, 1903). Warth (1895) details an account whereby sheets of granite as large as 200 m^2 and almost exactly 130 mm thick ($\pm$ 10 mm) were routinely quarried in Bangalore, India by slowly moving ($\sim$ 2 m/hr) an increasingly-long line of fire across the rock surface. The generally heterogenous nature of this particular granite and the sometimes inclined beginning surface did not hinder quarrying perfectly regular sheets. To understand stoping, it is beneficial to understand this quarrying practice.

It is not difficult to imagine the thermal propagation of an already existent crack, but it is less clear how, time after time, a crack can be thermally started at precisely the same depth ($\cong$ 130 mm). If the fire heats an area of granite for about an hour before it has been moved too far to be useful, it is easy to calculate that the rock could surely be heated to a depth of about 100 to 150 mm (Carslaw and Jaeger, 1959, p. 101). This isolated heating causes local expansion of the granite producing horizontal compressive stresses, buckling, and upward expansion, producing a vertical, tensional stress. This vertical stress increases as the heated zone thickens, eventually exceeding the granite's tensile strength and thereby producing a horizontal crack, which can be propagated by moving the fire.

This explanation can be quantitatively modeled by finding the vertical stresses (σ_{yy}) developed within a plate of thickness a experiencing a spatially periodic and steady temperature variation along its upper and lower surfaces (y = ± a/b) (Den Hartog, 1936); the fire can be taken as a single half-wavelength. Along the mid-plane of this plate the vertical stress is zero, just as it is near the base of the thermal front, and so the quarry problem can be modeled as half the plate model. The relevant equation of this work (eq 3) has apparently been misprinted, the corrected form is:

$$\sigma_{yy} = \left(\frac{E\alpha T\nu \sin(wx)}{1-\nu}\right)$$

$$\left[1 + \frac{wy\sinh(wy)\sinh(wa) - \cosh(wy)(\sinh(wa/2) + (wa/2)\cosh(wa/2))}{wa/2 + \sinh(wa/2)\cosh(wa/2)}\right] \qquad (49)$$

where wa = $\pi a/l$, l is the length of the fire or heated area, and a is twice the thickness of the heated granite. (The equations for Den Hartog's coefficients C_1 and C_2 should also each be multiplied by $-\nu/(1-\nu)$.) This equation can be greatly simplified by noting that since $a/l \sim 1/10$, then $\sinh(wy) \cong wy$, $\sinh(wa/2) \cong wa/2$, and $\cosh(wy) \cong 1 \cong \cosh(wa/2)$. Taking d = y = a/2 and the maximum temperature variation (that is, sinwx = 1), (49) becomes:

$$\sigma_{yy}^{max} = \left(\frac{E\alpha T\nu}{1-\nu}\right)\frac{\pi^2}{4}\left(\frac{d}{l}\right)^2 \qquad (50)$$

As the thickness of the heated zone (d) increases, so does the tension. This is, however, for a constant temperature throughout the plate, whereas in the quarry the temperature diminishes by T degrees over the distance d into the granite. By differentiating (50) with respect to d and treating T as a variable, it is easily found that for this variation (assumed linear) in temperature the stress is diminished by half. This is sensible, because only half the original amount of granite has been heated.

For a granite the first term amounts to about 4.1 bars/deg; with $d/l \sim 1/10$ to 1/8 and the temperature of the fire of 500° to 700°C the stress would be 20 to 50 bars. This is similar to the tensile strength of granite (Jaeger, 1962, p. 75).

This agreement may be somewhat fortuitous, because the tension is greatest near the surface while the rock actually breaks at some depth. Without a more exact knowledge of the actual quarry practice, however, a more detailed analysis is difficult.

It is clear from this natural example that, as also in the spherical shell model, the normal stress depends on the thickness of the thermal aureole. Because the flat granite was only heated locally, upward expansion may lift neighboring cool rock and fracture it parallel to the surface, and lateral compression may indicate buckling and fracturing. But because the thermal shell around the magma is spherical and its temperature field axisymmetric, the radial stress is always compressive. If stoped blocks cause irregular heating of wall rock, it too may similarly fracture. Although as the thermal aureole advances, the wall rock will first be fractured by tension along radials. And as this rock approaches the magma it will be compressed by the large stresses nearest the body. It is still not clear how the size of stoped blocks may depend on the mechanics of thermal shattering.

Block choking.—Because of their angularity and poor chance of repacking perfectly, stoped blocks collecting at the floor of the magma will occupy more than their original volume. This excess volume is a porosity that will be filled with magma. As the magma proceeds, its volume will diminish in direct proportion to this porosity. In the cavities formed by nuclear explosions it can be as large as 25 percent (for example, Coates, 1970).

The change in volume (V′) of magma free of blocks with ascent distance (Z) is proportional to the negative product of the volume of magma and some function (f(p)) of the porosity (p)

$$\frac{dV'}{dZ} = -f(p)V' \tag{50A}$$

That is, the volume of free magma decreases in proportion to its own volume and some function of the porosity. Integrating this equation at constant p from an initial volume V'_0 at $Z = Z_0$ to V′ at Z gives, upon some rearrangement

$$\frac{V'}{V'_0} = e^{-f(p)Z} \tag{50B}$$

Now to find f(p), we notice that after the body has ascended one body height, corresponding to $Z = Z_1$, the volume becomes $V'/V'_0 = 1 - 2p$. Using (50B) shows that $f(p) = -(\ln(1 - 2p))/Z_1$, and substituting this into (50B) we find, after some manipulation, that

$$\frac{V'}{V'_0} = (1 - 2p)^{Z/Z_1} \tag{50C}$$

For the tightest of packing of spheres $p \cong 0.25$, for example, the volume of magma free of blocks diminishes to about 6 percent of its original volume (V'_0) after ascending four body heights. The actual ascent distance is of course much less than four original body heights. If $p \cong 0.5$, which is about that for the poorest of packing of spheres, the body can ascend only a single body height. This process apparently severely restricts the distance to which a stoping magma can ascend from its source.

ZONE MELTING

In zone melting, magma moves by melting at its roof and solidification at its base. This general idea was apparently introduced by Pfann in 1952 (for example, 1959, 1962) for extreme refinement of silicon and germanium in the production of semi-conductors. In industry, the energy for melting is supplied externally, whereas in the migration of magma, heat must be supplied by the magma itself. Because the magma takes on the composition of the wall rock it is presently passing through, it is clear that this process is of limited importance in transporting island arc magma. If it were important, magma rising through continental crust would invariably erupt as rhyolite and in oceanic areas as tholeiite. It may, nevertheless, play a role in the production or accumulation of magma (Harris, 1957), and so for completeness this process is also considered.

Shimazu (for example, 1961) numerically investigated this process of magma transfer. He assumed a single phase solid with a definite melting point, and the initial body as an infinite sheet of a given thickness. A prescribed velocity of convection within the body promotes solidification at the base and fusion at the roof. Since the initial liquid mass has no permanent identity of its own, only its heat is followed. The mean temperature of a definite mass of magma is not monitored, but only the change in that mass having the given solidus temperature. In essence, the magma temperature is held constant, and its mass is varied to satisfy conservation of energy. If the sheet of magma has initial thickness h, any column of unit cross-sectional area contains energy totaling:

$$\rho_2 Cp_2 T_2 h = E \tag{51}$$

And with the temperature essentially held constant while h changes in response to loss of energy,

$$\rho_2 Cp_2 T_2 \frac{dh}{dt} = \frac{dE}{dt} = q_1 - q_3 , \tag{52}$$

where q_1 is the heat lost through the roof, and q_3 is the heat gained through the base by conduction. If melting and crystallization both involve the same changes in enthalpy (H) then these processes do not affect the size of the body. The body moves upward due to the thermal gradient and because of internal convection promoted by cooling at the top. The small loss of potential energy is dissipated by convection within the magma itself. Because the approaching roof-rock must be heated from the local geotherm temperature to its melting temperature while upon solidification at the floor this heat cannot normally be recovered, there is a continual loss of heat from the system. This heat loss is described by eq (20), without the gravitational term.

If the roof moves with velocity V_1 and the floor with V_2, $dh/dt = V_1 - V_2$, the rate of migration (V_1) of the upper face is determined by the balance between the heat lost upward to conduction (q_1), to heating the approaching rock ($\rho_1 Cp_1 \Delta T_1$; this important term seems to have been overlooked by Shimazu), and to fusion ($\rho_1 H_1$) and that supplied by magmatic convection ($\rho_2 Cp_2 T_2 W$) and conduction along the magma's adiabat (q_2):

$$q_1 + \rho_1 Cp_1 V_1 (\Delta T_1 + H_1/Cp_1) = \rho_2 Cp_2 T_2 W + q_2 \tag{53}$$

And similarly for the lower face:

$$q_2 + \rho_2 Cp_2 T_2 W = \rho_3 Cp_3 V_2 (\Delta T_2 + H_2/Cp_3) + q_3 , \tag{54}$$

where the indices 1, 2, and 3 refer, respectively, to the approaching roof rock, the magma, and the trailing rock. W is the internal convective velocity of the magma, and other symbols are as before. This internal convection transfers the heat released by freezing at the lower boundary to the upper boundary. When $W = 0$ the problem reduces to the classical Neumann problem (double-sided) of heat conduction as described by Carslaw and Jaeger (1959, p. 283).

For the upper face, the contribution of q_2 is much smaller than that of the others, and it may be dropped. Similarly for the lower face, q_2 and q_3 are small, and they tend to balance each other; the heat gained by cooling below the solidus (ΔT_2) is also small. With these approximations, the ascent velocities are found to be:

$$V_1 = \frac{W - q_1/\rho C p T_2}{\dfrac{\Delta T_1}{T_2} + \dfrac{H_1}{T_2 C p}} \tag{55}$$

$$V_2 = \frac{W T_2 C p}{H_2} \tag{56}$$

where $Cp_1 \cong Cp_2$ and $\rho_2 \cong \rho_1$ are assumed.

If the magma does not convect ($W = 0$), the upper face descends with a velocity essentially determined by the heat lost upward to diffusion. The lower face will not move at all; actually it may rise slowly because of the heat lost in maintaining the magma's adiabat (q_2), a term we have already discarded.

When there is internal convection ($W > 0$), and, for example when the thermodynamic quantities are similar to those of diopside (that is, $H \cong 0.358$ J/kg, $Cp \cong 1.17 \times 10^{-3}$ J/kg–deg) and the magma temperature $T_2 \cong 1250°C$, the lower face ascends with a velocity of order 4W. Provided that $H_1 \cong H_2$, the upper face will always rise more slowly than the lower face. This is primarily because a good deal of the heat supplied to this interface goes to heating the approaching wall rock to its solidus (that is, ΔT_1), especially so near the Earth's surface where ΔT_1 is large. It is also clear from (55) that the last quantity in the numerator will generally be very small ($\sim 10^{-11}$ m/s) relative to any sensible value of W. Neglecting this quantity reduces (55) to:

$$V_1 = \frac{W}{\dfrac{\Delta T_1}{T_2} + \dfrac{H_1}{T_2 C p}} \tag{57}$$

Here it is clear that at great depth where the rock is already near its solidus $\Delta T_1/T_2 << 1$, and if $H_1 \cong H_2$, then $V_1 \cong V_2$. With approach to the Earth's surface however, $\Delta T_1/T_2 \cong 1$, and the velocity slows by a factor of about five (that is, $V_1 \cong 4W/5$). Because of this strong variation in $\Delta T_1/T_2$, the actual ascent velocity of the upper face varies considerably with proximity to the Earth's surface. This detailed variation can be found by employing the equations describing T_s and T_m (that is, $\Delta T_1 =$

$T_s - T_m$) found near eq (21), but suffice it here to show only roughly how far the initial body can ascend. Since the mean temperature in the upper, say, 150 km of the Earth is no less than about 800°C, $\Delta T_1/T_2 \cong 3/4$ and $V_1 \cong W$. Then,

$$\frac{dh}{dt} = -3W \qquad (58)$$

and integrating from the initial body thickness h_0 to zero, its final thickness, over time t:

$$t \cong h_0/3W \qquad (59)$$

The thicker the initial body, the longer it lasts. For a typical body of thickness 10 and 50 km, convecting at the rate of 10 mm/y as assumed by Shimazu, the respective life times are about 300,000 yrs and 1.5 m.y. These times are much shorter than the times of, respectively, about 10 and 20 m.y. found by Shimazu. This difference is due to his omission of the ΔT_1-term in (53) which represents a large investment of energy to heat the approaching wall rock to its solidus.

The distance (L) the upper face travels before complete solidification is found from $L = V_1t$. For these life-times and the given rate of convection, L = 3 and 15 km. These are minimum estimates, because of the mean value assumed for ΔT_1. Overall, the distance traveled is essentially that described by (20) and as shown by figure 3.

Although these rough balances reveal the important features of this process, to be rigorous, the velocity of convection must be determined by the rate of heat transfer from the magma through the upper face as described by (30). And to approach reality, a laterally bounded body and a rock possessing a solidus and liquidus must be used. Since the mantle liquidus is believed to be about 1700°C (1 atm), it is doubtful that it could ever be exceeded, and thus zone melting reduces to a form of stoping and diapirism. For the migration of a laterally bounded granitic body in granitic crust, Ahren, Turcotte, and Oxburgh (1979) give results that agree with these.

DISCUSSION

General.—Since a characteristic body size (for example, radius) appears often in these formulations, it is of interest to understand theoretically what controls this parameter. Generally speaking, magma rises, because it is less dense than its surroundings. During this process of gravitational instability the magma is concentrated into a diapir that may continue to rise by diapirism or by stoping. The radius (a) of this initial diapir is related to the thickness (h_2) and viscosity (μ_2) of its source and the viscosity of the surrounding medium (μ_1) through the relation (Marsh, 1979b):

$$a = \frac{h_2}{2}\left[\frac{\mu_1}{\mu_2}\right]^{1/4} \qquad (60)$$

The conditions necessary for this equation to apply are rather general; the source can be of an irregular thickness and only locally continuous.

Since generally $\mu_1 >> \mu_2$, the body radius is apt to be much larger than the initial thickness of its source layer.

Considered alone (60) can not explicitly yield the body size unless a great deal more information is known about the source region. A more general investigation of magmatic instability beneath island arcs (Marsh, 1979b) has produced a set of five coupled equations that under certain circumstances can be inverted to yield explicit estimates of body size. These results imply an initial diapir radius of around 3-5 km, and these estimates roughly tally with pluton sizes found within batholiths (Gastil, Phillips, and Allison, 1975). These sizes imply volumes of about 100 to 400 km^3, and for the present discussion we will take these dimensions as typical of those of magmatic bodies beneath island arcs. Unless otherwise stated it is the initial ascension that is of interest in this discussion.

Diapirism.—The earlier kinematic heat transfer studies (Marsh, 1978; Marsh and Kantha, 1978) showed that to forstall solidification at depth the ascent velocity of a spherical body must be, depending on its size, about 10^{-5} cm/s ($a \cong 1$ km) to 10^{-7} m/s ($a \cong 6$ km). It was not known then, however, whether or not these velocities could actually be achieved. The drag models developed herein all give basically similar results (fig. 2). The drag depends critically on the viscosity of the wall rock *at* its contact with the magma. And within about a factor of ten it is this viscosity (that is, $\mu^* = A\mu_1$) that controls the ascent velocity. This viscosity can be made arbitrarily small through melting, but since the magma temperature is limited so is the reduction of viscosity.

The viscosity of peridotite just above its solidus is probably near that of the low-velocity-zone at the base of the lithosphere (that is, $\cong 4 \times 10^{20}$p; Cathles, 1975). If this is also the prevailing viscosity when the body is well within the lithosphere, the ascent velocity is unlikely to be larger than about 5×10^{-11} m/s ($\cong 0.15$ cm/yr). Ascent through the lithosphere, a distance of about 100 km, would take the unreasonably long time of about 70 m.y.

The maximum ascent velocity is determined by the extent of melting of the wall rock. Eggler (1977) has shown that in the presence of very small amounts of carbon dioxide and water the solidus of peridotite is buffered at about 1100°C to pressures in excess of 40 kb. The earlier thermal studies show that arc magma is certainly beyond this temperature prior to solidification, and so there will be some melting of the wall rock throughout the ascent. The extent of melting, however, is only a few percent (v.), until the dry solidus is exceeded (D. Eggler, personal commun., 1980). Beyond the dry solidus and depending on the exact rock composition, substantial quantities of melt ($\sim$ 25 percent/50°C) can be produced with relatively small changes in temperature (Mysen and Kushiro, 1977; Scarfe, Mysen, and Rai, 1979; Harrison, 1979). To lower the viscosity substantially from the low-velocity-zone value, then, the magma temperature probably must exceed the dry solidus of peridotite. The earlier thermal models showed this to be possible only in the upper half of the lithosphere (Marsh and Kantha, 1978, fig. 3). In this region the second body of magma — the first one has frozen by this point — can possibly move at a velocity of about 5×10^{-9} m/s ($\cong$ 15 cm/yr); a more exact value awaits knowledge of the viscosity near the dry solidus.

The total time taken to transfer magma by multiple diapirism through the lithosphere is the sum of the ascent times of all the bodies involved in forcing a path to the surface. That is, if the i^{th} body traverses a distance X_i of unprocessed lithosphere at a velocity V_i, the total ascent time (t) is:

$$t = \sum_i X_i/V_i \tag{61}$$

Since this is a harmonic sum in velocity, the smallest velocity over the largest distance contributes most to the ascent time. If two bodies, for example, each travel half the thickness of the lithosphere (that is, $X_1 = X_2 = L/2$) at velocities of, respectively, 5×10^{-11} and 5×10^{-9} m/s, the total ascent time is about 30 m.y.; where $L \cong 100$ km. That this is also unreasonably long time can be appreciated by noticing that upon formation of a new island arc (for example, Scotia), once the subducting plate gets to a depth of about 100 km magma appears at the surface within about 1 m.y. (Subduction at Scotia occurred at about 58.7 mm/yr for about 3.5 m.y. before the arc appeared, which is now about 4 m.y. old, and the plate was at a depth of about 100 km after about 2.5 m.y. (see references in Marsh, 1979b).) This represents a mean velocity of about 100 mm/yr ($\cong 3 \times 10^{-7}$ cm/s). In light of (61), the principal ascent velocity must also be of this order. If so, the effective viscosity (from fig. 2) must be on the order of 10^{17} for the entire ascent through the lithosphere; namely, at temperatures perhaps even below the dry solidus. This is a necessary condition for magma transfer by diapirism.

The actual ascent distance is limited by the amount of energy held by the magma (that is, the size of the body), for the entire column of roof rock must be heated to its solidus. At great depth this is relatively easy, but nearer to the surface this investment of energy is substantial. For the sizes mentioned already and beginning at a depth of, say, 120 km, a spherical body may ascend to within 60 km of the surface before solidification. A second identical body coursing the same path within about 100,000 yrs of the first has the advantage of a pre-heated passageway, and it can ascend to within about 20 km of the surface. At these shallow depths the magma may propagate dikes to the nearby surface and obey the model of Fedotov (1981). Thus at least two bodies of magma coursing the same path are necessary to transfer magma diapirically through the lithosphere. But this number depends on the size and shape of the bodies.

The volume of roof rock that must be heated can be made smaller if the shape of the body becomes that of a prolate spheroid, but there is no indication that this shape is dynamically favorable. The body, on the contrary, tends to become spherical or even slightly oblate.

For petrology it is important to know the mean magma temperature at all times during ascent. The earlier calculations on this account

(Marsh, 1978) generally assume that the Peclét number is large ($> \cong 10$), and thus in (26) $C_1 = 0$, $C_2 = 0.46$, and $n = 1/2$. The present results imply, however, that Pe may never be greater than about 5, and the corresponding Nusselt number is probably always in the range of about 1.5 to 2.0. Since Nu varies slowly with Pe in this range (fig. 4), the mean temperature is insensitive to the ascent velocity, and temperatures during ascent can be calculated with considerably more accuracy than previously believed. The equations given by Marsh and Kantha (1978) are directly applicable with the value of Jt_0 as given here by eq (28).

There still remains, however, an uncertainty in the relationship between Nu and Pe. The general form of this relation is as given by (26), but the value of the exponent n, as is shown by (26A and nearby), is determined by the shape of the velocity profile nearest the body. In constant viscosity flows n is either a half (liquid-liquid) or a third (liquid-solid), but here the variation of viscosity in the wall rock determines the velocity profile, and n may not be a half as generally assumed. This problem is considered by Morris (ms).

Non-Newtonian models.—Although these calculations of drag all assume a Newtonian viscous fluid, it is possible that during diapirism peridotitic wall rock behaves as a non-Newtonian substance. It might behave as a power-law fluid (Weertmen, 1978) or perhaps as a plastic material. In power-law fluids the shear stress (τ) is related to the strain rate ($\dot{e}$) as $\tau = m\dot{e}^n$; where $m\dot{e}^{n-1}$ is an effective viscosity and n is some fraction of order one third for the mantle. Using this relation in (2) a drag model can be constructed without difficulty using the shear flow analogy. Since the effective viscosity is generally significantly less than the Newtonian viscosity (Post and Griggs, 1973; Yokokura and Saito, 1978), at first appearance the drag seems significantly less for a power-law fluid. But the velocity gradients are much larger than for a Newtonian fluid, and this increases the drag. Together these effects tend to cancel, but early results show that the drag may be slightly reduced, by about 25 percent.

This can be seen by recalling that the strain rate ($\dot{e}$) within the softened zone about the body is given approximately by $\dot{e} \cong V_o/d$, where V_o is the fluid velocity in this zone of thickness d. This velocity relates through continuity to the ascent velocity V as $V_0 \cong aV/2d$, where a is the radius of the body (see near 8). The thickness d is a fraction ($f \cong 0.1$) of the thermal aureole d' (see near 10), and $d' \cong 2aPe^{-1/2}$, so $d = fd' \cong 2afPe^{-1/2}$. Finally, $V \cong f(8K\dot{e})^{1/2}$. Since the nonhydrostatic pressure is well known ($2\Delta\rho ga \cong 560$ bars), the strain rate can be estimated from the deformation maps for olivine given by Ashby and Verall (1977). Assuming a homologous temperature of, say, 0.8 implies from their figure 15 that the deformation will be as a power-law fluid where $\dot{e} \cong 10^{-9}\ s^{-1}$, which gives an ascent velocity of about 10^{-9} m/s. This is similar to that suggested by figure 2, and it emphasizes the strong effect of the thinness of the softened zone about the body. Here too the ascent velocity can be significantly increased only if the temperature of the wall rock can be greatly increased.

Plastic behavior implies an unyielding wall rock until some stress threshold is exceeded whereupon failure produces a volume of blocks bounded by slip surfaces. Movement takes place essentially along thin shear zones between blocks. For the simplest plastic material (that is, perfectly plastic) squeezed between opposing plates, as might take place at the head of a diapir, it is found (Jaeger, 1962, p. 148) that the shear stress is independent of the velocity of movement of the plates. This is a general feature of plastic flow solutions, in striking contrast to viscous flow where the shear stress is always proportional to the velocity. It is this feature of plasticity that makes it of interest in diapirism. But no definite results have yet been obtained.

Stoping.—Although it is commonly believed that stoping quickly contaminates magma thermally and chemically, the results shown by figure 6 suggest this is not necessarily true. Only if the average stoped block is less than about 3 m (in basalt) and 30 m (in granite) in radius is contamination significant. The mechanical means of weakening and fracturing the roof rock is certainly available through the thermal stresses accompanying intrusion; this has previously been noticed by McBirney (1959). And although the roof rock must be heated somewhat to cause fracturing, stresses ample for fracturing can be developed with only moderate changes in temperature ($\cong$ 100°C). Since the whole column of roof rock perhaps need not be heated much, the energy requirements may be much less severe than for diapirism. But this is not clear, because the ascent velocity of stoping is unknown. It has not been possible to limit greatly the rate of ascent on any reasonable grounds. If the minimum of sizes of blocks of contaminating rocks are used as estimates of the thickness of the thermal halo, which is dependent on the ascent velocity, then Pe $< \cong 1000$. For a body with a radius of 3 km, this limits the ascent velocity to less than 3×10^{-7} m/s. The rate of advance is determined by the rate of removal of blocks from the roof, and this stands as a major unsolved problem. But the ascent distance is probably limited to about several body heights, because stoped blocks rapidly congest the magma.

Field evidence of widespread stoping (either piecemeal or wholesale) remains as scanty as it was in Daly's time. Yet there are many volcanic plugs such as those of the Colorado plateau described by McBirney (1959) that have clearly stoped, in some form or another, their way to the surface. Blocks within the intrusion are commonly rare. The down-fallen blocks associated with caldera collapse certainly represent near surface stoping (Pitcher, 1978, 1979), but it is not clear if stoping on this scale could take place at depth away from Earth's surface. The commonly quoted results of Anderson (1936) only produce stress patterns favorable for stoping, when the magma is near the surface and when it is undergoing collapse, during venting and solidification. Thermal stresses seem to be generally far more important than those due to inflation and deflation.

GENERAL COMMENT ON MAGMA TRANSPORT BY DIAPIRS AND DIKE PROPAGATION

Diapirism through the lithosphere is a slow process, critically dependent on the diffusion of heat. It works best in unusually hot, viscous wall rock. Dike propagation is a much more rapid process, working best in elastic rocks. Diapirism concentrates buoyant mass into bodies with small ratios of surface area to volume. Dikes essentially disperse magma in bodies having unusually large ratios of surface area to volume. There are several geological manifestations of these general properties.

Dike propagation should commonly produce earthquakes, whereas diapirs probably should not. Should dikes be important in carrying magma in island arcs from the Benioff zone to the surface, a column of earthquakes should be observed. Although there are earthquakes near the volcanoes, in the uppermost 20 km, they are generally absent over the remainder of this region. And since this is apparently so in many island arcs, regardless of age, this absence is not an evolutionary feature.

The exact direction of dike propagation is less dependent on buoyancy than it is on the orientation of the regional principal stresses. Thus it is possible that all dikes emanating from a common source may have a variance of, say, $\pm$ 20° to the vertical. This variance, if continued from the Benioff zone to the surface, would scatter volcanism over an area of about 90 km in diameter. Diapirism, on the other hand, follows the vertical, and later diapirs are apt to follow the trails of earlier ones. Volcanism in island arcs is characteristically concentrated. Once formed, many volcanic centers in the Aleutian Islands have remained fixed over at least several tens of millions of years, with volcanism at any time commonly within a region of diameter of about 10 km.

Since dike propagation is able to transfer magma at speeds much larger than that of diapirism, fragments of wall rock that fall into the magma can more easily be brought to the surface by dike propagation. The slow process of diapirism, on the other hand, allows sufficient time for heavy pieces of peridotitic wall rock to settle from the magma. Of all the basaltic lava types, island arc lavas contain the fewest xenoliths of mantle rock. They are rare or "very rare" (A. R. McBirney, personal commun., 1980).

Lastly, if dike propagation is important in carrying island arc magma, as a rule regional fissure-type eruptions might be expected. Diapirs would favor central eruptions. Central eruptions are generally the rule.

In sum, experience suggests that the magma of island arc volcanism is transported by a diapiric process. There is no indication whatsoever that dike propagation is at all important except possibly near the surface. In non-island arc areas, like Hawaii, columns of earthquakes are observed traversing the lithosphere, fissure eruptions are common, and ultramafic xenoliths are not rare. These are the characteristics of magma transport by dike propagation that are absent in island arcs.

CONCLUSIONS

If magma is to ascend through the lithosphere as diapirs, the viscosity of the wall rock in contact with the magma must be of the order of 10^{16} to 10^{17} poise. Under these conditions a typical velocity of ascent is about 150 mm/yr. Unless the diapirs are unusually large (radius $> \cong$ 10 km), the first diapir will solidify at a depth of about 50 km, and a second diapir coursing the same path within a few hundred thousand years may approach within about 20 km of the surface. Repeated ascent of the same path is a necessary condition for successful diapirism. If later diapirs are about half the size of the initial bodies, they can, with all else equal, ascend about twenty-five times faster (that is, $\cong$ 4 m/yr).

If basaltic magma stopes its way through the lithosphere, the radius of the stoped blocks must be greater than about 3 m to prevent significant thermal and chemical contamination of the magma. The thermal stresses developed about the magma are sufficiently large ($\sim$ 4 kb/100°C) to induce marginal shattering. But it has not been possible to place useful bounds on the actual velocity of stoping. Because stoped blocks must occupy significantly more space than their original volume, they congest the body and severely limit the ascent distance.

ACKNOWLEDGMENTS

The essential concepts of this work have been considerably sharpened through discussions with Stephen Morris and S. H. Davis. A more analytical analysis of the diapir problem is being prepared by Morris as a dissertation. Parts of this study have been presented in lectures at a number of institutions: Pennsylvania State University, Department of Terrestrial Magnetism of Carnegie Institute, California Institute Technology, Georgia, Michigan State University, Massachusetts Institute Technology, V.P.I., Rutgers, Princeton, Northwestern, Chicago, and Utah. Critical reviews and discussions by S. Morris, A. R. McBirney, W. M. Elsasser, and S. A. Fedotov are appreciated. This work is supported by National Science Foundation grant EAR-8005109 to the Johns Hopkins University, and additional publication costs have also been supplied by the National Science Foundation.

REFERENCES

Acrivos, A. and Goddard, J. D., 1965, Asymptotic expansions for laminar forced-convection heat and mass transfer: Part 1. Low speed flows: Jour. Fluid Mechanics, v. 23, p. 273-291.

Acrivos, A. and Taylor, T. D., 1926, Heat and mass transfer from single spheres in Stokes flow: Physics of Fluids, v. 5, p. 387-394.

Ahren, J. L., Turcotte, D. L., and Oxburgh, 1979, On the upward migration of batholiths during solidification [abs.]: Am. Geophys. Union Trans., v. 60, p. 411.

Anderson, E. M., 1936, The dynamics of the formation of cone-sheets, ring-dykes and cauldron subsidences. Royal Soc. Edinburgh Proc., v. 56, p. 128-157.

Ashby, M. F. and Verall, R. A., 1977, Micromechanisms of flow and fracture, and their relevance to the rheology of the upper mantle: Royal Soc. London Philos. Trans. A., v. 288, p. 59-95.

Balk, R., 1937, Structural behavior of igneous rocks: Geol. Soc. America Mem. 5, 177 p.

Batchelor, G. K., 1967, An introduction to fluid dynamics: Cambridge, Cambridge Univ. Press, 615 p.

Birch, F., 1966, Compressibility; elastic constants, *in* Clark, S. P., Jr., ed., Handbook of physical constants: Geol. Soc. America Mem. 97, p. 97-174.

Buddington, A. F., 1959, Granite emplacement with special reference to North America: Geol. Soc. America Bull., v. 70, p. 671-747.

Carslaw, H. S., and Jaeger, J. C., 1959, Conduction of heat in solids, 2d ed.: Oxford, Clarendon Press, 510 p.

Cathles, L. M., III, 1975, The viscosity of the Earth's mantle: Princeton, N.J., Princeton Univ. Press, 386 p.

Coates, D. F., 1970, Rock mechanics principles: Ottawa, Information Canada, Mines Branch Mon. 874, 550 p.

Crank, J., 1956, The mathematics of diffusion: Oxford, Clarendon Press, 414 p.

Daly, R. A., 1903, The mechanics of igneous intrusion: Am. Jour. Sci., 4th ser., v. 16, p. 107-126.
——— 1914, Igneous rocks and their origin: New York, McGraw-Hill Book Co., 563 p.
——— 1933, Igneous rocks and the depths of the earth: New York, McGraw-Hill Book Co., 598 p.
Den Hartog, J. P., 1936, Temperature stresses in flat rectangular plates and in thin cylindrical tubes: Franklin Inst. Jour., v. 222, p. 149-181.
Eckert, E. R. G. and Drake, R. M., Jr., 1972, Analysis of heat and mass transfer: New York, McGraw-Hill Book Co., 806 p.
Eggler, D. H., 1977, The principle of the zone of invariant vapor composition: an example in the system CaO–MgO–SiO_2–CO_2–H_2O and implications for the mantle solidus: Carnegie Inst. Washington Year Book 76, p. 428-435.
Fedotov, S. A., 1981, Magma rates in feeding conduits of different volcanic centres: Jour. Volcanology and Geothermal Research, v. 9, 379-394.
Gastil, R. G., Phillips, R. P., and Allison, E. C., 1975, Reconnaissance geology of the State of Baja, California: Geol. Soc. America Mem. 140, p. 27-42.
Goodchild, J. G., 1892, Note on a granite junction in the Ross of Mull: Geol. Mag., v. 9, p. 447-451.
——— 1894, Augen-structure in relation to the origin of the eruptive rocks and gneiss: Geol. Mag., v. 1 (new ser.), p. 20-27.
Grout, F. F., 1932, Petrography and Petrology: New York, McGraw-Hill Book Co., 522 p.
——— 1945, Scale models of structures related to batholiths: Am. Jour. Sci., v. 243-A, Daly v., p. 260-284.
Haberman, W. L., and Sayre, R. M., 1958, Motion of rigid and fluid spheres in stationary and moving liquids inside cylindrical tubes: Washington, D.C., David Taylor Model Basin, Rept. 1143, 67 p.
Hadamard, J., 1911, Mouvement perament lent d'une sphere liquide et visqueuse dans un liquide visqueux: Acad. Sci. Paris, Comptes Rendu, v. 152, p. 1735-1738.
Happel, J. and Brenner, H., 1973, Low Reynolds number hydrodynamics: Lyden, Nordhoff Internat. Pub., 553 p.
Hardee, H. C., and Sullivan, W. N., 1974, An approximate solution for self-burial rates of radioactive waste containers: Albuquerque, N.M., Sandia Labs., Rept. SLA-73-0931, 19 p.
Harris, P. G., 1957, Zone refining and the origin of potassic basalts: Geochim. et Cosmochim. Acta, v. 12, p. 195-208.
Harrison, W. J., 1979, Partitioning of REE between garnet peridotite minerals and coexisting melts during partial melting: Carnegie Inst. Washington Year Book 78, p. 562-568.
Head, H. N. and Hellums, J. D., 1966, Heat transport and temperature distributions in large single drops at low Reynolds numbers: a new experimental technique: Am. Inst. Chem. Eng. Jour., v. 12, p. 553-559.
Ito, K., and Kennedy, G. C., 1967, Melting and phase relations in a natural peridotite to 40 Kilobars: Am. Jour. Sci., v. 265, p. 519-538.
Jaeger, J. C., 1962, Elasticity, fracture, and flow: London, Methuen and Co., 208 p.
Koide, H., and Bhattacharji, S., 1975, Formation of fractures around magmatic intrusions and their role in ore localization: Econ. Geology, v. 70, p. 781-799.
Kreith, F., 1973, Principles of heat transfer, 3d ed.: New York, Intext Educational Pub., 656 p.
Lawson, A. C., 1896, The eruptive sequence: Science, v. 3, p. 635-637.
Levich, V. G., 1962, Physicochemical hydrodynamics, Englewood Cliffs, N.J., Prentice-Hall Inc., 700 p.
Lighthill, M. J., 1953, Theoretical considerations on free convection in tubes: Quart. Jour. Mech. and Appl. Math., v. 6, p. 398-439.
Marsh, B. D., 1978, On the cooling of ascending andesitic magma: Royal Soc. London Philos. Trans. A, v. 288, p. 611-625.
——— 1979a, Island-arc volcanism: Am. Scientist, v. 67, p. 161-172.
——— 1979b, Island-arc development: some observations, experiments, and speculations: Jour. Geology, v. 87, p. 687-713.
Marsh, B. D. and Kantha, L. H., 1978, On the heat and mass transfer from an ascending magma: Earth Planetary Sci. Letters, v. 39, p. 435-443.
McBirney, A. R., 1959, Factors governing emplacement of volcanic necks: Am. Jour. Sci., v. 257, p. 431-448.

Moore, J. G., 1963, Geology of the Mount Pinchot quadrangle southern Sierra Nevada, California: U.S. Geol. Survey Bull. 1130, 152 p.
Morris, S., ms, 1980, An asymptotic method for determining the transport of heat and matter by creeping flows with strongly variable viscosity; fluid dynamic problems motivated by island arc volcanism: Ph.D. dissert., Johns Hopkins Univ., 154 p.
Mysen, B. O. and Kushiro, I., 1977, Compositional variations of coexisting phases with degree of melting of peridotite in the upper mantle: Am. Mineralogist, v. 62, p. 843-865.
Nadai, A., 1963, Theory of flow and fracture of solids: New York, McGraw-Hill Book Co., v. II, 705 p.
O'Brian, V., 1963, Slow forced scalar transfer from falling drops: Phys. Fluids, v. 9, p. 1356-1358.
Ostrach, S., 1964, Laminar flows with body forces, *in* Moore, F. K., ed., theory of laminar flows: Princeton, N.J., Princeton Univ. Press, p. 528-718.
Perry, R. H. and Chilton, C. H., 1973, Chemical engineers' handbook: New York, McGraw-Hill Book Co., p. 3-113.
Pfann, W. G., 1962, Zone melting: Science, v. 135, p. 1101-1109.
——— 1959, Zone melting: New York, John Wiley & Sons, 236 p.
Pitcher, W. S., 1978, The anatomy of a batholith: Geol. Soc. London Jour., v. 135, p. 157-182.
——— 1979, The nature, ascent and emplacement of granite magmas: Geol. Soc. London Jour., v. 136, p. 627-662.
Post, R. L. and Griggs, D. T., 1973, The earth's mantle: evidence of non-Newtonian flow: Science, v. 181, p. 1242-1244.
Roberts, J. L., 1970, The intrusion of magma into brittle rocks, *in* Newall, G. and Rast, N., eds., Mechanics of igneous intrusion: London, Gallery Press, p. 287-338.
Rybczynski, W., 1911 (quoted in Lamb, H., 1945, Hydrodynamics: New York, Dover Pub., p. 600).
Scarfe, C. M., Mysen, B. O., and Rai, C. S., 1979, Invariant melting behavior of mantle material: partial melting of two lherzolite nodules: Carnegie Inst. Washington Year Book 78, p. 498-501.
Shaw, H. R., 1974, Diffusion of H_2O in granitic liquids: Part I, experimental data; Part II. Mass transfer in magma chambers, *in* Hoffmann, A. W., Giletti, B. J., Yoder, H. S., Jr., Yund, R. A., eds., Geochemical Transport and Kinetics: Carnegie Inst. Washington Pub. 634, p. 139-170.
——— 1980, Fracture mechanisms of magma transport from the mantle to the surface, *in* Hargraves, R. B., ed., Physics of magmatic processes: Princeton, N.J., Princeton Univ. Press, p. 201-264.
Shimazu, Y., 1959, A thermodynamic aspect of the earth's interior-physical interpretation of magmatic differentiation process: Nagoya Univ. Jour. Earth Sci., v. 7, p. 1-34.
Stokes, G. G., 1851, On the effect of the internal friction of fluids on the motion of pendulums: Cambridge Philos. Soc. Trans., v. 9, p. 8; Collected Works, v. 3, p. 59-60.
Timoshenko, S. P. and Goodier, J. N., 1970, Theory of elasticity, 3d ed.: New York, McGraw-Hill Book Co., 567 p.
Warth, H., 1894, The quarrying of granite in India: Nature, v. 51, p. 272.
Weertman, J., 1978, Creep laws for the mantle of the Earth: Royal Soc. London Philos. Trans. A., v. 288, p. 9-26.
Yokokura, T., and Saito, M., 1978, Viscosity of the upper mantle as non-Newtonian fluid: Jour. Physics Earth, v. 26, p. 147-166.