DISCUSSION

TRANSPORT PHENOMENA IN HYDROTHERMAL SYSTEMS: COOLING PLUTONS—THE PATHLINE CONCEPT IN APPLICATION TO THE STUDY OF TRANSPORT PROCESSES IN HYDROTHERMAL SYSTEMS

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Norton and Knight (1977) and Norton (1978) have recently introduced the concept of pathlines in their studies of hydrothermal systems associated with cooling igneous bodies. Pathlines are supposed to represent the trajectories or paths of fluid packets moving through permeable rock. As they have pointed out, this interesting concept has potentially important applications to understanding processes such as the transport and deposition of ore-forming constituents in hydrothermal systems. However, pathlines as they have been defined and calculated in the above model studies do not represent the pathlines of fluid packets. The purpose of this note is to clarify the definition of fluid pathlines.

Pathlines have been defined by these authors as the paths of points or particles that move through a permeable medium with the Darcy or filter velocity (see for example, Scheidegger, 1960; Bear, 1972) $\bar{u}$ which is a function of the spatial coordinates $\bar{x}$ and time t. Pathlines defined in this way are calculated by equating the time derivative of the position vector of the particle to the Darcy velocity

$$\frac{d\bar{x}}{dt} = \bar{u} \tag{1}$$

and by integrating this with respect to time

$$\bar{x} = \int_{t_0}^{t} \bar{u}\,(\bar{x},t)\,dt + \bar{x}_0 \tag{2}$$

where $\bar{x}_0$ is the position vector of the particle at some initial time t_0.

The Darcy velocity vector represents the volume flow rate per unit area of fluid moving through a small elemental surface oriented perpendicular to the velocity vector. The physical meaning of the Darcy velocity can be understood by supposing that the actual fluid velocity in numerous pores intersecting the elemental surface is redistributed uniformly over the entire area of the surface in such a way that the product of the Darcy velocity and the area of the surface represents the volume flow rate of fluid moving through the surface. Therefore, the Darcy velocity is generally less than the average velocity of the fluid moving through individual pores within the permeable medium. It is incorrect to equate the average velocity of fluid particles or of a packet of fluid particles with the Darcy velocity.

Fluid velocities vary with position over the cross section of each pore intersecting the elemental surface area described above. For a viscous fluid, the velocity of the fluid vanishes on the solid surfaces bounding a particular pore and reaches a maximum in the interior of the pore cross section. An average fluid velocity can be defined by averaging fluid velocities over the cross section of pores intersecting the elemental surface area. Such an average might be constructed in a variety of ways. One simple way is to divide the volume flow rate of fluid crossing a small elemental surface perpendicular to the Darcy velocity vector by the area of pores intersecting the surface. In a statistically homogeneous permeable medium, the fractional area of such an elemental surface occupied by pores is equal to the porosity of the medium. Therefore an average fluid velocity defined in this way is simply the Darcy velocity divided by the porosity, n,

$$\bar{v} = \bar{u}/n \tag{3}$$

For small values of the porosity, as seems applicable in cracked igneous rock, this average velocity of fluid particles is many times larger than the Darcy velocity. It remains to be shown that pathlines defined with velocity $\bar{v}$ represent paths along which fluid packets move through a permeable medium. To establish this point, it is useful to consider the equations describing the transport of some molecular constituent within a permeable medium.

The conservation of a molecular constituent within a permeable medium may be represented by equations describing its conservation in the fluid and in the solid matrix

$$\frac{\partial}{\partial t}(n \rho_f \phi_f) = F + \alpha_f \nabla^2 \phi_f - \nabla \cdot \rho_f \bar{u} \phi_f \tag{4}$$

$$\frac{\partial}{\partial t}\{(1-n) \rho_r \phi_r\} = -F + \alpha_r \nabla^2 \phi_r \tag{5}$$

where ϕ is the concentration per unit mass of the constituent, ρ is the density, α is an effective diffusivity, and F represents the flux of the constituent per unit volume of the medium transferred from the matrix to the fluid. The subscripts f and r refer to fluid and rock matrix respectively. Each equation equates the time rate of change of the mass of the constituent within an elemental volume of the medium with terms describing the transport of the constituent out of the volume and its exchange between fluid and rock. The terms representing diffusion are written for simplicity in a form that assumes constant effective diffusivities. The effective diffusivity in the rock, α_r, can be approximated as (1−n) times the true molecular diffusivity of the constituent in the rock. The effective diffusivity of the fluid phase is the sum of n times the molecular diffusivity in the fluid and the hydrodynamic dispersion (Bear, 1972; Saffman, 1959).

Considering the simple case of no fluid-rock interaction and assuming that the diffusivities are small gives

$$\frac{\partial}{\partial t}(n \rho_f \phi_f) + \nabla \cdot \rho_f \bar{u} \phi_f = 0 \tag{6}$$

This equation may be rewritten using the equation for conservation of fluid mass in an elemental volume of the permeable medium

$$\frac{\partial}{\partial t}(n\,\rho_f) + \nabla \cdot \rho_f\,\bar{u} = 0 \tag{7}$$

to give

$$\frac{\partial \phi_f}{\partial t} + \left(\frac{\bar{u}}{n}\right) \cdot \nabla \phi_f = 0 \tag{8}$$

This equation states that there is no instantaneous change of ϕ_f following a path defined by

$$\frac{d\bar{x}}{dt} = \frac{\bar{u}}{n} = \bar{v} \tag{9}$$

If a fluid packet in the permeable medium is labelled by some concentration of a tracer that does not interact chemically with the matrix and that does not undergo molecular diffusion in the fluid, then neglecting hydrodynamic dispersion, the fluid packet so identified moves with an average velocity $\bar{v}$ given by (9). A meaningful definition of fluid pathlines is then obtained by integrating (9) in a manner analogous to (1). These pathlines clearly represent the average trajectories of fluid packets or more concisely fluid pathlines. It should be noted that Bear (1972) defines pathlines in this same way.

A fluid packet moving through permeable rock will be dispersed by both molecular diffusion and by hydrodynamic dispersion. However, if these effects are small, pathlines are appropriate for following the movement of dissolved constituents, such as ore forming cations, as they are transported in a hydrothermal system without precipitating or reacting with rock between a source region and a region of deposition. As Norton and Knight (1977) and Norton (1978) have suggested, the pathline concept may provide significant insight into the formation of ore deposits. However, incorrectly interpreting pathlines to be trajectories of a particle that moves with the Darcy velocity may be very misleading. Consider, for example, the pathlines in figure 24 of Norton and Knight (1977). These pathlines indicate that over the entire cooling history of the intrusion fluid particles move relatively short distances, not more than a few times the size of the intrusion. Only a limited number of fluid packets initially contained within the permeable country rock can interact with high temperature rock in and around the intrusion. Those that do interact with high temperature rock do so only once.

As was shown above, fluid packets in low porosity rock in fact move at an average velocity many times greater than the Darcy velocity. If the porosity of the rock is 1 percent, fluid packets will move through distances on the order of 100 times the distances suggested by the Norton and Knight models. More fluid packets will therefore have the opportunity to interact with hot rock in and around the intrusion, and some will be re-

cycled through the rock many times before the intrusion has cooled. This clearly gives a very different picture of the mode of fluid transport and rock-water interaction than that proposed by Norton and Knight.

REFERENCES

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REPLY

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Parmentier presents equations that represent the maximum possible length of pathlines for chemical components in natural systems. His formulation explicitly describes fluid flow for *nonreactive* and *nondiffusing* components. This formulation will not represent the geological condition where fluid flow causes changes in rock composition.

Fluid pathlines, sourcelines, and sourceregions of "fluid packets" as presented by Norton and Knight (1977) and Norton (1978) are a first approximation to the analogous features for individual chemical components. The imprecision of using Darcy velocity as part of this approximation is obvious from Norton and Knapp (1977, eq 31).

The velocity ($\bar{v}_\epsilon$) of a concentration anomaly of the ϵ^{th} component through the matrix of a flow system is a fraction of the true velocity, $\bar{v}_{true}$, of the multicomponent fluid:

$$\bar{v}_\epsilon = \left\{ \frac{\Delta C_{\epsilon f}}{\phi_f \Delta C_{\epsilon f} + \phi_m C \Delta_{\epsilon m}} \right\} \phi_f \bar{v}_{true}, \text{ where}$$

ϕ_f and ϕ_m are the volume fractions of fluid in the flow channels and matrix blocks, respectively, $C_{\epsilon f}$ and $C_{\epsilon m}$ the concentration, moles cm^{-3} of the ϵ^{th} component in the flowing fluid and matrix blocks, respectively, and Δ is the finite difference operator. This relationship derived from the general transport equation (Norton and Taylor, 1979, eqs 1 and 8), also comprises the theories of chromatography and infiltration metasomatism (Wilson, 1940; DeVault, 1943; Korzhinskii, 1968; and Hofmann, 1972).

As can be seen in eq (1), the concentration front of the ϵ^{th} component moves through the rock at the true velocity *only* when $\Delta C_{\epsilon m} \equiv 0$, for example, when *no* transfer of the ϵ^{th} component occurs between the flowing fluid and the surrounding matrix blocks. Similarly, the concentration front moves at the Darcy velocity *only* for the condition $(1-\phi_f)$