[AMERICAN JOURNAL OF SCIENCE, VOL. 274, APRIL, 1974, P. 414-442]

THE DEVELOPMENT OF MEANDERS IN NATURAL RIVER-CHANNELS

EDWARD J. HICKIN

Department of Geography, Simon Fraser University,
Burnaby 2, B.C., Canada

ABSTRACT. Many laterally migrating channels develop flood-plain surfaces with ridge and swale arrays. The planimetric orientation of contemporary ridge and swale systems accords with the convex bankline of the present channel bends; the older arcuate ridge-swale pattern at the rear of the flood plain faithfully records the former courses of the river.

Microenvironments in the ridge and in the swale zones are sufficiently different to give rise to distinct bands of plant communities which are easily distinguished on aerial photographs.

Orthogonals (erosion pathlines) can be generated in the ridge-swale field of individual point-bar complexes to indicate the pattern of channel migration and meander development. The erosion pathlines also provide a basis for comparing downvalley and crossvalley migration rates. Indeed, along the erosion pathlines alone can the true migration rates be measured. Furthermore, the orthogonal field provides a definition of the erosional axis of a point-bar complex—a parameter based on process rather than on morphology.

Investigations of the meander growth-patterns of the Beatton River in northeast British Columbia indicate that there is a critical value of the ratio radius of channel curvature to channel width (r_m/w) which, once reached by a developing meander, exerts considerable control over subsequent direction and rate of lateral migration. Furthermore, the critical value of r_m/w, which is readily identifiable from the pattern of flood-plain surface deposits, appears to be much less variable in nature than is a similar parameter described by Leopold and Wolman in 1960. On the Beatton River the critical value of r_m/w averages 2.11, and standard deviation is only 0.13.

The pattern of meander growth is discussed in terms of the dynamics of open-channel flow. Some implications of the study for river engineering and palaeohydrological investigations are also presented.

INTRODUCTION

The primary aim of this paper is to consider the problem of interpreting meander growth-patterns from the surface configuration of flood plains. It has long been recognized that many flood plains exhibit corrugated surfaces formed as the channels migrated laterally downvalley and toward the concave banks; the succession of ridges and swales that form the corrugations have been termed *meander scrolls* or *scroll bars* (Lobeck, 1939). The Mississippi flood plain has been closely studied in this respect (for example, see Fisk, 1952). It is generally accepted that the undular surface results from juxtaposition of successive levees (ridges) and backswamps (swales) as the river migrates laterally.

Although meander scrolls are often associated with large high-order channels on gentle slopes, there are a number of notable exceptions. One is the Beatton River on the Alberta Plateau of northeast British Columbia, the flood plain of which has been selected as a testing area for some of the hypotheses proposed in this study.

The Beatton River has its source near the settlement of Sikanni Chief and flows some 500 km at an average slope of 0.0016 in a southeasterly direction to the Peace River at Fort St. John (see fig. 1). The 16,000 km^2 basin was selected for study because of its very well-developed

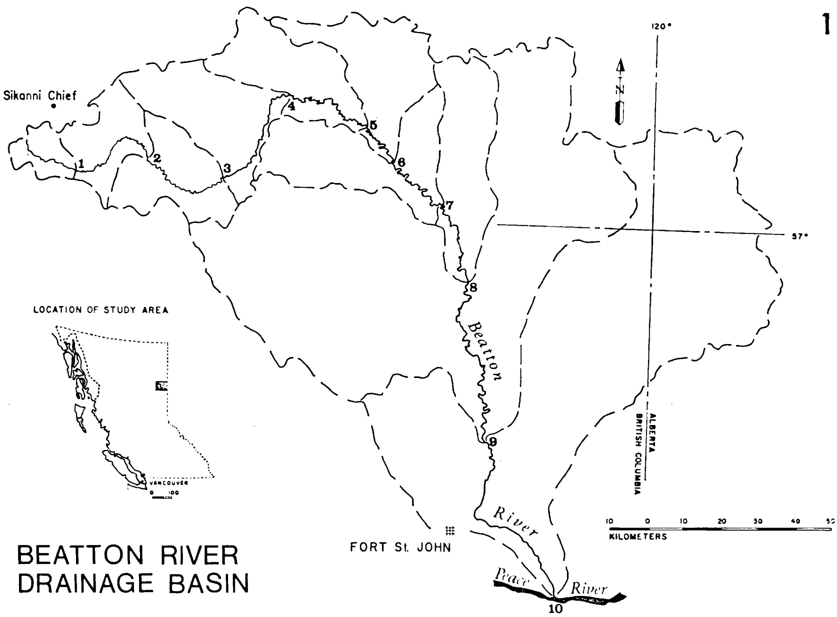


Fig. 1. Location map of the field area—Beatton River drainage basin.

flood-plain scrolls and because of the availability of large-scale topographic map and aerial photograph coverage. The Beatton River flows over surficial glacial and fluvio-glacial material for most of its length; its channel is cut in silty sands with some gravel. Underlying the surficial material are the Cretaceous Kaskapau and Dunvegan Formations, represented by intercalated fine to medium sandstones, siltstones, shales, and silty shales with minor lenses of coarse sandstones and fine conglomerates. Lateral migration of the channel seems to have typified the post-glacial period; the river is extremely sinuous in plan, as numerous cut-offs and flood-plain scrolls testify.

BEATTON RIVER SCROLL-BARS

A detailed description of the morphology, structure, composition, and origin of some typical scroll bars on the Beatton River, in preparation by the writer, will not be presented here. The origin of the scroll bars seems clear. During low-frequency flows (say, bankfull discharge) the weak material in the concave bank is subjected to vigorous erosion. The eroded bank-material is transported by secondary currents into the zone of low shear stress and velocity at the convex side of the channel. Here the material is deposited when the velocity of the turbulent eddies associated with the primary transporting current falls below the settling velocity of the sediment particles. When the high flow subsides, the bar deposited at the convex side of the channel is left exposed as a ridge, is readily colonized by vegetation, and becomes the new bank. During the low-flow stage the stream slowly erodes the base of the concave bank and removes any slumped material, steepening the side slope.

Subsequent high flows will once again vigorously erode the concave bank, tending to widen the channel, while the secondary currents deposit a new bar streamward of the previous ridge at the convex side of the stream. Thus an equilibrium channel-width is maintained (see fig. 2A). Since scroll patterns are preserved on many flood plains, it follows that overbank deposition subsequent to their formation is not sufficient to obliterate the form of the lateral-accretion deposits.

The foregoing sequence implies that an array of ridges and swales on a flood plain will best be formed where the hydrologic regime involves a marked seasonal cycle of high to low flows. Constant discharge would produce a level and continuous flood-plain surface. If there is no marked difference between high and low flows, or if the channel material is not easily eroded, then the development of scroll bars will be suppressed. There seem to be two principal reasons for the occurrence of

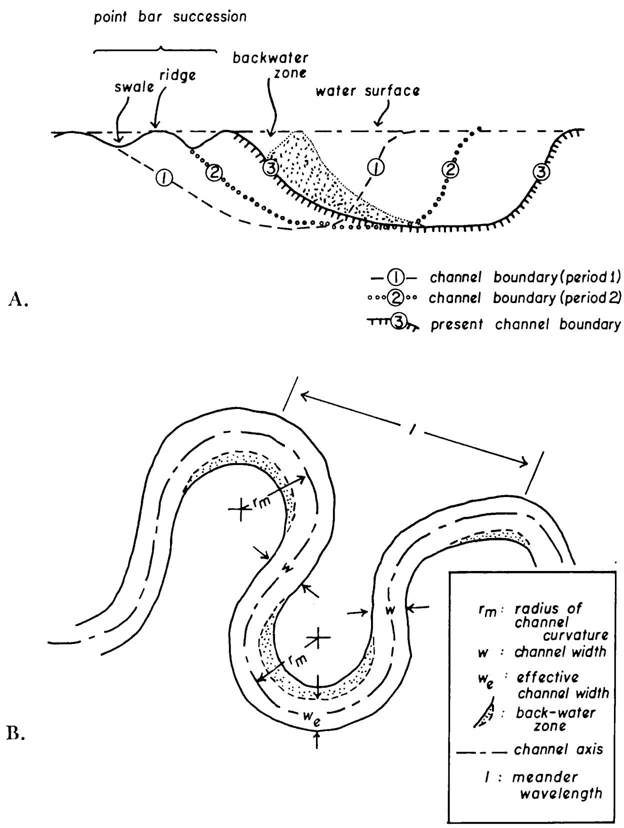


Fig. 2. (A) Point-bar formation in a laterally migrating channel; and (B) the planimetric geometry of a meandering channel.

very well-defined scroll bars in the Beatton Valley. Firstly, as noted, the channel material is very easily eroded. Very steep concave banks with considerable slump material at their bases are very common; very high suspended sediment concentration gives the river the appearance of flowing mud, and regularly-spaced migrating boils at the stream surface suggest considerable bedload transport through dune trains. Secondly, the Beatton River, like almost all channels on the Alberta Plateau, is charged with snow melt during the spring. Streamflow records for the gauging station at Fort St. John indicate that most of the total yearly discharge occurs in a few months centered on June; during the remainder of the year the rate of flow is relatively small. Bankfull discharges at reach 5 (see fig. 1) and at Fort St. John are respectively 200 m^3/sec and 400 m^3/sec. Annual peak-daily discharges at Fort St. John average 440 m^3/sec. Furthermore, a fact not revealed by the annual hydrograph is that, during the winter months, river ice virtually halts all but minor erosion in many reaches of the channel.

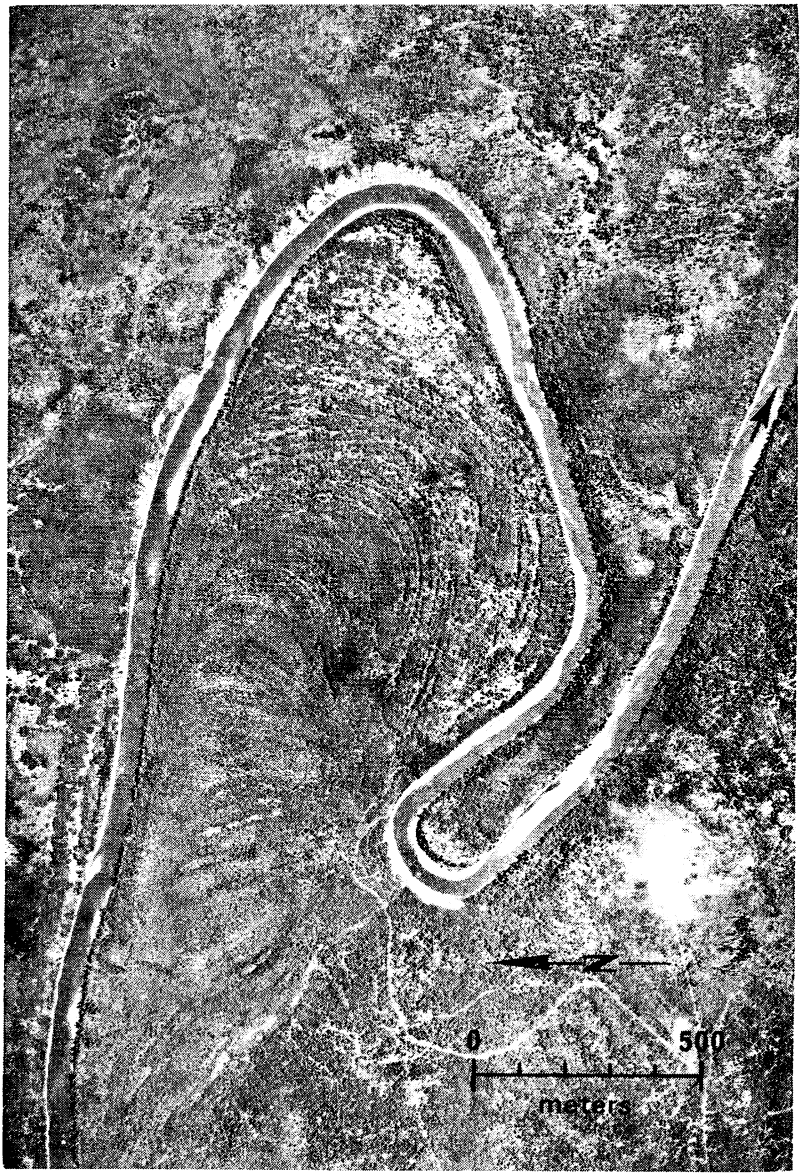
An aerial photograph of a typical meander bend with well-developed scroll bars is shown in plate 1; for purposes of discussion this meander will be named the Umbach channel-bend. Since the ridges and swales of flood plains support distinctly different vegetation communities (see Gill, ms), individual scroll bars appear readily on aerial photographs.

SCROLL BARS AS A HISTORICAL RECORD OF MEANDER DEVELOPMENT

Scroll bars locate accurately the convex bank of a developing meander in space and in time, recording channel erosion and deposition. For any flood plain of the type in question, the ridges (or swales) can be assigned an order away from the present channel such that T_1, T_2, T_3, T_4 . . . . T_n represents the set of ridges where T_1 is the most recent ridge deposit and T_n is the oldest ridge at the rear of the flood plain. The reverse ordering system is more attractive in terms of time sequence but is not practicable, since the oldest flood-plain ridges are the most difficult to define. The ordering assigns accurate relative ages to all parts of the flood plain and permits relative age correlations within and also between meander bends. Certainly it is evident from aerial photographs that, in many places on the Beatton River, there is a strong relation, among the ridge-spacing patterns on adjacent meander bends.

The regularity of the scrolls at the Umbach meander and the strong annual peaks in the hydrographs make it tempting to assign a constant time interval of one year to successive ridges. This simple relation is particularly attractive, since absolute dates could be assigned to any part of the flood plain by simply counting the number of annual ridges away from the present channel. Unfortunately, in the examples investigated so far, this simple sequence does not apply. In the case of the Umbach meander bend shown in plate 1, earlier aerial photographs indicate that the location of the channel has been virtually static since the last period of marked lateral migration ending in about 1949. Absolute ages of the ridge systems can be established from an examination of

PLATE 1



The Umbach channel-bend. Note the regular pattern of scroll bars and the present zone of most active lateral migration (E). British Columbia Government air photo BC7184-123.

the structure and age of the vegetation that they support. Flood plains formed by stream migration provide ideal conditions for the development of vegetation successions, and dendrochronological methods promise not only to provide absolute ages of the ridges but also information about hydrological conditions (for example, see Sigafoos, 1964; Everitt, 1968; Parker and Henoch, 1971).

EROSION PATHLINES

A direct indication of the paths followed by channel reaches as the stream migrates laterally is given by the set of orthogonals through the arcuate meander scrolls. Ideally, orthogonals are defined by the characteristic lines through the surfaces corresponding to the ridge patterns on the point-bar complex.[1] However, the functions defining the surfaces are generally exceedingly complex. Moreover, individual ridge traces on aerial photographs often wrongly appear discontinuous, complicating an already tedious problem of digitizing the scroll bars for analysis. A compromise graphical solution is shown in figure 3B. From point P_1 on T_1 two normals, N_1 and N_2, are constructed to T_1 and T_2 respectively. Point P_2 is located at the intersection of T_2 and the vectorial resultant of P_1N_2 and P_1N_1. The deflection angle θ from the normal N_1 at P_1, to P_2 on T_2 is given by cos P_1d/P_1P_2.

This procedure for locating P_2 is obviously most accurate where the interval P_1P_2 and the deflection angle are both very small: these conditions are invariably fulfilled in the case of the Beatton River ridge systems. For all cases considered to date, the scale of the aerial photographs from which the ridge traces have been abstracted is such that

[1] A general mathematical solution, based on figure 3A, defines two successive surfaces (ridges) such that:

$$T_1 = f(x,y) \quad (1)$$

$$T_2 = f(x,y) \quad (2)$$

The characteristic or orthogonal is some unknown function such that

$$x = X(t)$$

$$y = Y(t) \text{ where t is time}$$

It follows that the vector

$$\vec{(t)}_x = \frac{dX}{dt} = \lambda \frac{\partial f}{\partial x} \quad (3)$$

and that

$$\vec{(t)}_y = \frac{dY}{dt} = \lambda \frac{\partial f}{\partial y} \quad (4)$$

P_1 in figure 3A has the coordinates $x_1 = X(t_1)$ and $y_1 = Y(t_1)$, and the required coordinates of P_2 are $x_2 = X(t_2)$ and $y_2 = Y(t_2)$. Integrating equations (3) and (4) yields:

$$x_2 = X(t_1) + \lambda \int_{t_2}^{t_1} \frac{\partial f}{\partial x} [X(t), Y(t)]\, dt \quad (5)$$

and

$$y_2 = Y(t_1 + \lambda \int_{t_2}^{t_1} \frac{\partial f}{\partial y} [X(t), Y(t)]\, dt \quad (6)$$

Once the nature of functions T_1 and T_2 is known, equations (5) and (6) provide a unique solution for the coordinates at P_2.

the simple graphical technique is just as accurate as the solution offered by a more refined mathematical model.

Erosion pathlines give several types of information about river behavior. First, they provide an immediate visual impression of the meander growth-pattern; for example, figure 4A is an erosion-pathline map of the channel bend shown in plate 1. The map provides an indication of the pattern of variation in erosion direction over the flood-plain surface; any point on the channel can be traced back across the flood-plain surface to the original uninflected channel.

Secondly, the erosion pathlines provide the only valid basis for studies of lateral erosion-rates on flood plains, since along the orthogonals alone can the true erosion distance be measured.

Thirdly, the erosion pathlines provide an objective means of defining the *erosional axis* of a meander. The erosional axis of a meander is defined as that orthogonal along which the average rate of channel migration is greatest; that is, the longest orthogonal on the erosion-

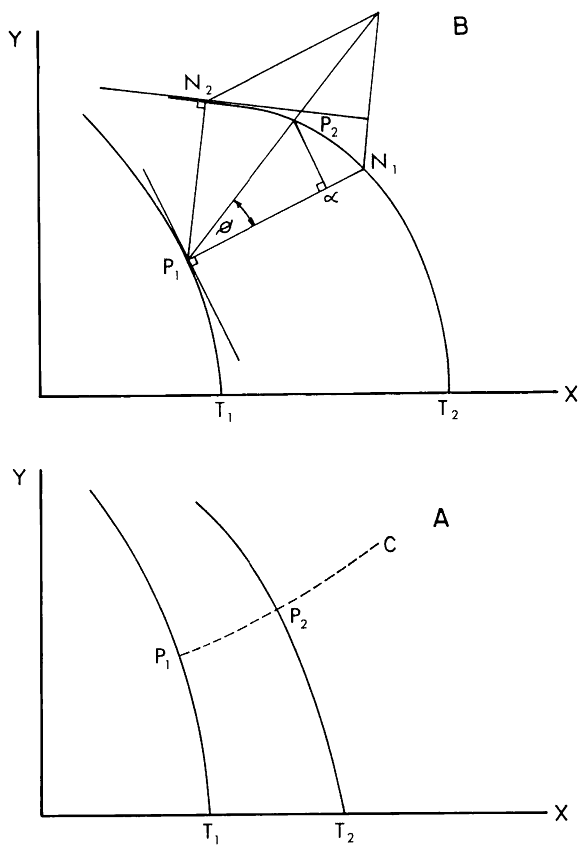


Fig. 3. Nomenclature for the analysis of orthogonal location on a point-bar surface.

pathline map. It can be located readily by plotting axial channel distance (from some arbitrary datum) versus orthogonal length; an example is shown in figure 4B. The erosional axis can be used as a datum on which measurements of meander geometry can be based.

Fourthly, the erosion pathlines can also be used to provide quantitative information about the relative importance of downvalley and crossvalley erosion in the development of a meander. For example, in figure 4C the orthogonal defining the erosional axis in figure 4A has been rotated so that the abscissa lies along the alignment of the original straight channel. It can be seen that, during the initial phase of meander development, crossvalley erosion in a direction normal to the original channel-axis (normal-directed erosion) was very much greater (100 m) than downstream-axial erosion (10 m). However, as lateral erosion continued, the rate of normal-directed erosion declined rapidly. The erosional axis can be represented by a simple exponential function and is regraphed on a full log-scale in figure 4D; the least-square best-fit curve is given by $L_v = 42\ L_a^{0.42}$ where L_v and L_a are respectively the normal-directed erosion and the downstream axial-erosion.

It follows that:

$$L_a = 0.024\ L_v^{2.38} \tag{7}$$

Differentiating equation 8 yields:

$$\frac{dL_a}{dL_v} = 0.056\ L_v^{1.38} \tag{8}$$

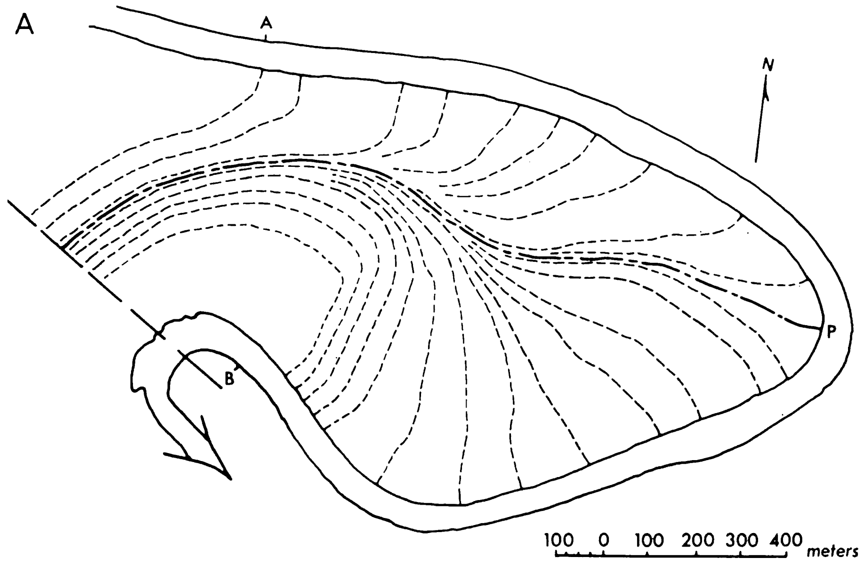
Equation 8 indicates that the rate of downstream axial-erosional increases with respect to the rate of normal-directed erosion as the amplitude of the meander increases.

However, the point bar shown in plate 1 is a relatively simple case; plate 2 and figure 5 show a somewhat more complicated case (Big Arrow channel-bend) than the one previously considered. Downstream axial-erosion by the upstream limb of the meander has destroyed much of the original ridge and swale topography; the ridge lines no longer parallel the channel on the upstream limb, and the erosional axis cuts across the point-bar surface rather than trending with the geometric long-axis of the feature. Nevertheless, the graph of relative erosion rates (see fig. 5), based in part on the extrapolated erosional axis, is essentially exponential in form and is thus similar to that shown in figure 4.

Clearly, many types of complications can occur. Meanders formed by upstream axial-erosion and those with multiple erosional-axes will be the subjects of later discussion.

EQUILIBRIUM CONDITIONS IN MEANDER BENDS

It has long been recognized that there exists in meandering channels a strong relation among meander wavelength, radius of channel curvature, channel width, and the scale of the flow system, usually represented by a discharge parameter (for example, see Inglis, 1949; Leopold and



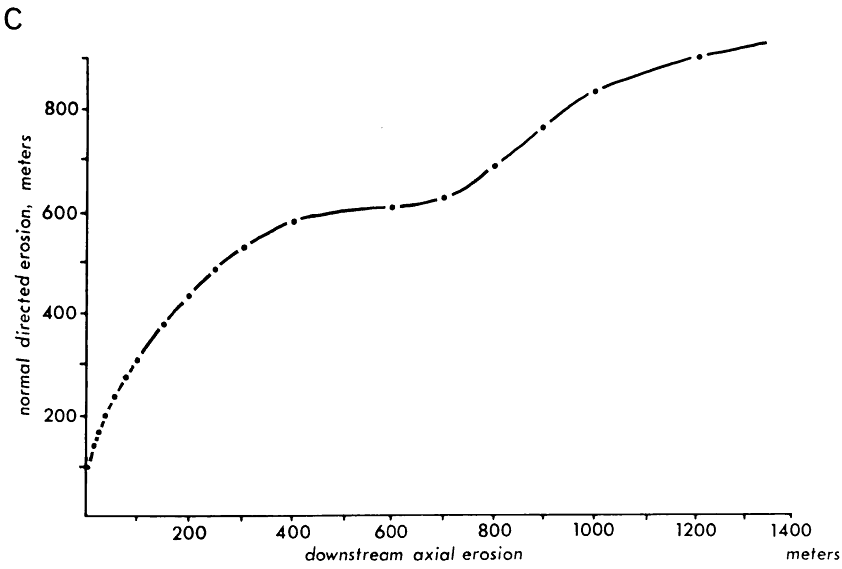
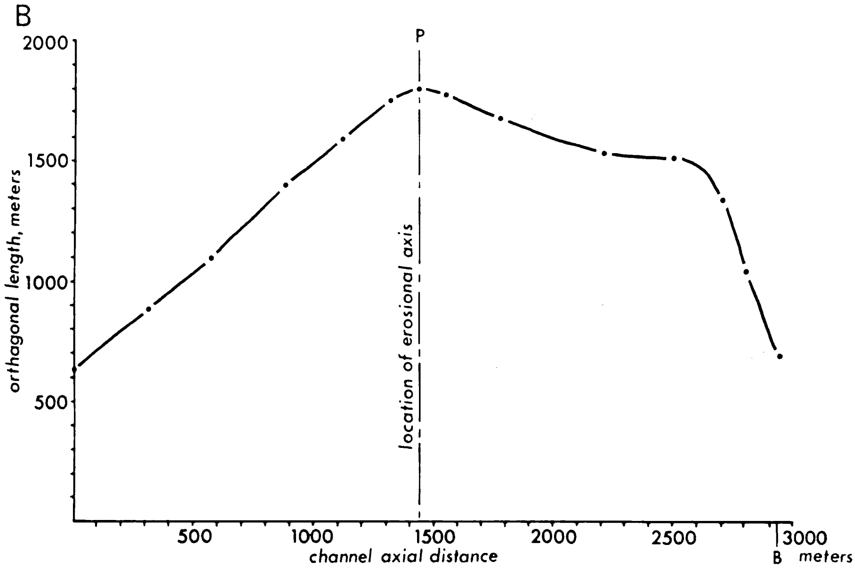


Fig. 4. (A) The orthogonal pattern; (B) erosional-axis location; and (C and D) direction of lateral migration for the Umbach channel-bend.


B
2000
1500
1000
500
orthagonal length, meters
P
location of erosional axis
500
1000
1500
2000
2500
3000
channel axial distance
B meters

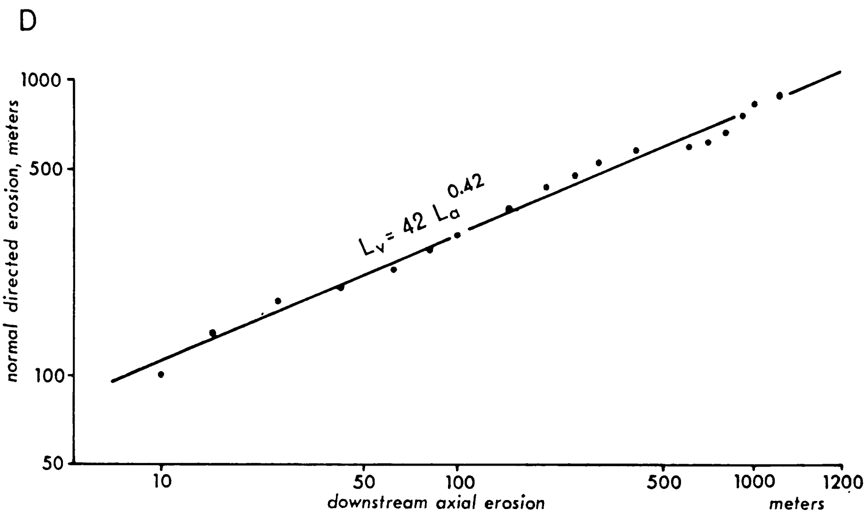
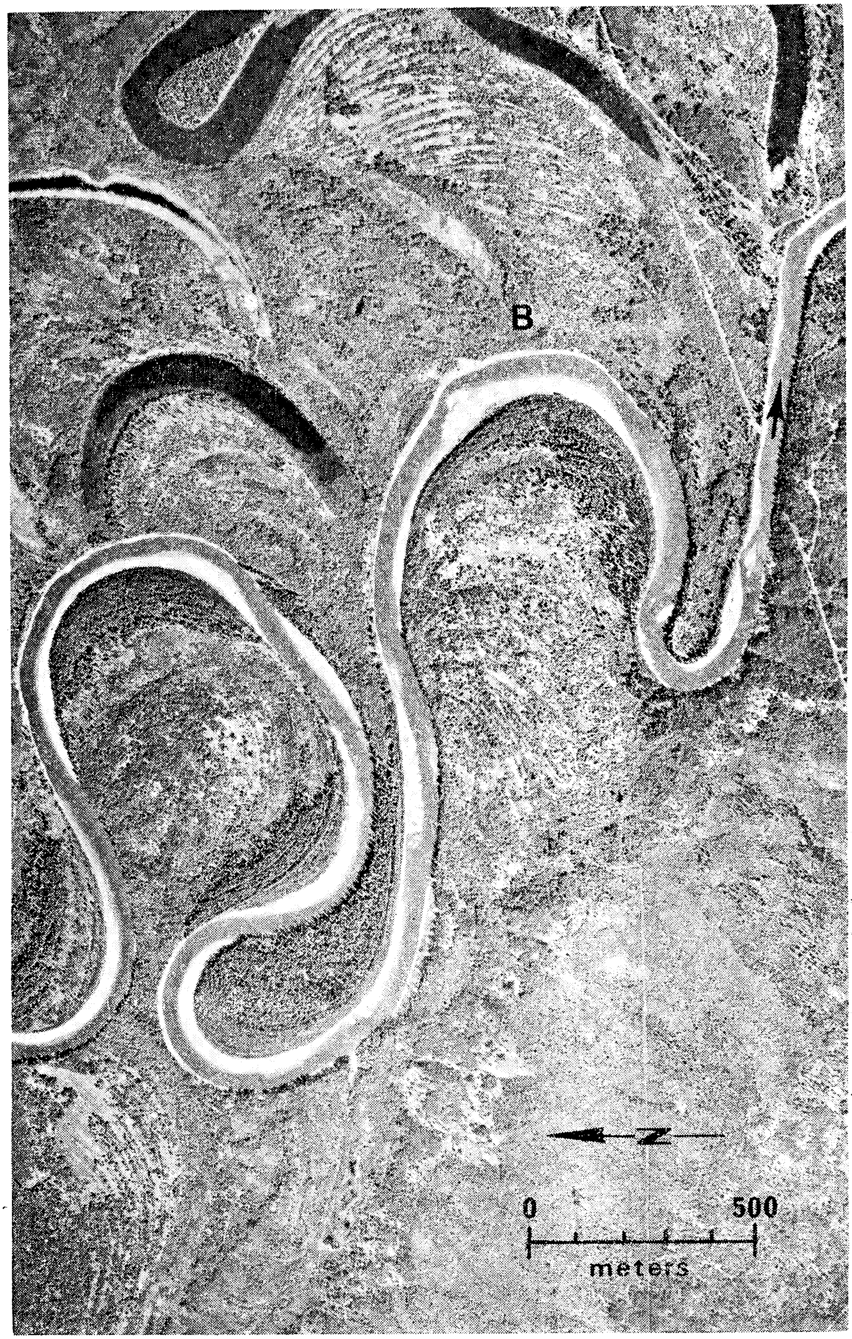

D
1000
500
100
50
normal directed erosion, meters
$L_v = 42\ L_a^{0.42}$
10
50
100
500
1000
1200
downstream axial erosion
meters

PLATE 2

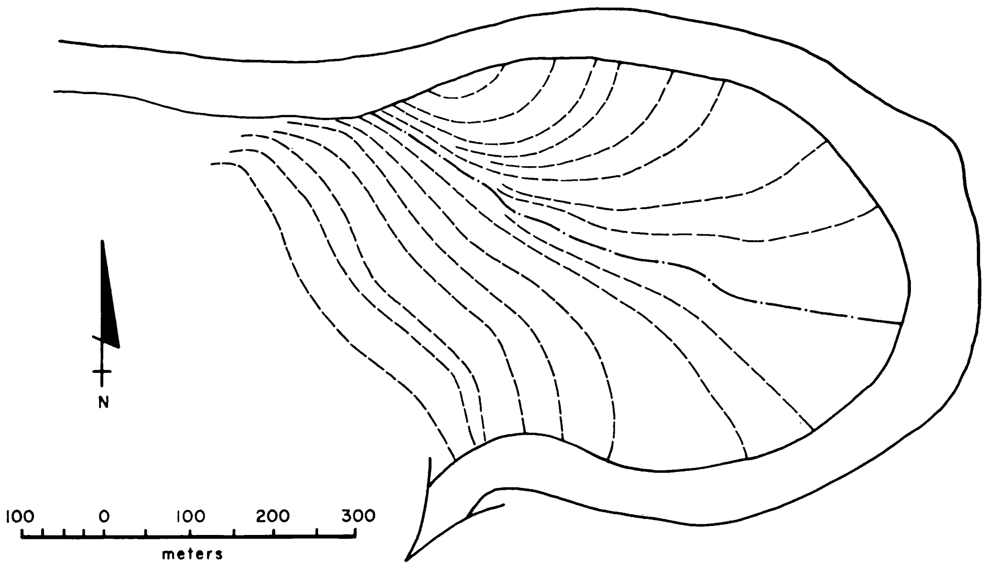


Big Arrow channel-bend (B). Note the general correspondence of vegetation patterns among individual meander loops. British Columbia Government air photo BC7185-109.

Wolman, 1960; and Dury, 1965). It has been reported (Leopold and Wolman, 1960) that, for a sample of 50 rivers differing in size and physiographic province, a large proportion of bends have a value for the ratio radius of curvature (r_m) to channel width (w) in the range 2 to 3, with two thirds of the values in the range of 1.5 to 4.3 (see fig. 2B for definition of channel parameters). This constancy implies that an equilibrium value of r_m/w exists in natural channels, although the process responsible for the equilibrium condition is not well understood. Nevertheless, it does seem likely that the character of flow resistance is an important factor controlling this aspect of open-channel geometry. Certainly studies of flow through closed conduits at bends indicate that minimum flow resistance due to curvature occurs when the ratio radius of curvature to pipe diameter is near a value of 2.0 (for example, see Hofmann, 1929). The reasons for the pattern of flow resistance variation with changing channel radius are extremely complex. During the early stages of meander development, when values of r_m/w are high, flow through a channel bend is subject to increasing transverse acceleration which results in increasing superelevation of the water surface and a deepening of the channel near the concave bank. If the increased local velocities and flow depths do not result in the growth of closely-spaced, high-amplitude bed forms, there will be a decrease in relative roughness and a concomitant decrease in skin resistance. On the other hand, the decrease in r_m/w will result in an increase in internal distortion resistance, although the experimental work of Leopold and others (1960) suggests that this component of resistance is probably negligible for values of r_m/w greater than 5.0. However, the same experiments suggest that, as r_m/w falls below a value of 5.0, internal distortion resistance increases rapidly until the development of spill resistance produces a discontinuity in the total resistance/channel curvature function. Bagnold (1960) suggests that the disintegration of the separation zone at the convex bank of an open channel occurs at values of r_m/w below 2.0, rapidly increasing total resistance to flow through the bend. He argues convincingly that with r_m/w at 2.0, the strong separation zone at the convex bank effectively reduces channel width, while a decrease in r_m/w below 2.0 results in a breakdown of flow into large eddies which markedly increase the resistance to flow through the bend. Although there is a paucity of direct measurements of shear distribution in open channel bends, those that are available (for example, Ippen and Drinker, 1962; Yen, 1965; and Apmann, 1972) do seem to indicate that, although maximum shear rates increase with decreasing values of r_m/w, the rate of change in shear increases markedly below a value of about 2.0.

Evidence from the Beatton River area suggests not only that the value of the curvature ratio exerts considerable control over the pattern of meander development but also that the critical value of r_m/w can be readily determined from an examination of flood-plain surface deposits. Furthermore, there is strong evidence to suggest that the critical value of r_m/w is much less variable in nature than has been previously recognized.

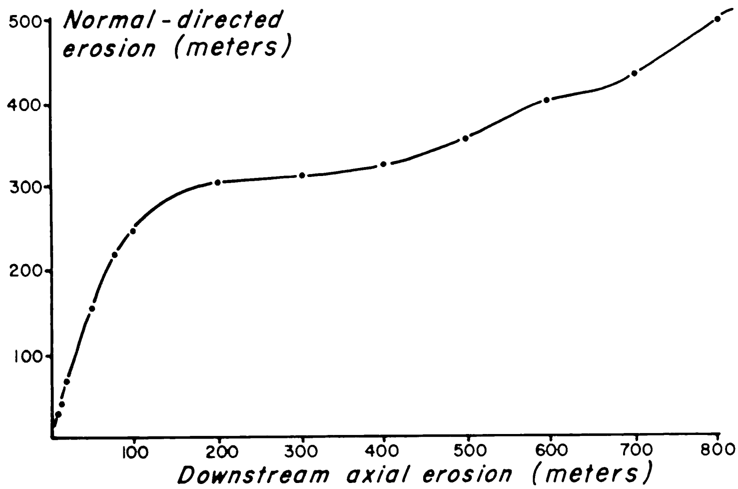
Inspection of figures 4D and 5B indicates that, although the erosional axes conform in general to exponential functions, there are marked internal divergences at specific points on those curves. The inflections in the curves represent abrupt and significant changes in the direction of lateral erosion and should not be dismissed as statistical noise; it is the writer's belief that such direction changes are a response to variation in flow resistance due to channel curvature changes during the development of the meander. The relation of radius of ridge and



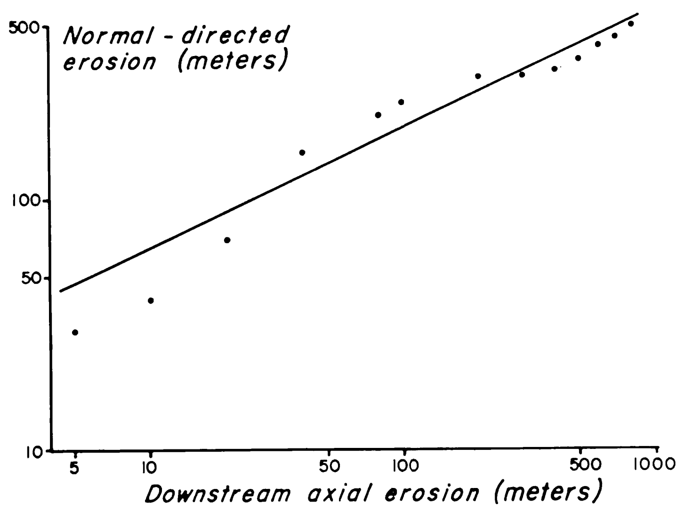
A.

Fig. 5. (A) The orthogonal pattern; and (B and C) the direction of lateral migration for the Big Arrow channel-bend.

swale curvatures (that is, prior channel curvatures) to distance along the erosional axis of the Umbach meander bend is graphed in figure 6A. During the early development of the meander, the radius of channel curvature was predictably quite large, with values of the order of 1000 m. But the growth of the channel bend rapidly reduced the radius of curvature to about 200 m, after which the rate of decline in r_m decreased to yield a minimum r_m of 110 m at the 500-m mark. Beyond the 500-m mark the radius of channel curvature fluctuates but continues to settle back to the 110-m value corresponding to a minimum flow-resistance



B.



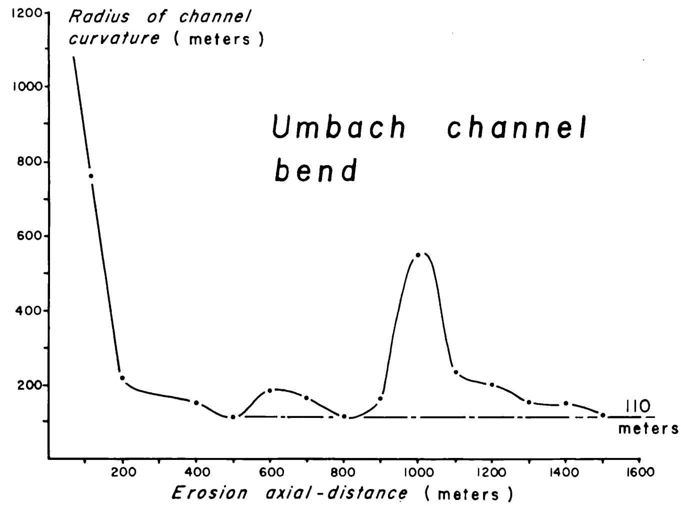
C.

through the channel-bend. The average channel width at the meander in question is 53 m; the critical r_m/w value is therefore 2.08.

If the corresponding plot of r_m/w and distance along the erosional axis is constructed for the Big Arrow meander, the same type of curve results (fig. 6B). The minimum radius of curvature, at 123 m, is slightly larger than that in the previous example, but the channel width is also larger, and r_m/w, at 2.05, is virtually unchanged.

However, since the principal erosion-axes of channel bends need not always pass through all the channel segments of minimum curva-

A.



C.

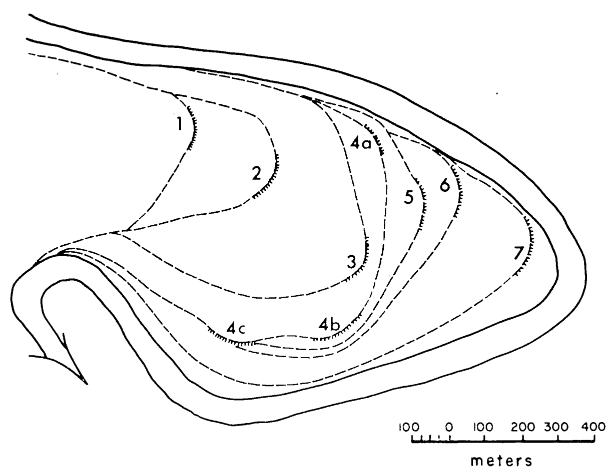
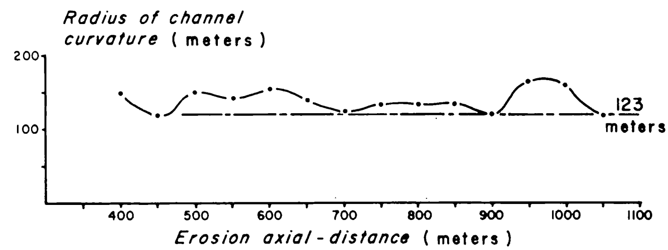


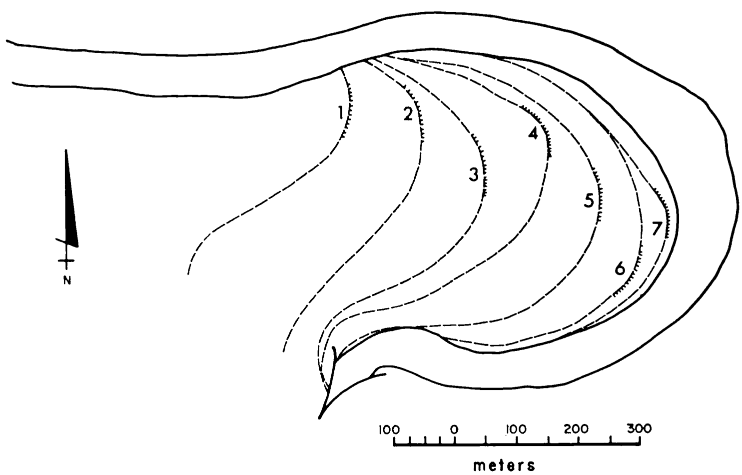
Fig. 6. The relation of channel curvature to distance along the erosional axes of (A) the Umbach and (B) Big Arrow channel bends; respective critical curvature segments are shown in (C) and (D).

ture, the plots in figures 6A and B provide less meaningful data than does a close examination of the complete flood-plain surface. Figure 6C shows a map of the Umbach channel bend in which the flood-plain surface has been segmented in accordance with the locations of the principal minimum-curvature reaches. That is, each segment boundary represents the limit of a channel migration phase in which a reach of the channel achieved a minimum curvature; immediately streamward of the segment boundary, the radius of channel curvature increases as a result of a change in the direction and/or location of the primary axis of erosion. Thus, the channel migrated laterally until a critical radius of curvature developed on boundary 1. In response to the rapidly increasing flow resistance at the channel bend, subsequent lateral erosion decreased the channel curvature by markedly displacing the zone of maximum migration in the downstream direction. Lateral erosion continued along

Big Arrow channel bend



B.



D.

TABLE 1
Critical values of r_m and r_m/w at each of the segments on the Umbach channel-bend shown in figure 6

No.	r_m	r_m/w
1	105	1.98
2	110	2.08
3	100	1.89
4 a	125	2.36
4 b	118	2.22
4 c	125	2.36
5	121	2.28
6	124	2.34
7	119	2.25

the new erosional axis until a new critical curvature developed, and the radius, once again, was increased by downstream displacement of the erosional axis.

The radius of curvature of boundary 4 in figure 6C exhibits minima at three distinct reaches. It is common for secondary erosional axes to be present in the one migration phase, but less usual for the channel segments at all erosional axes to reach the same minimum value of curvature simultaneously.

The apparently tight bends near the northern limb of the meander are not the result of normal channel-migration but rather have been produced by the truncation effects of a slight southward and then subsequent northward shift of the channel.

The critical radius of curvature for each of the nine segments marked in figure 6C and the corresponding values of r_m/w are listed in table 1. The values average 2.19 and range from 1.89 to 2.36; the standard deviation of the array is 0.16; that is, seven of the nine values vary by no more than 7.5 percent above or below the mean value of 2.19. The variation of r_m/w is remarkably small and entirely consistent with the concept of the critical minimum in channel-curvature condition.

Similarly, a flood-plain segment map can be prepared for the Big Arrow channel-bend (fig. 6D); curvature data appear in table 2. Values of r_m/w in this case are somewhat more variable than those derived for the Umbach meander, ranging from 1.53 to 2.25 about a mean of 1.93. Standard deviation for the r_m/w array is 0.21, representing a variation

TABLE 2
Critical values of r_m and r_m/w at each of the segments on the Big Arrow channel-bend shown in figure 6D

r_m/w	r_m	No.
1	123	2.05
2	115	1.92
3	135	2.25
4	92	1.53
5	125	2.08
6	110	1.83
7	112	1.87

of ±11 percent about the mean. Nevertheless, the data once again strongly support the hypothesis that a sensibly constant critical r_m/w condition is influencing the pattern of meander growth in the Beatton River. Although the discussion of processes and morphologic relations has thus far been restricted to only two examples of point bar complex, the influence of the critical r_m/w condition on meander development is evident on all point bars of the Beatton River and its tributaries which have a distinct ridge and swale surface configuration.

OBSERVATIONS ON THE GENERAL PATTERNS OF MEANDER DEVELOPMENT

Like most actively migrating meander trains, the Beatton River meander-trace is exceedingly complex in pattern. Nevertheless, the foregoing analysis of critical channel curvatures does provide a basis for interpretation. The most commonly occurring meander patterns in the Beatton River are of the types illustrated in figure 7.

Type A bends are those with a single channel loop in which the axis of symmetry is curved in the downstream direction. There are two principal modes of formation of the type A channel bend. The first is one in which the bend forms by radially-outward erosion from two fixed points of inflection (case A_1 in fig. 7); the limbs of ridges on the flood-plain surface are sub-parallel to the existing channel (accordant ridge system). The second mode involves subsequent modification of an existing bend by erosion (see case A_2 in fig. 7). The down-stream limb of the bend migrates in the upstream direction, destroying part of the original flood-plain surface. The erosion pathlines indicate an apparently asymmetric pattern of deposition; only the ridge limbs on the upstream arm of the bend are accordant. The second mode is much less common than the first; the Big Arrow channel-bend is an example of a type A bend, in this case almost exclusively the product of the A_1 mode. More commonly, type A bends are formed by a combination of both modes, with the first maintaining a marked dominance.

Type B channel bend is a development of the type A meander form; again, there are two primary modes of origin. The first, and by far the more common mode, is represented by case B_1 in figure 7. When the A_1 channel bend reaches a critical curvature, a lobe of deposition develops on the downstream limb of the bend, thus increasing the radius of channel curvature. The presence of this secondary erosional-axis is usually indicated by an abrupt direction change in the erosional pathlines; however, the ridge system is essentially accordant. The second mode is illustrated by case B_2 in figure 7. Lateral erosion into the existing flood-plain by the downstream limb of the channel continues, until a critical curvature is achieved. A secondary erosional-axis develops to increase the radius of curvature and thus produces the hooked or recurved channel-pattern. This mode of origin is usually recognized by the typically discordant ridge-system of the flood-plain and by the accordant ridge-system of the downstream penetration lobe. The Umbach channel-bend is an example of the type B meander pattern.

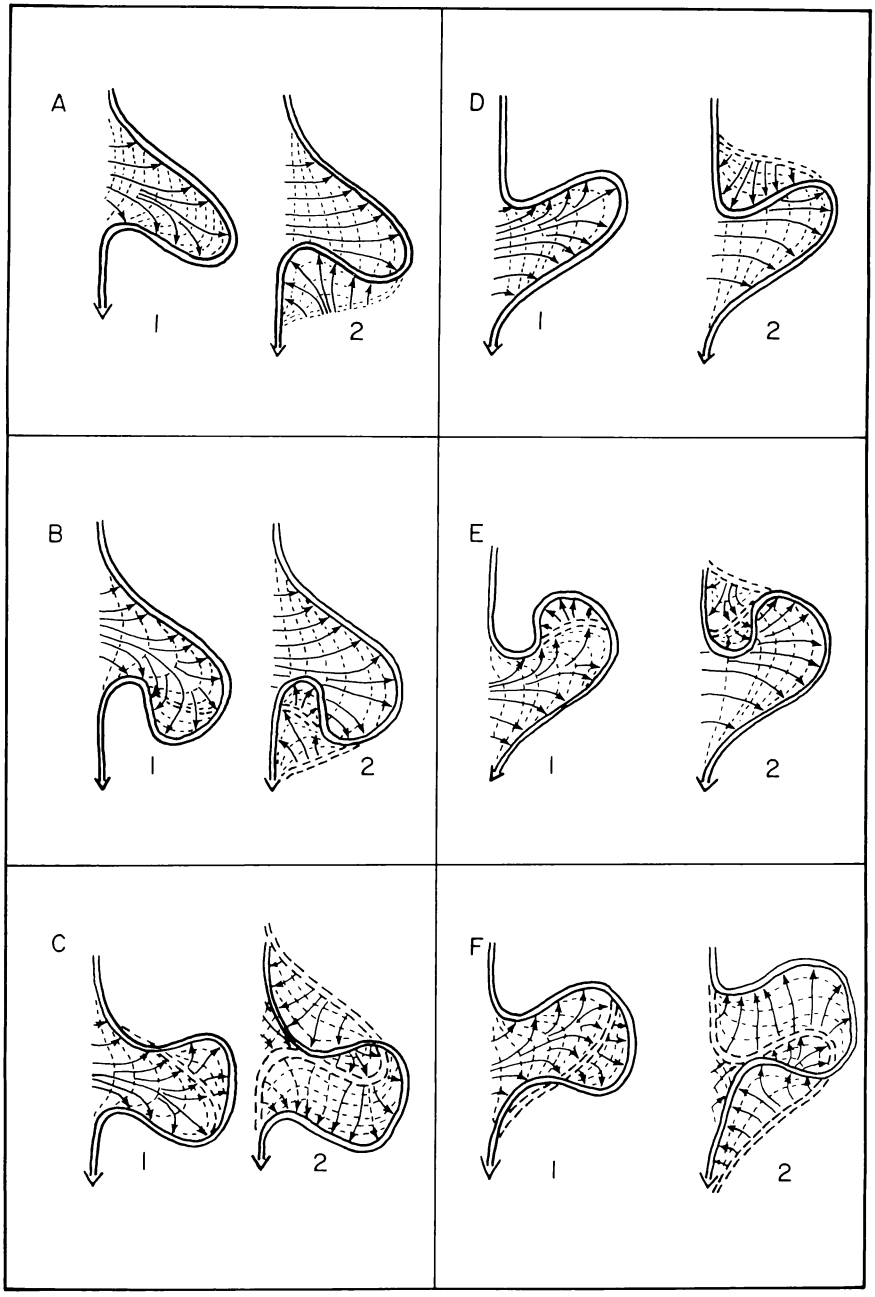


Fig. 7. The development of typical meander patterns on the Beatton River.

The type C channel bend is also a development of the type A pattern. In figure 7, case C_1 represents a channel bend that develops a depositional lobe on the upstream limb of the original channel bend; the secondary erosional-axis is a response to the critical curvature condition on the primary erosional-axis. The same channel pattern is more commonly the result of differential rates of downstream migration by the original type A bend (see case C_2). Migration of the upstream limb of the original bend proceeds initially at a faster rate downstream than that of the downstream limb, resulting in a reduction of channel curvature to the critical condition. The channel responds by increasing the rate of downstream migration by the downstream channel limb, thus decreasing the radius of curvature. If the upstream limb is migrating very rapidly relative to the downstream limb, the typical "goosenecks" and cutoffs are formed.

Channel types D, E, and F are the upstream-oriented counterparts of types A, B, and C. The origins are identical except that the second mode is the important process in the formation of types D and E; the first mode is generally more common than the second in type F meanders.

Clearly, more complex forms than those in figure 7 may have very complicated histories. For example, the channel bend in plate 4 developed from a type A_1 meander. When a critical curvature developed on the primary erosional axis, a second erosional axis developed to produce a type B_1 meander. Subsequent migration by the upstream limb of the channel destroyed much of the initial flood-plain surface and served to decrease the radius of curvature of the northern part of the channel to a critical value. The channel responded by developing a third erosional-axis producing a depositional lobe of the D_1 type. "T" formations of the type in plates 3 and 4 are common on the Beatton River.

Still more complex meander patterns, in which downstream migration has completely destroyed all traces of the original flood-plain surface representing the early stages of development, may have histories that are virtually impossible to interpret in their entirety. However, remnants of flood-plain deposits left by a developing channel bend are usually preserved to provide evidence of its history. Any reconstruction of former channel-bend traces must take cognizance of the following four general principles of meander development.

1. During the early stages of meander development, the points of channel inflection are sensibly fixed, and lateral migration of the channel bend invariably results in a reduction of the radius of channel curvature along the principal erosional-axis.

2. When the ratio radius of bend curvature to channel width approaches a value of about 2.0, the rate of lateral migration at that channel segment will be reduced to a minimum; elsewhere on the bend, differential rates of channel migration will act to increase the radius of curvature at the segment in question.

PLATE 3

A typical "T" meander pattern on the Beatton River. Note the development of the "goose-neck" and the various phases of lateral migration indicated by the vegetation pattern. British Columbia Government air photo BC7303-052.

PLATE 4

A "T' meander pattern produced by the formation of upstream and downstream channel lobes on a simple type A bend. Channel migration has destroyed much of the original point-bar surface; note the truncated scroll-bars. British Columbia air photo BC7185-065.

3. The radii of curvature of adjacent meander loops tend to be inversely related; that is, an increase in the radius of curvature of a given bend often results in a decrease in the radius of the adjacent meander bend. Consequently the rate and direction of lateral erosion on a given channel bend are not independent of the erosional activity in adjacent bends.

4. Although it is generally accepted that a maximum rate of concave bank erosion occurs just downstream of the axis of symmetry of a point bar, it should nevertheless be recognized that, in many cases, a channel will respond to a critical curvature condition by eroding the concave banks on the upstream limb of a channel bend.

The ideas on meander development outlined above do not conflict with those suggested by Langbein and Leopold (1966). Those authors, arguing from the work on most probable fixed-length paths between two points by Von Schelling (1951, 1964), demonstrate that the planimetric geometry of a river meander can be defined in terms of a random-walk model as:

$$\phi = \omega \text{ s in } \frac{s}{M} 2\pi \qquad (9)$$

where ϕ is the flow direction at a point s, ω is the maximum angle the meander describes relative to the axis of the meander train, and M is the channel length of a meander (all angles are in radians). Equation 9 describes a path referred to by Langbein and Leopold as a *sine-generated curve* (see fig. 8A). This curve minimizes the sum of the squares of the changes in flow direction in each successive unit length and is considered by those authors to underlie the stable form of meanders.

Figure 7C suggests that certain meanders, by responding to critical r_m/w conditions, may develop the sine-generated form. But the present study also suggests that, in the free-meandering state, the process of channel adjustment is a discontinuous one in which phases of adjustment occur in response to the development of critical curvatures. It seems likely that, because the rate of lateral migration in the free-meandering state is so great, the channel planimetric geometry only responds to marked differences in shear rates along the channel. Furthermore, free-meandering channels invariably possess readily movable beds, and initial resistance variations can more easily be reduced by adjustment in bedform configuration than by major changes in the direction of lateral migration.

By implication, incised channels or those cut in resistant materials should conform much more closely to the sine-generated form than do free meanders. And indeed, as a group, they do so. Langbein and Leopold (1966) also concluded that "during incision, irregularities apparently tend to be averaged out and only the regular (sine-generated) form is preserved", although they do not recognize the reasons outlined in the present study. Incised channels have comparatively slow

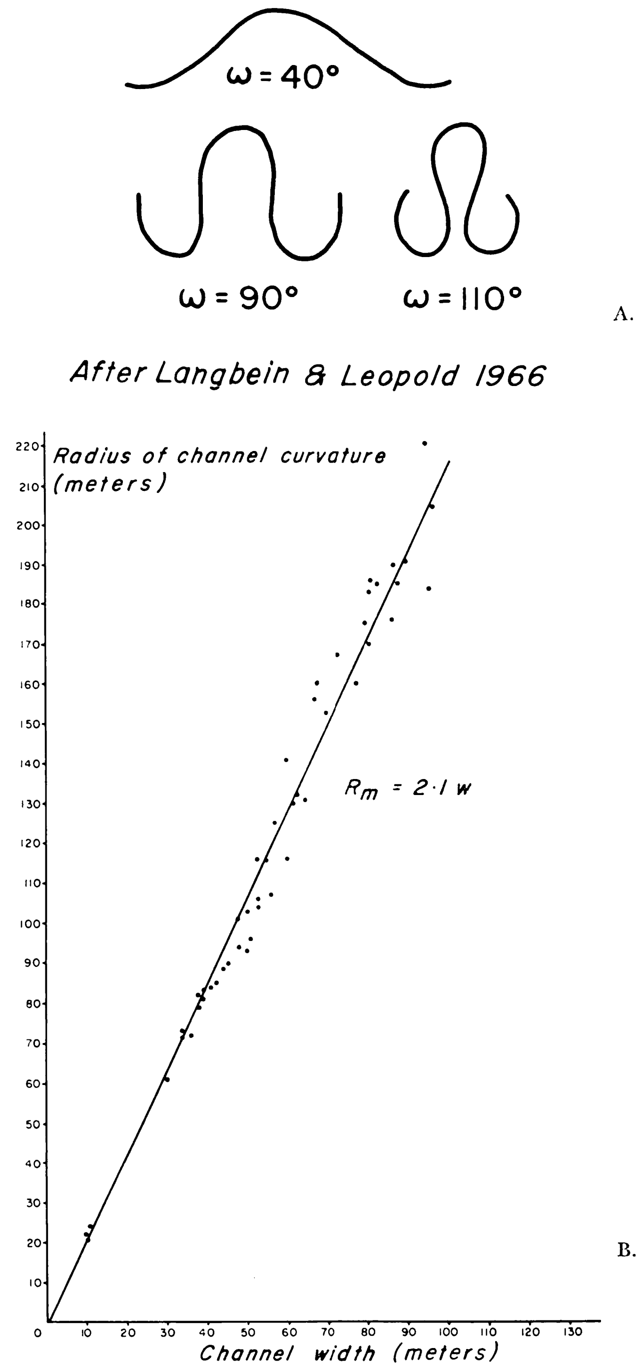


Fig. 8. (A) Some sine-generated curves; and (B) the relation of critical r_m to w for 50 point-bars on the Beatton River.

rates of lateral migration, and there is sufficient time to adjust to relatively small variations in shear stresses induced by channel curvature differences; there is also generally little scope for reducing variance in resistance by bedform adjustment.

IMPLICATIONS OF THE ANALYSIS

The present study is essentially a contribution to the general body of theory relating to river-channel behavior and flood-plain formation. But the work also has several important implications for applied studies in the areas of river engineering and of paleohydrology.

One of the many channel stabilization problems facing the river engineer is the control of lateral migration at river bends. The problem is of particular concern on channels like that of the Beatton River in which rates of lateral migration are great. The traditional approach of the engineer has been to build semi-empirical models of flow fields in channel bends in order to predict the distribution of shear stress and thus the potential areas of maximum erosion along the channel boundary. The recent study of flow processes in open-channel bends by Apmann (1972) is a typical example of this type of analysis. Without doubt, detailed analysis of flow in curved channel-reaches is vital to an understanding of erosion problems. But alone it is not adequate. The present study indicates that unless the channel reach is also considered in the context of the history of development of the whole channel, an understanding of channel behavior at bends will remain incomplete. In the foregoing analysis it has been demonstrated from historical evidence that marked changes in the pattern of erosion and of deposition in channel bends occur once the critical value of r_m/w has been achieved. This process of channel adjustment can also be seen in contemporary channels. For example, the Umbach channel-bend (pl. 1) at the erosional axis is in the critical curvature state, and the zone of most active lateral-migration has been displaced downstream (note the bank erosion and deposition of an extensive bar on the opposite convex side of the channel). A similar situation occurs at the Big Arrow channel bend (pl. 2); the contemporary channel is responding to the critical curvature by eroding laterally the upstream limb of the channel. Many current engineering solutions would locate incorrectly potential danger zones at the point of minimum curvature or just downstream of it (for example, see Apmann, 1972).

In the Beatton River examples, the change in direction of channel erosion will have little impact on man. However, there are many other cases where settlement has been established on the banks of river bends now at the critical r_m/w condition. For example, the settlement of Medicine Hat in the southeast of Alberta is built just downstream of a channel bend of the South Saskatchewan River which is in the critical-curvature condition. The present study suggests that the maximum rate of lateral erosion in the immediate future will not occur at the point of minimum radius of curvature but rather on the limbs of the channel bend. If the northern limb migrates, considerable damage to productive farm land will occur; erosion on the southern limb will threaten the residential area of Medicine Hat. There is some evidence to suggest that erosion is already starting to alter the alignment of both limbs. There is a need for a shift in emphasis of river engineering studies from detailed analyses of the contemporary water-sediment system to a more holistic approach in which bank-erosion problems are seen in the context of the total pattern of river behavior.

If the critical value of r_m/w is sensibly constant, then the ideas developed in this study can also be applied to paleohydrological problems. For example, on plains of alluvial deposition, it is often possible to identify patterns of ancient scroll-bars on cutoffs. An analysis of the scroll-bar pattern should indicate the critical r_m value and in turn the bankfull width of the prior channel. Furthermore, the backswamp deposits usually contain large amounts of organic matter which may be used to obtain accurate radiocarbon dates for the abandonment of the prior channel. Estimates of bankfull discharges based on r_m/w values, together with absolute dates, will yield a valuable historical record of hydrologic regimes.

Analysis of stream underfitness is another area where the critical r_m/w concept may provide insight into past hydrologic and climatic conditions.

The extensive work by Dury (1964) on channel underfitness has shown that many meandering valleys were once active channels formed by rivers much larger than those presently flowing through them. Dury has exploited the relation between drainage area (and bankfull discharge) and meander wavelength (see Friedkin, 1945; Inglis, 1949; Leopold and Wolman, 1960) to provide an index of river underfitness based on the ratio valley-meander wavelength/river-meander wavelength. However, on streams such as the Beatton River, the channel trace is often exceedingly tortuous and irregular, making measurement of meander wavelength virtually impossible. Nevertheless, a meaningful measure of the scale of the meander system is given by the magnitude of the critical r_m values; the corresponding wavelength is obtained from the relation $4r_m = l$ (see fig. 2).

It has been suggested by Speight (1965) that much of the subjectivity of meander wavelength measurements can be eliminated by considering all inflections in the stream trace. In an attempt to take into account the total oscillatory behavior of a river reach he analyzed series of closely spaced flow-direction measurements on several streams in Australia and in New Guinea for autocorrelation and for the spectrum of meander intensity versus frequency of oscillation per unit distance. But the results failed to reduce the external "noise" sufficiently to yield meaningful wavelength relations. Similarly, application of power spectral analysis to the meandering trace of the Murray River in New South Wales, Australia, has produced anomalous results (J. Urquhart, Australian

Natl. Univ., personal commun.). However, such results are to be expected; the present study indicates that the size of channel bends is often influenced as much by the nature of lateral migration as by the critical radius of channel curvature (and thus by the meander wavelength). Power spectral analysis provides the only effective way to describe the complex oscillatory behavior of a meandering channel, but using it to define a simple index of the meandering scale is an abuse of the technique. The critical r_m parameter remains the most useful expression of the scale of a meander system.

The analyses outlined above rest heavily on the assumption of temporal and spatial constancy for the critical values of r_m/w. Although it has been demonstrated that the values are sensibly constant within individual point bars, it has yet to be shown that constancy applies to a whole drainage net. In order to establish the degree of r_m/w variation in this broader context, aerial photographs of 50 individual point-bars located along the full length of the Beatton River were examined to obtain the appropriate data; these are listed in table 3 and graphed in figure 8B.

Clearly, along the full length of the Beatton River the critical values of r_m/w vary little; the mean value is 2.11, and the standard deviation is only 0.13 or ±6.16 percent. The graph of r_m versus w in figure 8B indicates that the data are slightly more variable for large values of r_m. This increased variability is, in part, the result of the greater degree of river incision in the downstream reaches where r_m is relatively great; the channel width is somewhat reduced where the channel is entrenched. Consequently, some meander scrolls on the downstream point-bar complexes are probably related to a larger channel-width than that exhibited by the corresponding contemporary channel.

Nevertheless, the data strongly support the assumption of constancy for critical values of r_m/w on the Beatton River.

Furthermore, preliminary investigation of rivers in other areas (Battle River and South Saskatchewan River, southeast Alberta; Hay River, northeast Alberta; Mississippi River, near Memphis, Tennessee; Mekong River, Thailand—see Ohya, 1961) suggest that a critical r_m/w of about 2.0 may have a very wide application indeed.

However, there is an obvious need for extensive field investigations to test the ideas presented in this study. It is not known whether the principles of meander growth developed for the Beatton River have a general application. The influence of bed and bank materials on the critical value of r_m/w is not known. The influence of channel incision on the critical value of r_m/w should also be the subject of further investigation. There is also a need to develop criteria for measuring channel curvature on rivers that migrate without depositing scroll bars. Although many problems are yet to be resolved, this area of enquiry promises to yield fruitful results, not only in the general area of fluvial geomorphology but also in the field of river engineering.

TABLE 3
Critical values of r_m and r_m/w for fifty point bars on the Beatton River

Map location (1:50,000)	Reach no. (see fig. 1)	Photo no.	r_m (meters)	w (meters)	r_m/w
94 G/2 E 234266	1	A12146-229	22	10	2.20
94 G/2 E 235265	1	A12146-229	21	10	2.10
94 G/2 E 236264	1	A12146-229	24	11	2.18
94 G/2 E 238268	1	A12146-229	21	10	2.10
94 G/2 E 243267	1	A12146-229	22	10	2.20
94 G/1 W 445303	3	A12160-255	72	34	2.12
94 G/1 E 457291	3	A12160-255	73	34	2.15
94 G/1 E 459297	3	A12160-255	72	36	2.00
94 G/1 E 478286	3	A12146-295	81	39	2.08
94 G/1 E 485266	3	A12146-221	83	39	2.13
94 G/1 E 489273	3	A12146-221	84	41	2.05
94 G/1 E 565226	3	A12146-295	85	42	2.02
94 H/4 W 612243	3	BC7302-150	61	30	2.03
94 H/4 W 644272	4	BC7302-158	79	38	2.08
94 H/4 W 734339	4	BC7184-69	82	38	2.16
94 H/4 E 760427	4	BC7185-155	89	44	2.02
94 H/4 E 763479	4	BC7182-049	90	45	2.20
94 H/5 E 763495	4	BC7182-049	94	48	1.96
94 H/5 E 775491	4	BC7182-049	106	53	2.00
94 H/5 E 786505	4	BC7177-018	107	56	1.91
94 H/5 E 811503	4	BC7177-019	93	50	1.86
94 H/5 E 815500	5	BC7177-210	96	51	1.88
94 H/5 E 831497	5	BC7182-054	101	48	2.10
94 H/5 E 845494	5	BC7182-054	103	50	2.06
94 H/5 E 848857	5	BC7182-056	116	55	2.11
94 H/6 W 937487	5	BC7182-062	130	62	2.09
94 H/3 W 027427	5	BC7185-106	104	53	1.96
94 H/3 W 035420	6	BC7185-106	116	53	2.20
94 H/3 E 075390	6	BC7185-065	125	57	2.19
94 H/3 E 090375	6	BC7184-123	116	60	1.93
94 H/3 E 127322	6	BC7182-134	170	81	2.09
94 H/3 E 118321	6	BC7182-134	175	80	2.19
94 H/3 E 176293	7	BC7302-107	160	68	2.35
94 H/3 E 185281	7	BC7302-201	160	78	2.05
94 H/3 E 204278	7	BC7302-201	156	67	2.39
94 H/2Pv 219255	7	BC7302-210	132	63	2.09
94 H/2Pv 237235	7	BC7302-207	167	73	2.29
94 H/2Pv 237235	8	BC7303-052	153	70	2.19
94 H/2Pv 270210	8	BC7303-059	186	81	2.30
94 H/2Pv 269197	8	BC7303-059	183	81	2.26
94 A/15W 305151	8	BC7303-147	141	60	2.35
94 A/15W 317062	8	BC7303-274	131	65	2.01
94 A/10W 354877	9	BC7304-178	184	96	1.92
94 A/10W 354863	9	BC7304-178	176	87	2.02
94 A/10E 394810	9	BC7295-131	185	83	2.23
94 A/10E 392783	9	BC7295-097	185	88	2.10
94 A/10E 390757	9	BC7295-020	190	87	2.18
94 A/10E 396756	9	BC7295-020	191	90	2.12
94 A/7 E 418638	9	BC7299-201	205	97	2.01
94 A/7 E 405395	10	BC7300-135	221	95	2.33

ACKNOWLEDGMENTS

The writer is indebted to numerous colleagues for their helpful comments on an earlier version of this paper, particularly to M. Church of the University of British Columbia and to G. H. Dury of the University of Wisconsin.

The illustrations were kindly prepared by D. Dairon of Simon Fraser University.

This study represents part of a project currently being undertaken at the Department of Geography, Simon Fraser University, and is funded by the National Research Council of Canada.

REFERENCES

Apmann, R. P., 1972, Flow processes in open channel bends; Am. Soc. Civil Engineers Proc., v. 98, HY 5, p. 795-809.

Bagnold, R. A., 1960, Some aspects of river meanders: U.S. Geol. Survey Prof. Paper 282-E, 10 p.

Dury, G. H., 1964, Principles of underfit streams: U.S. Geol. Survey Prof. Paper 452-A, p. 1-67.

——— 1965, Theoretical implications of underfit streams: U.S. Geol. Survey Prof. Paper 452-C, 43 p.

Everitt, B. L., 1968, Use of cottonwood in an investigation of the recent history of a flood plain: Am. Jour. Sci., v. 266, p. 417-439.

Fisk, H. N., 1952, Mississippi River Valley geology relation to river regime: Am. Soc. Civil Engineers Trans., v. 117, p. 667-689.

Friedkin, J. F., 1945, A laboratory study of meandering of alluvial rivers: U.S. Waterways Eng. Expt. Sta., 40 p.

Gill, D., ms, 1970, Vegetation and environment in the Mackenzie River Delta, N.W. Territories—A study of sub-arctic ecology: Ph.D. dissert., Univ. of British Columbia, Dept. Geography, 694 p.

Hofmann, Albert, 1929, Loss in 90-degree pipe bends of constant circular crosssection: Univ. Munich Tech. Hydraulic Inst. Bull. 3, (English translation of the original, pub. 1935 by Am. Soc. Mech. Engineers, New York), 45 p.

Inglis, C. C., 1949, The behaviour and control of rivers and canals: Poona, India Central Waterpower Irrig. and Navigation Research Sta., Research Pub. 13, v. 1 and 2, 486 p.

Ippen, A. T., and Drinker, P. A., 1962, Boundary shear stresses in a curved trapezoidal channel: Am. Soc. Civil Engineers Proc., v. 88, HY 5, p. 143-179.

Langbein, W. B., and Leopold, L. B., 1966, River meanders-theory of minimum variance: U.S. Geol. Survey Prof. Paper 422-H, p. 1-15.

Leopold, L. B., Bagnold, R. A., Wolman, M. G., and Brush, L. M., 1960, Flow resistance in sinuous or irregular channels: U.S. Geol. Survey Prof. Paper 282-D, p. 111-134.

Leopold, L. B., and Wolman, M. G., 1960, River meanders: Geol. Soc. America Bull., v. 71, p. 769-794.

Lobeck, A. K., 1939, Geomorphology: New York, McGraw Hill, 291 p.

Ohya, M., 1961, Geographical study of floods in the Basin of the Mekong Tributaries: Japanese Govt., Reconnaissance team Rept., September, 1961.

Parker, M. L., and Henock, W. E. S., 1971, The use of Englemann Spruce latewood density for dendrochronological purposes: Canadian Jour. Forest Research, v. 1, no. 2, p. 90-98.

Sigafoos, R. S., 1964, Botanical evidence of floods and flood plain deposits: U.S. Geol. Survey Prof. Paper 485-A, 35 p.

Speight, J. G., 1965, Meander spectra of the Angabunga River: Jour. Hydrology, v. 3, p. 1-15.

Von Schelling, H., 1951, Most frequent particle paths in a plane: Am. Geophys. Union Trans., v. 32, p. 222-226.

——— 1964, Most frequent random walks: Schenectady, N.Y., General Electric Co. Rept. 64GL92, p. 8.

Yen, B. C., 1965, Characteristics of subcritical flow in a meandering channel: Univ. Iowa Inst. Hydraulic Research p. 14.