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DISCUSSIONS

A DIGITAL COMPUTER PROCEDURE FOR PREPARING BETA DIAGRAMS*

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INTRODUCTION

Robinson, Robinson, and Garland (1962, 1963) have recently described a procedure for preparing beta diagrams by the use of a digital computer. One of us (Noble) has been working on statistical techniques for the analysis of folded and rotated structures. In order to compare the results obtained with those obtained by the beta diagram technique, a procedure for computing the data necessary to construct beta diagrams has been developed independently by Noble and programmed for digital computer use by Eberly. In addition to the theoretical applications mentioned above, the procedure has been successfully used to analyze field data (Noble, ms).

The procedure here described differs principally from that of Robinson in that circular, axially symmetrical, counting areas are used. Based on the same principal as the standard manual method of counting-out, this method is theoretically more sound than the use of irregularly shaped counting compartments bounded by lines of equal azimuth and equal declension. The resultant diagrams are directly comparable to diagrams produced by hand counting. Indeed, because the slight bias introduced by the inherent distortion near the edges of an equal-area net is eliminated, the diagrams should, in theory, be slightly more accurate than those counted by hand.

In addition, the procedure differs in that the pattern produced by the centers of the counting areas does not have the pronounced axial symmetry of the array used by Robinson (1963, fig. 2C).

COORDINATE NOTATION

Computer calculations are made using vectors described by direction cosine notation. Input data in spherical coordinates using the angles ϕ and θ are converted to direction cosine notation by the computer using the relations

$$X = \sin\phi \cdot \cos\theta \qquad (1),$$
$$Y = \sin\phi \cdot \sin\theta \qquad (2),$$
$$Z = \cos\phi \qquad (3),$$

where ϕ is the angle measured from the z axis and θ is the angle measured clockwise from the x axis in the xy plane.

S VECTORS

The orientation of the form-surfaces used in constructing the beta diagram is described by positive unit vectors, termed S Vectors, each normal to

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a given form-surface. The S Vectors are described by spherical coordinates using the angles ϕ and θ. Strike and dip information can easily be converted to the spherical coordinate notation used by the convention:

ϕ = angle of dip of form-surface, measured from the horizontal,

θ = direction of dip of form-surface, measured clockwise from north.

Both ϕ and θ are positive.

BETA VECTORS

Beta Vectors are the mathematical equivalents of the points of intersection of the great circles drawn in cyclographic projections. Since a given Beta Vector is perpendicular to both of the S Vectors that generate it, Beta Vectors can be computed by solving the three simultaneous equations:

$$X_{s_i} \cdot A + Y_{s_i} \cdot B + Z_{s_i} \cdot C = 0 \qquad (4),$$

$$X_{s_j} \cdot A + Y_{s_j} \cdot B + Z_{s_j} \cdot C = 0 \qquad (5),$$

$$A^2 + B^2 + C^2 = 1 \qquad (6),$$

for A, B, and C where X_{s_i}, Y_{s_i}, Z_{s_i}; X_{s_j}, Y_{s_j}, Z_{s_j} are the direction cosines of the two S vectors and A, B, and C are the direction cosines of the resultant Beta Vector. For n S Vectors, no two of which are identical, there are $n(n-1)/2$ Beta Vectors. A Beta Vector cannot be computed for two identical S Vectors. For this reason, S Vector pairs with identical direction cosine values are rejected, and the total number of Beta Vectors is reduced by one for each pair of identical S Vectors.

G VECTORS AND COUNTING PROCEDURE

The counting procedure is analogous to the method used in graphic preparation of petrofabric diagrams (Turner and Weiss, 1963). An array of 351 positive unit vectors, termed G Vectors, is used to represent points dis-

TABLE 1

Table of G Vector groups and Beta Vector intervals

G Vector groups			Beta Vector intervals		
Group (K)	ϕ	$\Delta\theta$	Interval	Intervals(ϕ)	Intervals($\cos\phi$)
	0°	0°			
1	7.5	60	A	0°00′ - 23°06′	1.0000 - 0.9197
	15	30			
2	22.5	20	B	14°24′ - 30°36′	0.9687 - 0.8606
3	30	15	C	21°54′ - 38°06′	0.9279 - 0.7868
4	37.5	12	D	29°24′ - 45°36′	0.8714 - 0.6995
5	45	10	E	36°54′ - 53°06′	0.7999 - 0.6002
6	52.5	9	F	44°24′ - 60°36′	0.7147 - 0.4907
7	61	9	G	52°54′ - 70°06′	0.6034 - 0.3565
8	70	7.5	H	61°54′ - 78°06′	0.4713 - 0.2059
9	80	6	J	71°54′ - 90°00′	0.3110 - 0.0000
10	90	5	K	81°54′ - 90°00′	0.1412 - 0.0000

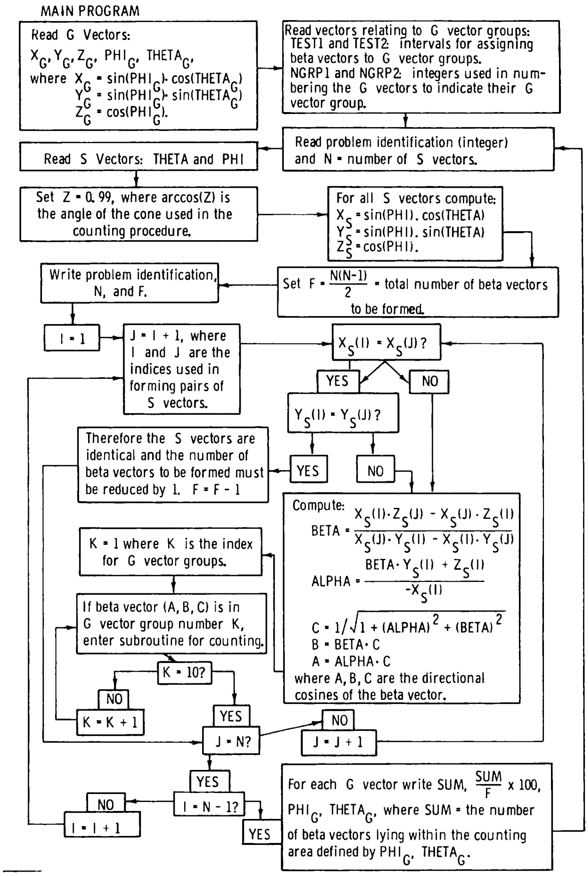

Fig. 1. Flow sheet for the main part of the FORTRAN program.

tributed approximately equally over the surface of the hemisphere.[1] Vector density is slightly greater than the equivalent density obtained by using a centimeter grid in conjunction with a 20 cm net. The orientation of the vectors used to form the net is given in table 1.

The G Vectors are used as the axes of cone-shaped volumes to determine the relative density of Beta Vectors over the hemisphere. It can easily be shown that a cone subtending one-hundredth of the solid angle value represented by a hemisphere is generated when the generating line is separated by an angle of arccos 0.99 from the axis of the cone. Thus, if a Beta Vector is separated from a G Vector by an angle of less than arccos 0.99, it will fall within a 1 percent portion of the hemisphere centered on the given G Vector. If the Beta Vector is separated from the G Vector by an angle of greater than arccos 0.99 it will lie outside the 1 percent area.

[1] A vector array in which the vectors are distributed with exactly equal density can undoubtedly be formulated. Such a net would be esthetically more pleasing than the one used here, but it would not contribute significantly to the accuracy of the calculations.

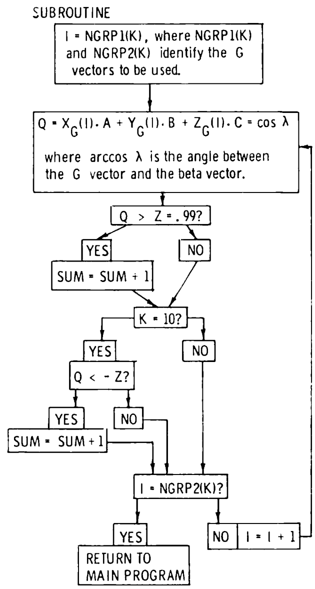

Fig. 2. Flow sheet for the subroutine of the FORTRAN program.

The comparison of Beta and G Vectors is performed by multiplying the direction cosine values of the Beta Vector by those of the G Vector according to the equation:

$$X_G \cdot A + Y_G \cdot B + Z_G \cdot C = \cos\lambda \qquad (7),$$

where arccos λ is the angle between the G Vector and the Beta Vector. Cos λ is compared with the number 0.99 to determine whether or not the Beta Vector lies within the 1 percent counting area.

G Vectors on the equator of the hemisphere ($\theta = 90°$) present a special problem, as the areas represented by these particular vectors are half that of the other areas. This is overcome by combining diametrically opposed half-areas on the equator. This is done mathematically by allowing that a given Beta Vector lies within the counting area if cos λ is within either the interval [–1.0, –0.99] or the interval [0.99, 1.0].

To speed computation, the G Vectors have been divided into ten G Vector groups (table 1). With the exception of group one, all the vectors in a given group have identical values of Z_G. Any Beta Vector falling within a 1 percent counting area centered on any G Vector in a given group will have a value of C in the interval:

$$[\cos(\arccos C + \arccos 0.99), \cos(\arccos C - \arccos 0.99)]$$

Using this relation, 10 Beta Vector intervals, each corresponding to a G Vector group, have been calculated (table 1). In machine computation, A Beta Vector is placed in the appropriate Beta Vector interval(s) before comparison is made using equation (7).

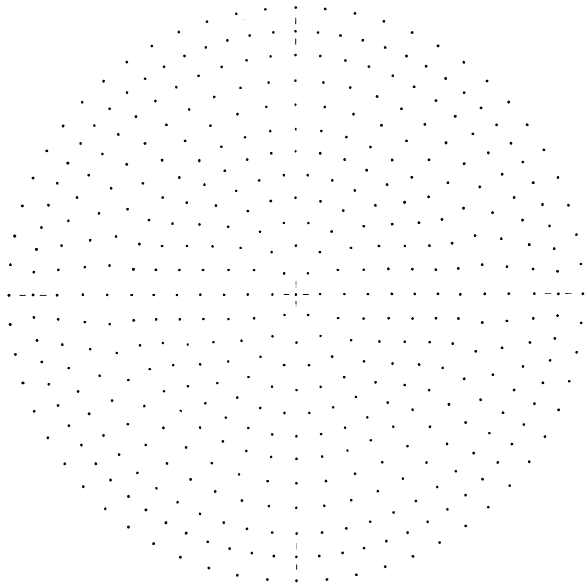

Fig. 3. G Vector template for contouring beta diagram density data. Template constructed from data in table 1 using the Lambert equal-area projection.

DIGITAL COMPUTER COMPUTATION

The procedure described above has been programmed in FORTRAN for use on the IBM 7090 digital computer. Flow sheets for the program are given in figures 1 and 2. The complete FORTRAN program and the format for the G Vector, S Vector, and identification cards may be obtained from the American Documentation Institute, Washington, D. C.[2]

The data contained in the IBM punched card deck, which consists of, in order, (a) program cards, (b) G Vector cards, (c) two TEST cards which contain data defining the Beta Vector intervals, (d) two NGRP cards which contain integers used in numbering the G Vectors to indicate the group in which they belong, (e) an identification card, and (f) S Vector cards, are placed on magnetic tape and read into the computer. Direction cosine values are computed for the S Vectors using equations (1), (2), and (3). The value of $n(n-1)/2$ ($=F$) is then computed. After checking that the two vectors are not identical, a Beta Vector is computed using the first two S Vectors by solving equations (4), (5), and (6). Using direction cosine C, the Beta Vector interval(s) in which it belongs. A Beta Vector is then combined with all the G Vectors in the associated G Vector Group(s) using equation (7), a record being made when $\cos \lambda$ falls within the interval [0.99, 1.0], or in the interval [–1.0, –0.99] for G Vector Group 10. The remaining Beta Vectors are computed and processed in the same manner. When this has been done, the number of Beta Vectors lying within arccos 0.99 of each G Vector has been ob-

[2] To secure this material, order Document 8048 from the *Chief, Photoduplication Service, Library of Congress, Washington 25, D.C., Auxiliary Publication Project*, remitting $1.25 for microfilm (35 mm) or $1.25 for photocopies.

tained. Each sum of Beta Vectors is then divided by the total number of Beta Vectors, $n(n-1)/2$ minus the number of pairs of identical S Vectors, and multiplied by 100 to give a percentage.

For each G Vector the total number of Beta Vectors associated with it (SUM) and the percentage of the total number of Beta Vectors this represents (SUM x 100/F) are printed followed by ϕ and θ of the given G Vector. The G Vector data are preceded by the identification of the particular S Vector set and the integers N and F.

Problem size is limited only by the size of the memory bank of the computer used and by the amount of machine time available. A computation with 100 S Vectors consumes less than 2 minutes of machine time on the IBM 7090 computer, using a binary deck previously compiled.

CONTOURING

The percentages (SUM x 100/F) obtained from the computer are plotted by hand on an equal area G Vector projection (fig. 3)[3] and contoured in the conventional manner. To obtain control at the margins of the hemisphere, the percentages for $\phi = 80°$ are also plotted on the outermost ring of points diametrically opposite (plus or minus 180°) from their position on the 80°G Vector ring. As all G Vectors having $\phi = 90°$ lie on a great circle, the beta percentage values of G Vectors with $\phi = 90°$ and θ differing by 180° are equal. The resultant beta diagram is an upper hemisphere projection; a lower hemisphere projection is obtained by rotating the diagram 180° in the plane of the projection.

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[3] The G Vector projection is constructed from the data given in table 1. On the Lambert equal-area projection the radius of a small circle of co-latitude ϕ is equal to

$$R \sin \phi / \sqrt{2}$$

where R is the radius of the great circle $\phi = 90°$. A limited number of projections 20 cm in diameter have been prepared and are available from the senior author on request.