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A THEORY OF SIMILAR FOLDING IN VISCOUS MATERIALS

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ABSTRACT. A sequence of horizontal layers of alternating stiff and soft viscous materials will deform under horizontal compression both by shortening and thickening and by buckling. The relative importance of the two effects is determined by the viscosities, thicknesses, and inclination of the layers. Experiments on models suggest that the folding will be similar; then the inclination of the beds will be constant over the whole length of a limb. If for a given period of deformation, α_1 and α_2 are the initial and final inclinations, and R_1 and R_2 the initial and final lengths of any section of a limb, then calculations assuming constant viscosity give

$$\left(\frac{R_2}{R_1}\right)^2 = \left(\frac{E + C\cos 2\alpha_2}{E + C\cos 2\alpha_1}\right)^{E/E+C} \cdot \left(\frac{\sin\alpha_1}{\sin\alpha_2}\right)^{E-C/E+C} \cdot \frac{\cos\alpha_1}{\cos\alpha_2}$$

where E and C are constants depending only on viscosities and thicknesses. This permits the total compression to be split into a part taken up by rotation of the layers and a part taken up by shortening and thickening. Further, each layer undergoes a shear deformation

$$\beta = \arctan\left[-2\tan\alpha_2/\mu(E + C)\right],$$

valid for $\alpha_1 << \alpha_2 < 60°$ and $\mu_2 << \mu_1$ where μ = viscosity of the layer. Thirdly, the principal stresses within a layer are inclined to the layer at angles $\theta = \frac{1}{2}\arctan[\tan 2\alpha/\mu(E\text{-}C)]$. The first and third equations can be modified to take account of (1) compression not perpendicular to the axial planes, (2) anisotropic materials of a laminar kind.

INTRODUCTION

Goguel (1948) has discussed folding of rocks on the assumption that behavior is ideally plastic. In this paper a calculation is presented on the assumption that behavior is ideally viscous. The purpose is not to advocate any view as to which ideal is more closely approached by actual rocks. Rather, it is to pursue the geometrical consequences of the viscous assumption, with a view to finding a criterion by which the assumption can be tested.

The calculation concerns an idealized analogue of a rock undergoing similar folding. Ideally, subsequent observations will show what differences exist between real rocks and the analogue. Such a conclusion is not even in sight at present, but the calculation is intended as a contribution to progress in this direction.

THE ANALOGUE

The calculation relates to an extended sequence of alternating layers in which similar folding has already been initiated. The following assumptions are made:

1. The sequence is a ..12121212.. sequence in which all layers of set 1 are identical, and all layers of set 2 are identical with each other but different from those in set 1.

2. The material of both sets is homogeneous, incompressible, and Newtonian, so that once viscosities μ_1 and μ_2 are assigned, the difference between the materials of the two sets is fully described.

(It subsequently proves possible to adapt the calculation to anisotropic materials with uniaxial symmetry, such as flaky rocks might imitate, but even

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then it is still assumed that the viscosity coefficients are constant with respect to both stress and strain.)

3. Cohesion is maintained at all interfaces.

4. Within any layer, departures from affine deformation are negligible over the greater part of each limb, that is, particles which are in a straight line at one moment remain in a straight line at subsequent moments of the deformation.

It is well known that in rocks material from the softer layers migrates into the fold-noses more actively than material from the stiffer layers. Presumably the flow is more rapid in the center of the soft layers so that particles forming a straight line across the layer at one moment form a curve at later moments. Assumption (4) requires that this effect be negligible, a condition that is satisfied when the length of the straight limbs is large in comparison with the length of the curved nose-section.

Other kinds of non-affine deformation are also conceivable; but observations on kink-bands suggest that in the geologic situation the calculation simulates, affine deformation is commonly approximated. The kink-bands, that is, transverse bands in which the mica flakes have a uniform orientation slightly different from that outside the bands, commonly have straight edges, except in the nose regions where slight curvature attributable to the flow just discussed is often seen. Such consistently straight edges are unlikely to be preserved if deformation is not effectively affine.

5. Strain is confined to planes perpendicular to the fold axes. If all properties of the assemblage are uniform parallel to the fold axes and the assemblage is infinite in this direction, the condition is necessarily fulfilled by the analogue; the extent to which rocks behave as if infinite and uniform is a question not within the scope of this paper.

Using assumption (4), it is convenient to partition the assemblage for descriptive purposes into limb regions and nose regions, limb regions being regions where departures from affine deformation are negligible and nose regions being regions where they are not. The limb regions are bounded by lines such as PQPQ in figure 1. As deformation proceeds, the lengths PQ and their inclinations change, subject only to the condition that PP and QQ remain parallel to each other and to their initial direction; otherwise folding will not

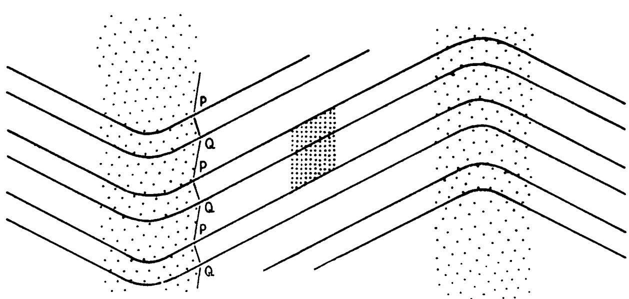

Fig. 1. Similar folding of the kind discussed, showing division into limb regions (unornamented) and nose regions (open stipple). The close-stippled area is shown again in figure 2.

be similar. The calculation concerns such changes within the limb regions only; the nose regions are not discussed.

Geometry of the deformation.—Orthogonal axes Ox and Oy will be used, such that Oy is parallel to QQ. Consider an infinitesimal deformation of the close-stippled area in figure 1 (enlarged in fig. 2). The conditions of incompressibility and plane strain require areas in the xy plane to be conserved.

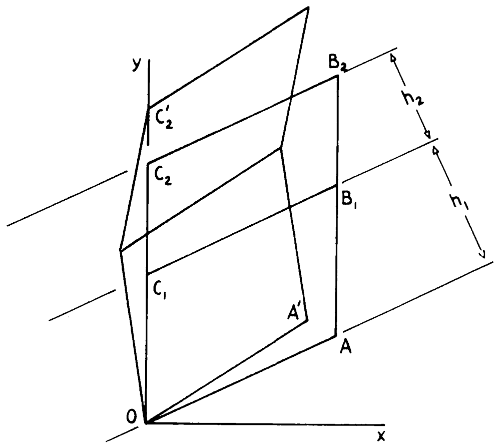

Fig. 2. A representative area of a limb profile before a slight deformation ($OAB_1C_1B_2C_2$) and after.

Then the most general deformation of OAB_1C_1 can be analyzed into a pure shear with OA as a principal axis, a simple shear parallel to OA, and a rotation. Let the pure shear strain be $d\varepsilon$ along OA ($d\varepsilon$ positive when OA increases); let the simple shear strain be $d\gamma_1$ (γ_1 positive when anticlockwise): and let the rotation be $d\alpha$ ($\alpha = \angle AOx$, positive when measured anticlockwise from OA). The three displacements of C_1 due to these changes have components parallel to Ox given by $2h_1$, $d\varepsilon \sin\alpha$, $-h_1 d\gamma_1 \cos\alpha$, and $-h_1\, d\alpha \sec\alpha$ respectively. Now if $B_1C_1B_2C_2$ is similarly deformed by strains $d\varepsilon$, $d\gamma_2$, and $d\alpha$, the x-displacements of C_2 with respect to C_1 are $2h_2\, d\varepsilon \sin\alpha$, $-h_2\, d\gamma_2 \cos\alpha$, and $-h_2\, d\alpha$ $\sec\alpha$. The net x-displacement of C_2 with respect to 0 is therefore a combination of six terms, and to preserve QQ irrotational this net displacement must be zero. Rearranging the six terms gives

$$2(h_1 + h_2)\, d\varepsilon \sin\alpha - (h_1\, d\gamma_1 + h_2\, d\gamma_2) \cos\alpha - (h_1 + h_2)\, d\alpha \sec\alpha = 0 \quad (1).$$

Internal stress distribution.—We now ask: what stresses must be present in the limbs for the deformation to follow the geometrical specifications given?

Let the state of stress at a point within OAB_1C_1 be specified by σ_s, σ_{p1} and τ as in figure 3. If deformation is to be affine, the strain rates $d\varepsilon/dt$ and $d\gamma/dt$ must be the same at every point in OAB_1C_1. If forces due to gravity and inertia are neglected and the flow is planar and incompressible, these in turn require that σ_s, σ_{p1}, and τ be constant throughout OAB_1C_1. This may be verified formally using equations §19.(8) and (13) and §39.(3) and (4) in Jaeger (1962, p. 71, 139).

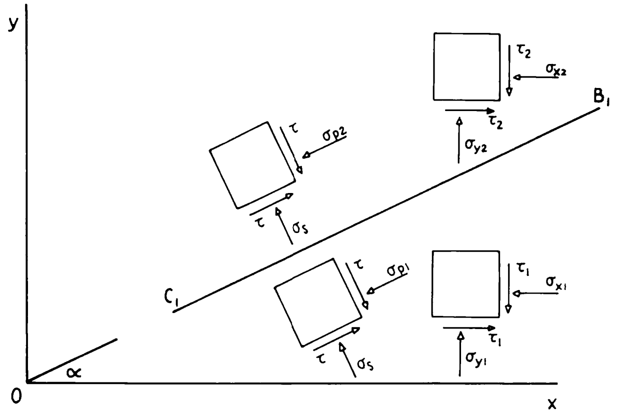

Fig. 3. Alternative descriptions of the state of stress on either side of B_1C_1 in figure 2. The stresses shown are those which must be imposed by the environment on the sample squares outlined, in order to produce the deformation discussed.

If the interface remains coherent, it also follows that the shear stress and perpendicular stress are identical either side. The state of stress within $B_1C_1B_2C_2$, which is similarly uniform, can therefore be specified by σ_s, σ_{p2}, and τ. Coherence at the interface also requires that $d\varepsilon/dt$ is the same in both sides so that σ_{p1} and σ_{p2} are related by

$$\frac{\sigma_s - \sigma_{p1}}{\mu_1} = \frac{4\,d\varepsilon}{dt} = \frac{\sigma_s - \sigma_{p2}}{\mu_2}$$

or

$$\mu_2\,(\sigma_s - \sigma_{p1}) = \mu_1\,(\sigma_s - \sigma_{p2}).$$

$\sigma_s - \sigma_{p1} = 4\mu_1 \dfrac{d\varepsilon}{dt}$ is derived by applying Jaeger's equations §9.(13) (1962, p. 71) to plane strain of an incompressible fluid.

External stresses.—We wish to use these properties of the stress distribution to solve equation (1) for $d\varepsilon$, $d\gamma_1$, and $d\gamma_2$ separately in terms of α. At once we have $\mu_1\,d\gamma_1 = \tau\,dt = \mu_2\,d\gamma_2$, enabling, say, $d\gamma_2$ to be eliminated; but the relation between $d\varepsilon$ and $d\gamma_1$ depends on the relation between the difference $\sigma_s - \sigma_{p1}$ and τ. It is the purpose of this section to show that the relation between these two is fixed by the orientation of the principal stresses of the external stress field to which the whole layered assemblage is subject.

The relation of the internal and external stress fields is illustrated in figure 4, a representative part of which has already been shown in detail in figure 3. The mechanism by which the uniform stresses Σ_x, et cetera, are modified to the discontinuous stresses σ_{x1}, σ_{x2}, et cetera, although interesting, is not relevant to the present calculation. Resolving Σ_x, Σ_y, and T we have firstly—

$$\Sigma_x = \frac{h_1\,\sigma_{x1} + h_2\,\sigma_{x2}}{h_1 + h_2},$$

$$\Sigma_y = \frac{h_1\,\sigma_{y1} + h_2\,\sigma_{y2}}{h_1 + h_2},$$

$$T = \frac{h_1\,\tau_1 + h_2\,\tau_2}{h_1 + h_2};$$

secondly,

$$\sigma_{x1} = \sigma_s \sin^2 \alpha + \sigma_{p1} \cos^2 \alpha - 2\tau \sin \alpha \cos \alpha,$$

$$\sigma_{y1} = \sigma_s \cos^2 \alpha + \sigma_{p1} \sin^2 \alpha + 2\tau \sin \alpha \cos \alpha,$$

$$\tau_1 = \tau \cos 2\alpha - \frac{1}{2}(\sigma_s - \sigma_{p1}) \sin 2\alpha,$$

and similarly for σ_{x2}, σ_{y2}, and τ_2. If ϕ is used for the inclination of the maximum external stress to Ox so that $T = \frac{1}{2}(\Sigma_x - \Sigma_y) \tan 2\phi$ (ϕ positive when measured anticlockwise from Ox) and M is used for a weighted mean viscosity defined as $(\mu_1 h_1 + \mu_2 h_2)/(h_1 + h_2)$, rearrangement gives

$$\frac{\tau}{\sigma_s - \sigma_{p1}} = \frac{M}{2\mu_1}.\tan(2\alpha - 2\phi) \qquad (2).$$

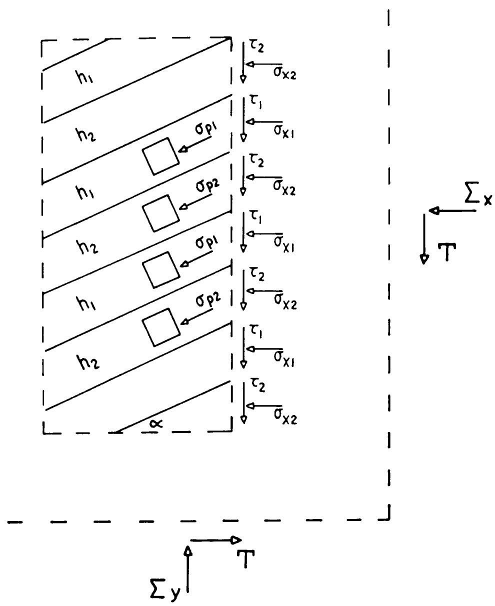

Fig. 4. Summary of stresses in several adjacent layers. Although $\sigma_{x1} \neq \sigma_{x2}$, the stresses shown could generate approximately uniform stress Σ_x at a distance that is large compared with h_1 and h_2 (Ramberg, 1960).

RESULTS

Since $d\gamma_1/d\varepsilon = 4\tau/(\sigma_s - \sigma_{p1})$, equation (2) may be used to express $d\gamma_1$ in terms of $d\varepsilon$. For any value of ϕ, therefore, the geometrical equation (1) can be used to express any one of the variables $d\varepsilon$, $d\gamma_1$, and $d\gamma_2$ in terms of $d\alpha$. In accordance with assumption (2), the geometrical properties of this infinitesimal deformation are not affected by the time over which it takes place: doubling the external stresses will double the internal stresses and halve the time for any specified small change. This does not prevent the four geometrical variables from being expressed in terms of time; it only leads us to expect that the same stress quantity $\Sigma_x - \Sigma_y$ will occur as a factor in all the expressions. The remainder of the text simply sets out some of the results derivable algebraically on the foregoing foundations.

For further condensation, let

(I) $\Sigma_x - \Sigma_y = \Sigma$,
(II) $T/\Sigma = u$ so that $\tan 2\phi = 2u$,
(III) $T \tan 2\alpha = \Delta$.

Then,
$$d\alpha = \frac{\Sigma}{4}\left\{E \sin 2\alpha + C \sin 2\alpha \cos 2\alpha - 2u\,(E-C) \sin^2 2\alpha - 2uE \cos^2 2\alpha - 2uE \cos 2\alpha\right\} dt \tag{3}$$

$$d\varepsilon = -\frac{\Sigma}{4M}(\cos 2\alpha + 2u \sin 2\alpha)\, dt \tag{4}$$

$$d\gamma_1 = -\frac{\Sigma}{2\mu_1}(\sin 2\alpha - 2u \cos 2\alpha)\, dt \tag{5}$$

$$\sigma_s = \sin^2\alpha\,(\Sigma_x + \Delta) + \cos^2\alpha\,(\Sigma_y - \Delta) \tag{6}$$

$$\sigma_{p1} = [A(\Sigma_x + \Delta) + B(\Sigma_y - \Delta)]/[h_1\mu_1 + h_2\mu_2] \tag{7}$$

$$\tau = -\frac{\Sigma}{2}(\sin 2\alpha - 2u \cos 2\alpha) \tag{8}$$

and if θ = inclination of the principal stresses within a layer to the direction of the interfaces,
$$\theta_1 = \frac{1}{2}\arctan\left[\frac{M}{\mu_1}\tan(2\alpha - 2\phi)\right] \tag{9}$$

where $A = \cos^2\alpha\,\mu_1(h_1 + h_2) - \sin^2\alpha\, h_2(\mu_1 - \mu_2)$,
$B = \sin^2\alpha\,\mu_1(h_1 + h_2) - \cos^2\alpha\, h_2(\mu_1 - \mu_2)$,
$C = E - 1/M$,
$E = (h_1\mu_2 + h_2\mu_1)/\mu_1\mu_2(h_1 + h_2)$.

Corresponding results for set 2 of the assemblage are generated from equations (5), (7), and (9) by interchanging 1 and 2 in all suffixes.

The physical significance of C and E deserves comment. E is a weighted mean fluidity analogous to the weighted mean viscosity M, using fluidity as a reciprocal of viscosity. C may be regarded as a fluidity contrast—as μ_1 approaches μ_2, C approaches zero, and as one set of layers becomes negligibly fluid, C approaches the fluidity of the other set, modified by the thickness ratio.

It is clearly a convenience to discuss situations in which $u = 0$, that is in which $\phi = 0$ and the maximum external stress is perpendicular to the axial planes, as a special case of these general relations.

Compression and rotation in the orthogonal situation.—When $u = 0$, equations (3) and (4) give $d\varepsilon = -(\cot 2\alpha)\, d\alpha/M(E + C \cos 2\alpha)$. We wish to derive the finite strain that accompanies a finite change in α. Since the net result of two successive finite strains involves their product as well as their sum, $d\varepsilon$ is not a suitable quantity for integration; but we may replace $d\varepsilon$ by dR/R where R is the distance between any two particles whose join is parallel to the layering. Changes in R being purely additive, a finite change is obtained exactly upon integration. Thus, writing $\int_1^2$ for integration over a finite change from state 1 to state 2, $\int_1^2 dR/R = [\log R]_1^2 = \log(R_2/R_1)$.

To perform the corresponding operation on the right hand side of the equation for $d\varepsilon$, it is necessary to note that M, E, and C do not change with α. M, E, and C involve the layer thicknesses that do change with α, but only in the form of the thickness-ratio, which, in the circumstances considered, is constant. The result,

$$[\log R]_1^2 = \frac{1}{2}\left[\frac{E}{E+C}\log\,(E + C\cos 2\alpha) - \frac{E-C}{E+C}\log\sin\alpha - \log\cos\alpha\right]_1^2,$$

is conveniently rewritten

$$\left(\frac{R_2}{R_1}\right)^2 = \left(\frac{E + C\cos 2\alpha_2}{E + C\cos 2\alpha_1}\right)^{E/E+C} \cdot \left(\frac{\sin\alpha_1}{\sin\alpha_2}\right)^{E-C/E+C} \cdot \left(\frac{\cos\alpha_1}{\cos\alpha_2}\right) \quad (10).$$

Graphically, this result can be shown by the locus of a point whose polar coordinates are R, α as in figure 5. In geological terms, if one point on a bedding plane is used as the reference point 0, then another point on the same bedding plane will travel past 0 along a curve such as those drawn, so that the distance between them diminishes in the early stages when the folding is open and increases again when the folding is sufficiently acute. The changes with time, plotted in figure 6 for one of the type-cases in figure 5, show that the rate of rotation has a well-defined maximum somewhat before the inclination of 45° is reached. A feature of figure 5 is that it does not assume Σ = constant, although it does assume that $\phi = 0$ throughout, that is, the principal stresses remain fixed in direction. To plot figure 6, Σ has been assumed constant.

Compression and rotation in oblique situations.—When $u \neq 0$, that is, when folding is in progress on axial planes not perpendicular to the maximum external stress—a probable situation in secondary folding on the limbs of primary folds, for example—then equations (3) and (4) are less easy to manipu-

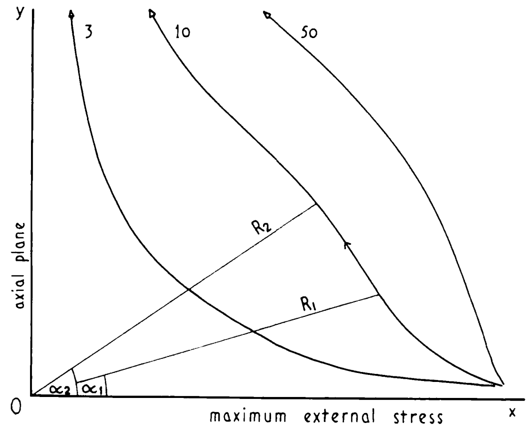

Fig. 5. Change of length of a fold limb with change of inclination, in a situation where the maximum external stress is perpendicular to the axial plane. Curves are calculated for layers of thickness ratio ½, viscosity ratio 50, 10, and 3 as marked. All curves are asymptotic to Ox in accordance with the fact that, the straighter the layers are initially, the greater the compression they will suffer in folding to a given angle.

late. However. when the viscosity contrast between the sets of layers is large, C approaches E and approximately

$$d\alpha = \frac{\Sigma E}{4}(1 + \cos 2\alpha)(\sin 2\alpha - 2u \cos 2\alpha)\ dt \qquad (11).$$

If this approximate t, α relation is combined with the exact t, $d\varepsilon$ relation (4), an integrated result relating a finite change in length to a change in α is obtainable. as at (10), (but at this point it is more concise to express log R than to write all the terms as exponents):

$$\log R = \frac{1}{2ME}\left\{ r + \log\left|\frac{1 + \cos 2\alpha}{\sin 2\alpha - 2u \cos 2\alpha}\right|\right\} + k \qquad (12),$$

where

$$r = \frac{1}{4u^2}\log_e\left|\frac{1 + \cos 2\alpha}{\sin 2\alpha - 2u \cos 2\alpha}\right| + \frac{\tan\alpha}{2} - \frac{\sqrt{4u^2 - 1}}{2u^2}\cdot \begin{matrix}\tanh^{-1}\\ \coth^{-1}\end{matrix}\left(\frac{1 + 2u \tan\alpha}{\sqrt{4u^2 + 1}}\right) \qquad (13)$$

(except when u = 0), and $t = 4r/\Sigma E$. In the expression for r, the $\tanh^{-1}$ or $\coth^{-1}$ is used according as the argument is greater or less than one.

Curves based on these equations are given in figures 7 and 8. As with the t, α relation, the curves in figure 8 are exact only when C = E, whereas the curves in figure 7 result from applying the t, α data to the situation $h_1 = \frac{1}{2} h_2$, $\mu_1 = 10\mu_2$, when C = 0.64E. However, the broken line in figure 6 shows that the t, α curves for the orthogonal situation are fairly insensitive to changes in C/E or μ_1/μ_2; so figure 7 may not be far from the exact behavior for the μ, h values discussed.

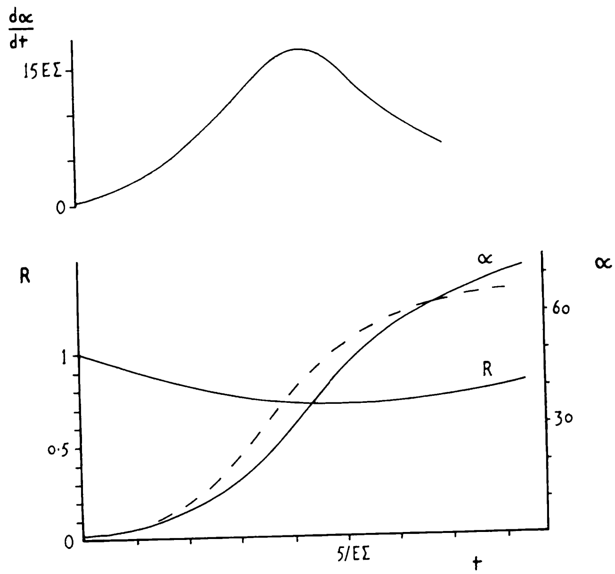

Fig. 6. Development of a fold with time, where the maximum external stress is perpendicular to the axial plane, calculated for thickness ratio ½, viscosity ratio 10. (R and α as in fig. 5, Σ = stress difference $\Sigma_x - \Sigma_y$, E = mean fluidity defined on p. 758). The broken line shows the change of α with t for very large viscosity contrasts (see text, p. 760).

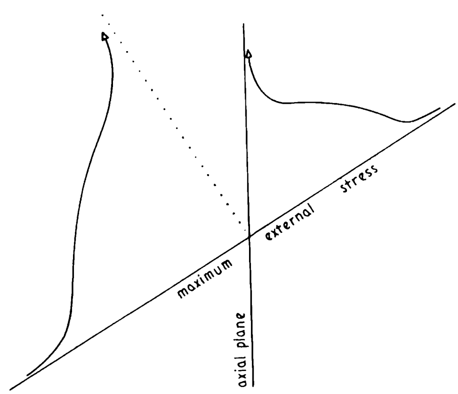

Fig. 7. Change of length of a fold limb with change of inclination, in a situation where the maximum external stress is oblique to the axial plane. Curves are calculated for layers of thickness ratio ½, viscosity ratio 10, and obliquity 31° as an example. The open sector signifies that as long as the angle between the axial planes and the maximum external stress remains constant, the folds will not close; in reality, this angle is likely to change as acuteness becomes extreme.

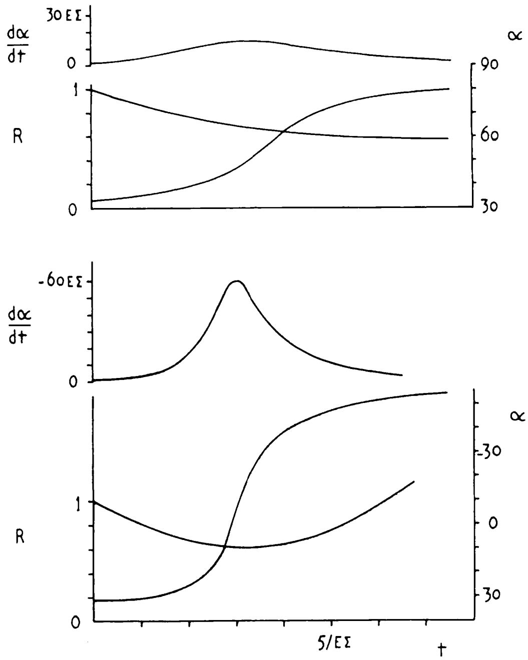

Fig. 8. Development of the same fold as in figure 7 with time, notation as in figure 6. On one limb rotation is slow and compression is prolonged; on the other limb rotation, once initiated, proceeds rapidly, and thereafter the limb suffers extension.

Compression and rotation in anisotropic materials.—Up to this point, μ_1 has been used to relate both compressive stress to compressive strain and shear stress to shear strain. While valid if the medium envisaged is isotropic, this double use is not appropriate for anisotropic media. Following Jaeger's rigorous approach to uniaxially anisotropic media (1962, p. 65 [ii]) but transforming from elastic to viscous media and continuing to assume plane strain, we find a second coefficient necessary and sufficient: μ will still be used to relate compressive stress and strain, but G is now introduced to relate shear stress to shear strain: thus for example

$$\frac{d\varepsilon}{dt} = \frac{\sigma_s - \sigma_{p1}}{4\mu_1} \qquad \text{as before}$$

but $\qquad d\gamma_1/dt = \tau/G_1.$

Of the equations (3) to (5), (4) is unchanged by the transformation and (5) is transformed by simply writing G for μ. (3), however, involving both compression and shear, transforms to

$$d\alpha = \frac{\Sigma}{4} \sin 2\alpha [E' + C' \cos 2\alpha]$$

$$\text{where } E' = \frac{h_1 G_2 + h_2 G_1}{G_1 G_2 (h_1 + h_2)}, \; C' = \frac{h_1 G_2 + h_2 G_1}{G_1 G_2 (h_1 + h_2)} - \frac{h_1 + h_2}{h_1 \mu_1 + h_2 \mu_2} \qquad (14).$$

Bearing in mind that E, C, and M are constants, we conclude that all results stated for isotropic media carry over directly to axially anisotropic media by putting E′ for E and C′ for C. Moreover, all assemblages with the same value for E′/C′ will follow the same paths in the R, α diagram. For the orthogonal situation, therefore, figure 9 is a summary of all possible travel paths. For

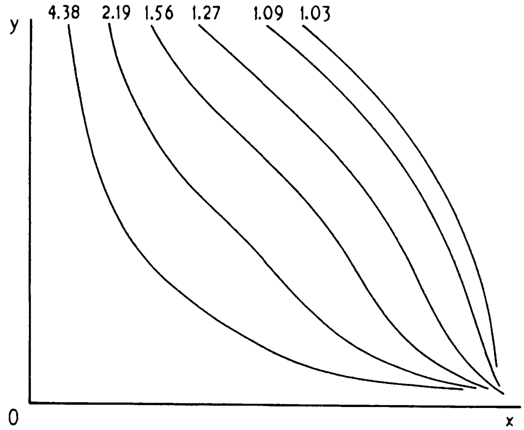

Fig. 9. Change of length of limb with inclination, where maximum external stress is perpendicular to the axial plane, generalized for uniaxially anisotropic layers of any thickness ratio. Each curve is marked with the value of E′/C′ to which it is appropriate; E′ and C′ are defined on p. 762.

any combination of μ_1, μ_2, G_1, G_2, h_1, and h_2, the ratio E′/C′ can be calculated and an appropriate curve interpolated.

It will be remembered that some results for oblique situations were quoted only for $\mu_1 >> \mu_2$ so as to ensure $E/C \cong 1$. It should be noted that with anisotropic media, the condition that $E'/C' \cong 1$ no longer requires $\mu_1 >> \mu_2$: it is more important that μ_1 and $G_1 >> G_2$. The results therefore seem particularly likely to be applicable to assemblages of alternating sandy and micaceous layers.

Amounts of shear.—Equations (3) and (5) in the orthogonal situation $u = 0$ give $d\gamma_1 = 2\, d\alpha/\mu_1[E + C \cos 2\alpha]$. If there is any kind of marker-line across the bedding whose inclination to the perpendicular to the bedding is β_1, then $d\gamma_1 = d(\tan \beta_1)$ and

$$\tan \beta_1 = -\frac{2}{\mu_1}\int d\alpha/[E + C \cos 2\alpha]$$

$$= \frac{-2}{\mu_1\sqrt{E^2 - C^2}} \arctan\left(\sqrt{\frac{E - C}{E + C}} \tan \alpha\right) + K$$

if the effect of compression on β_1 is for the moment neglected. For $X < 0.4$, $\arctan X = X$ within 5 percent, and this condition is satisfied for example if $\alpha < 60°$ and $E/C < 1.1$. Within these limits, therefore,

$$\tan \beta_1 = K - 2 \tan \alpha/\mu_1(E + C)$$

and if $\beta = 0$ when $\alpha = 0$, $K = 0$ and (15).

$$\tan \beta_1 = -2 \tan \alpha/\mu_1(E + C)$$

Curves in figure 10 show the change of β_1 and β_2 with α under these assumptions.

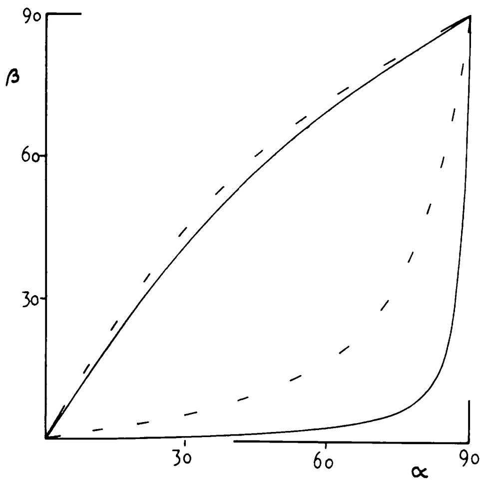

Fig. 10. The amount of shear suffered by the less competent layers (upper curve) and the more competent layers (lower curve) during rotation from near straight to inclination α. Curves are calculated for the orthogonal situation, with thickness ratio ½ and viscosity ratio 50, when $E/C = 1.09$. Curves are also shown (broken) for viscosity ratio 10, $E/C = 1.6$, although the approximations made are not good for such a large value of E/C.

(The initial condition here specified seems natural at least in the case where the axial plane of a minor fold on the limb of a major fold is accepted as a marker for shear of the major fold limb; see figure 11. The fact that β_2 is always a little greater than α is possibly related to the often-observed convergence of crenulation-cleavage toward the fold-noses in shaly layers.)

Equation 15 is adaptable to axially anisotropic materials by putting E' for E and C' for C, and figure 9 shows that for $E'/C' < 1.1$ as specified, the amount of compression is so small that its effect on β probably is negligible. Equation 15 therefore appears suitable for comparison with observations on natural rocks. Unfortunately, equation 15 is more susceptible than any other result to error due to variation in thickness among the members of the set of layers. This matter is discussed more fully in the next section.

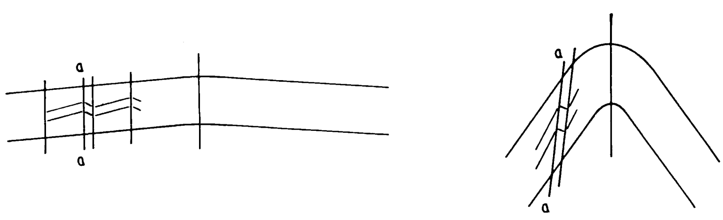

Fig. 11. The axial plane of minor folds on the limb of a major fold (a) at inception, when aa is likely to be approximately perpendicular to the bedding, (b) after accentuation of the major fold. If the axial plane aa is a valid marker for shear on the major-fold limb, its convergence toward the fold axis in (b) will be $\beta_2 - \alpha$ (whose value is 10° for $\alpha = 60°$ in the example shown in fig. 10).

DISCUSSION

Discussion is confined to the question: can the assumptions made about the analogue be plausibly extended to rocks? For answer, the assumptions are divided into two groups:

group 1: assumptions about visible properties—uniform thickness, et cetera—where departures from the assumption can be measured;

group 2: assumptions about invisible properties such as viscosity, incompressibility, cohesion, and plane strain.

Taking the second group first, emphasis is laid on the calculation as an impartial test. Its purpose is to discover whether these assumptions are valid, by making deductions from them that can be subjected to field test.

Naturally, the field test will be a good test of the assumptions in group 2 only if conditions agree with the assumptions in group 1. These are:

(A) Folding is similar. This actually embraces several assumptions, some of which are not critical to the calculation. It is not necessary to suppose the profile of all bedding planes to be congruent, but it is necessary that every alternate bedding plane be roughly congruent with its fellows; in particular, it is important that there be no systematic change of profile up the sequence, so that the axial planes remain parallel.

(B) The thickness of every member of either set is the same. This is the requirement most difficult to match in the field. Fortunately, the R, α relations are not sensitive to error in this respect—even if one layer is over-thick, it has to rotate in conformity with its neighbors and cannot rotate in accordance with its own erratic thickness. By contrast the β, α relation is sensitive to thickness errors; an overthick layer can and will shear through a different angle from its thinner fellows. Progress therefore requires that field check be concentrated on the R, α relation or that the β, α relation be modified to take account of the departure of one layer from the mean thickness for its set.

(C) Deformation is affine. It is to some extent possible to pick field examples where this property is demonstrable, for example by the traces of crenulation-cleavage. The curvature seen on these near fold noses may be due to non-affine deformation; at least, where no curvature is seen, affine deformation is prima facie acceptable.

(D) The sequence is separated by transitions, of negligible thickness, into layers of rock-type 1 and layers of rock-type 2, there being no difference in mechanical properties between one layer and another of the same set. It is possible to check a field exposure for sharpness of boundary between layers. It is not easy to establish identity of mechanical properties, so that to some extent the assumption of uniformity should appear in group 2; but it seems likely that exposures can be found where the assumption of uniformity is so little open to error, in comparison with the other assumptions of group 2, that its inclusion in group 1 is more realistic.

Finally it may be noted there is no expectation of showing that the viscosity of a rock is totally independent of stress; enough experimental work has been done to show that this is a most unlikely result. The question open to test is whether, over the range of stresses suffered by the rock during its main deformation episode, the viscosity was or was not effectively constant. The answer might well be as informative about the magnitude and behavior of the stresses as about the mechanical properties of the rocks.

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