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THE COOLING OF IRREGULARLY SHAPED IGNEOUS BODIES

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ABSTRACT. Numerical values are calculated for the temperature in a cooling extrusive sheet, on the assumption that the thermal properties of liquid and solid magma and country rock are the same. The progress of specified isotherms are illustrated for bodies of the following simple geometric shapes: a flow terminated at a vertical plane, a flow and an intrusive sheet with an abrupt change in thickness, a flow with nonuniform initial temperature, a flow and an intrusive sheet with a feeder. Judging from these data, the common assumption that joint columns in cooled igneous bodies are perpendicular to the isotherms is not always valid, especially in regions of sharp curvature of the isotherms and in the last regions to solidify.

Temperatures at the contact between magma and country rock are greatest at internal corners of the magma, and an increase in metamorphism may be expected as such a corner is approached. The effect of latent heat is to raise the temperature at the contact, to slow up the early stages of cooling, and to increase the asymmetry of the temperature distributions, but the pattern of the isotherms is not greatly altered. For calculations such as these, radiation at the upper surface of a flow and pre-heating of the country rock can apparently be neglected.

INTRODUCTION

Calculations of the temperatures in cooling intrusive and extrusive sheets have been in the literature for many years. Formulae for the extrusive sheet were given by Ingersoll and Zobel (1913) and were fully discussed by Lovering (1935), who also considered intrusions in the form of rectangular cylinders, rectangular parallelepipeds, and spheres, as approximations to stocks and laccoliths. In this early work it was assumed that the country rock was initially at a constant temperature which could be taken to be zero, and that the magma was initially at a uniform temperature T_o. A number of other simplifying assumptions were also made; namely: (i) the effect of the emission of latent heat was neglected, (ii) liquid and solid magma and country rock were all assumed to have the same thermal properties, (iii) the effect of the pre-heating of the country rock that would occur if magma flowed over it for some considerable time before quiescence was neglected, (iv) for extrusives the upper surface was assumed to be at zero temperature. These assumptions are in fact reasonably well justified for the purpose of obtaining a rough quantitative estimate of temperatures in a cooling igneous mass. If they are not made, alterations in detail occur; in fact, calculations allowing for all these effects can be made without difficulty, though at the price of additional complexity. This matter is discussed in the last section.

A rather different approach to the problem of the cooling sheet was made by Lane (1898), who used an approximate method that is, in effect, more complicated and less satisfactory than those referred to above, and his results are used by Tomkeieff (1940) in a discussion of columnar jointing. The discussion of jointing requires consideration of the cooling of masses of rock of relatively complicated forms. Following Iddings (1886, 1909) and James (1920) geologists have generally assumed that joint columns run normal to the isotherms in the cooling mass, and attempts have been made to estimate the shape of the isotherms, and hence of the intrusion, from directions of jointing. Tomkeieff has injected the additional suggestion that the rate of movement of the isotherms as well as their shape has an important effect on jointing.

Clearly, even for the development of qualitative ideas on jointing such as those mentioned above, some knowledge of the actual shape of the isotherms is needed, and, of course, calculation of temperatures is a necessary preliminary for calculation of thermal stress. The object of this paper is primarily to give accurate drawings of the isotherms in some simple special cases on the simple assumptions stated above. The effects of latent heat, pre-heating of the country rock and so on will usually be to raise temperatures (in a manner discussed in the last section) but they will not greatly affect the *pattern* of the isotherms. Simple rectangular shapes have been chosen since they lead to simple mathematical formulae. It may be expected, however, that if their corners were rounded off slightly (for example, to the shape of the curves $\tau = 0.01$ in the figures) the general pattern would not be changed significantly.

There is one further assumption which must be stated precisely; this is that cooling of the magma takes place by conduction and not by the development of systematic convection cells. One presumes that in thin sheets convection cells do not develop, but in large bodies such as the Skaergaard intrusion they certainly do; in sheets whose thicknesses are several hundred meters they might be expected, but field evidence for their existence is lacking. This paper explores the thermal consequences of the assumption that cooling is by conduction; the thermal consequences of the existence of convection cells would be different and must be studied independently. By comparing consequences of the two possible assumptions it may be possible to make deductions about the actual mechanism of cooling.

ISOTHERMS FOR TWO-DIMENSIONAL REGIONS

In all cases it is assumed, as stated in the Introduction, that the country rock is at zero temperature and that the magma is suddenly emplaced at a uniform temperature T_o. The temperature T at any point in the magma at time t after emplacement can be calculated from formulae given later, and the isotherms at any time can then be drawn.

As remarked by Lovering (1935), sheets of all thicknesses may be described by the same set of curves if, instead of the actual time t, a quantity τ proportional to it is used which is defined by

$$\tau = \kappa t/d^2 \,, \tag{1}$$

where t is the time after emplacement, d is the thickness of the sheet (or a specified portion of it) and κ is the thermal diffusivity of the material, which is defined as $\kappa = K/\rho c$, where K is the thermal conductivity, ρ the density, and c the specific heat. The quantity τ defined by (1) is in fact dimensionless, and the units would normally be c.g.s., calorie and °C, in which the value of κ for solid dolerite is about 0.007. If we introduce the more convenient units $D = d/100$ for the thickness of the sheet in meters and $t_y = 3.17 \times 10^{-8}t$ for the time in years after emplacement, and use the value of κ for solid dolerite given above, (1) becomes

$$\tau = 22t_y/D^2 \ . \tag{2}$$

Thus, for example, $\tau = 0.01$ corresponds to a time t_y after extrusion of 0.045 years for a flow 10 meters thick, or 1.1 years for a flow 50 meters thick.

Figures 1 to 8 show the position of a specified isotherm at equally spaced intervals of time for a variety of shapes of the igneous body. The chosen isotherms are 0.8 T_o and 0.6 T_o, which, for an initial temperature of T_o = 1200°C would be 960°C and 720°C.

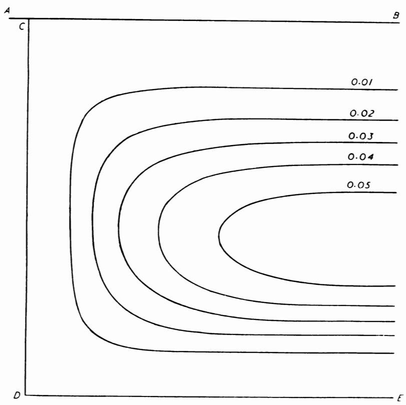

Fig. 1

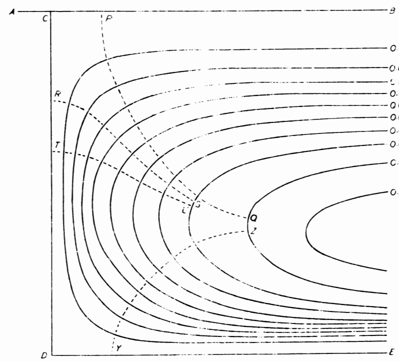

Fig. 2

Fig. 1. Progress of the 0.8 T_o isotherm for an extrusive sheet initially at temperature T_o occupying the region BCDE of thickness *d*. The free surface BC is maintained at zero temperature and the region below the surface ACDE contains country rock initially at zero temperature. The numbers on the curves are values of $\tau = \kappa t/d^2$.

Fig. 2. Progress of the 0.6 T_o isotherm for the situation of figure 1.

Case (*i*) *A flow terminated at a vertical plane* (*figs. 1 and 2*).—Here the magma occupies the semi-infinite strip BCDE of thickness *d*, and the country rock is in the region below ACDE. The upper surface ACB of both country rock and magma is at zero temperature. Figure 1 shows the position of the isotherm 0.8 T_o at equally spaced intervals of time specified by the values of τ on the curves, and figure 2 shows corresponding curves for the 0.6 T_o isotherm. Isotherms for an extrusive that fills a valley with vertical walls may be obtained by combining two figures of this type. As remarked earlier, if the valley, instead of having vertical walls, has a shape similar to the isotherms for $\tau = 0.01$ or 0.02, the general pattern of the isotherms is not likely to be altered greatly.

Case (*ii*). *A change in the thickness of a flow* (*figs. 3 and 4*).—Here the flow is supposed to have thickness *d* in the region to the right of DE and thickness ½*d* in the region to the left. There is country rock below the boundary CDEF, and the upper surface AB is always at zero temperature. Figures

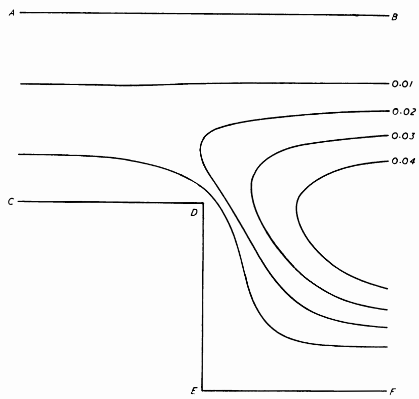

Fig. 3

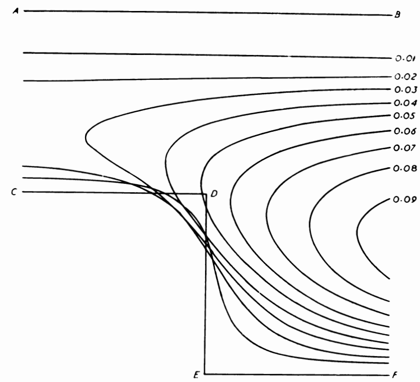

Fig. 4

Fig. 3. Progress of the 0.8 T_o isotherm for an extrusive sheet initially at temperature T_o whose thickness is d in the region BF and $\frac{1}{2}d$ in the region AC. The free surface AB at zero temperature and the region below CDEF contains country rock initially at zero temperature. The numbers on the curves are values of $\tau = \kappa t/d^2$.

Fig. 4. Progress of the 0.6 T_o isotherm for the situation of figure 3.

3 and 4, respectively, show the position of the 0.8 T_o and 0.6 T_o isotherms for equally spaced values of the time specified by $\tau = \kappa t/d^2$. Their behavior near the point D in figure 4 is noteworthy. For small values of τ, 0.01 and 0.02, the 0.6 T_o isotherm moves into the country rock, corresponding to heating of the country rock near D above this temperature, and it subsequently moves away from it in the general process of cooling.

Isotherms for a flow with a localized thickening, as in figure 9, may be obtained by combining two figures of the present type.

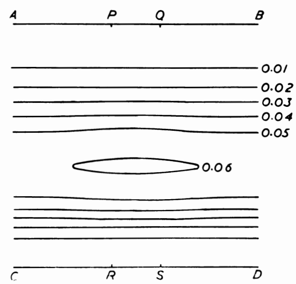

Fig. 5. Progress of the 0.8 T_o isotherm in an extrusive sheet of thickness d in which the surface AB is at zero temperature and the region below CD is country rock initially at zero temperature. The magma in the region PQRS is initially at a temperature of 1.05 T_o and that in the regions QBDS and APRC at temperature T_o. The numbers on the curves are values of $\tau = \kappa t/d^2$.

Case (iii) The effect of non-uniformity of the initial temperature of the magma (fig. 5).—It is well known that in actual flows the temperature of the magma varies considerably. The effect of such variations may be studied by supposing part of the flow to be at a slightly higher temperature than the remainder. In figure 5 the flow is assumed to have uniform thickness d, to be in

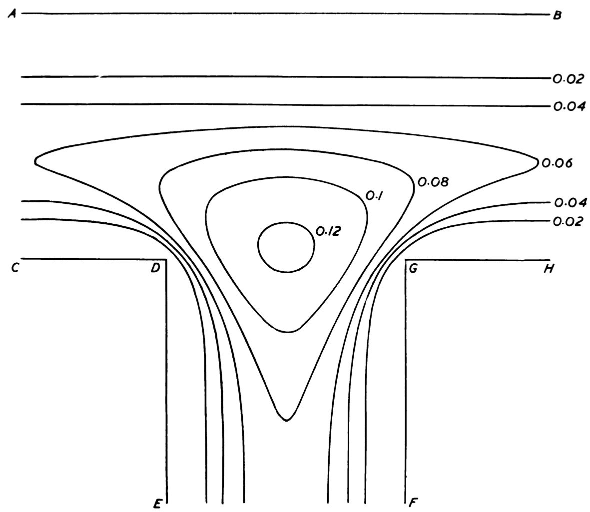

Fig. 6. Progress of the 0.8 T_0 isotherm in an extrusive sheet ABHC of thickness d with a feeder EDGF of the same thickness. The surface AB is at zero temperature, the magma is initially at temperature T_0, and the country rock in the regions CDE and HGF is initially at zero temperature. The numbers on the curves are values of $\tau = \kappa t/d^2$.

contact with country rock along CD, and to have its upper surface AB kept at zero temperature. The initial temperature of the flow is assumed to be T_0 except in the rectangular region PQRS, where PQ $= d/5$, in which it is 1.05 T_0, that is, about 50°C hotter than the rest of the flow. Successive positions of

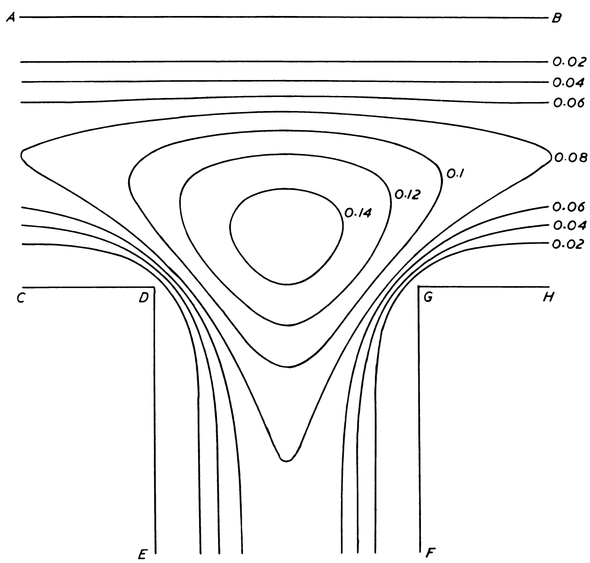

Fig. 7. Progress of the 0.8 T_0 isotherm in an intrusive sheet, ABHC of thickness d with a feeder DGFE of the same thickness. The numbers on the curves are values of $\tau = \kappa t/d^2$.

the 0.8 T_0 isotherm are shown in figure 5. The effect of the localized hot-spot PQRS is to give a closed isotherm near the center of the flow. If instead of the hot-spot's extending right across the flow it had been near the center of the flow only, there would still have been a closed isotherm near the center; on the other hand, if the hot-spot had been near the upper or lower surfaces of the flow, there would merely have been a slight distortion of the isotherms.

Case (iv) A flow with a feeder (fig. 6).—Here, for simplicity, the feeder DEFG and the flow ABHC are taken to have the same thickness d. Figure 6 shows successive positions of the 0.8 T_0 isotherm for this case. The hot-spot at the intersection, and its projection into both the flow and the feeder, are noteworthy.

Case (v). An intrusive sheet with a feeder (fig. 7).—The only change from Case (iv) is that the region above the upper surface AB is assumed to consist of country rock initially at zero temperature. Successive positions of the 0.8 T_0 isotherm are shown in figure 7; they are seen to be very similar in type to those of figure 6, showing that this very considerable change in conditions at the upper surface has little effect on the pattern of movement of the isotherm.

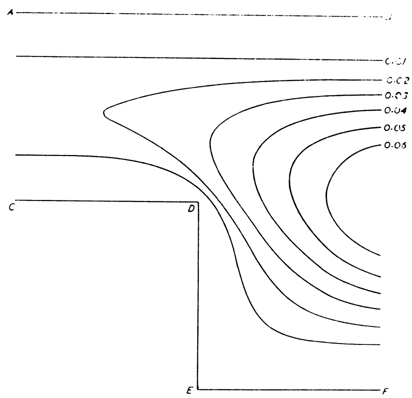

Fig. 8. Progress of the 0.8 T_0 isotherm in an intrusive sheet, initially at temperature T_0, whose thickness is d in BF and $\frac{1}{2}d$ in AC. The numbers on the curves are values of $\tau = \kappa t/d^2$.

Case (vi). A change in thickness of an intrusive sheet (fig. 8).—The only change from Case (ii) is that the region above the upper surface AB is assumed to contain country rock. Successive positions of the 0.8 T_0 isotherm are shown in figure 8.

Figures 1 to 7 illustrate the way in which selected isotherms move in idealized situations of moderate complexity. They are perhaps deceptively simple, since attempts to guess the behavior of the isotherms frequently lead to results very different from the calculated ones. In all cases, the isotherms and their movement are determined by the equation of conduction of heat, which excludes certain types of behavior. When hypothetical isotherms are drawn (for example, to predict directions of jointing), these sets of isotherms, and their movement, must be possible ones for the whole region under consideration. For example, Waters (1960), wishing to explain certain inclined joint columns as being caused solely by inclined isotherms, shows portions of hypothetical isotherms (his fig. 1), but the writer cannot envisage a complete set of isotherms covering the region in question that will have the required shape, which must be done before Waters' explanation can be regarded as justified.

COLUMNAR JOINTING

It is not the object of this paper to discuss the mechanism of columnar jointing, but certain observations follow from the movement of the isotherms as shown in figures 1-7. As remarked earlier, if it is assumed that columnar jointing is the result of cracking caused by tensile thermal stresses due to contraction, these stresses are controlled by temperature and hence specified by the isotherms. There is no theoretical reason why the maximum tensile stress should be parallel to the isotherms,[1] though this is a reasonable working hypothesis when their curvature is not too great. Drawing curves normal to a series of positions of a moving isotherm, as in the dotted curves of figure 2, becomes relatively difficult as the isotherms contract onto a local hot-spot. Thus, when such a point is approached, the joints may tend to depart from the directions normal to the isotherms. The patterns shown in figure 2 may be looked at in two ways: on a relatively small scale the set PQ, RS, TU, are reminiscent of the common "fan" structure, whereas on a larger scale the sets PQ, YZ suggest the "basin" structure in which horizontally lying joints occur, (cf. Iddings, 1886). This point is referred to below in connection with figure 9.

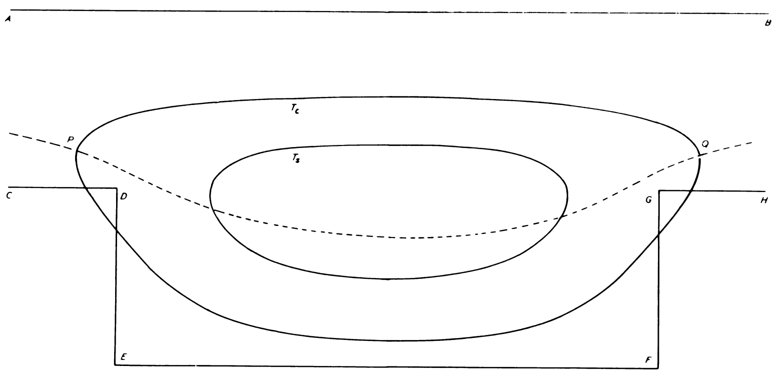

Fig. 9. The relative position of the isotherms $T_s = 0.8\ T_o$ and $T_c = 0.6\ T_o$ at a time given by $\kappa t/d^2 = 0.04$ for a flow filling a valley. The flow is initially at temperature T_o and its maximum thickness is d.

It is extraordinarily difficult to describe the stress-situation likely to exist in a cooling igneous mass. Judging by observations on modern flows, cracks are formed at a relatively high temperature (above red heat), and for simplicity it might be assumed that there is some temperature T_c at which cracking occurs, so that cracks might be expected to extend from the surface to the T_c isotherm. There will also be an isotherm T_s at which solidification is complete.

[1] In certain cases, such as the cooling of spheres or circular cylinders by radial flow of heat, the principal axes of stress are, by symmetry, in and perpendicular to the isotherms provided there are no mechanical constraints applied. In general, however, there are such constraints; for example, if two generators of a circular cylinder are held in fixed positions, the stress-situation in the cylinder would be entirely different from that in the unconstrained cylinder, although the temperature distributions would be identical.

Taking $T_c = 0.6\ T_0$, $T_s = 0.8\ T_0$, these isotherms for a flow shaped as in figure 9 are shown in figure 9 for $\tau = 0.04$ (as actual values of T_c and T_s in the present context, the figures $0.6\ T_0$ and $0.8\ T_0$ are too low; they were chosen to give diagrams at reasonably widely spaced intervals of temperature).

The process of jointing may then be envisaged as follows: Cracks will start at both the upper surface and the lower contact and at any time will have moved in to near the T_c isotherm; as long as some magma remains liquid, the two systems of cracks will remain virtually independent, the stresses at the tip of each crack being determined by the thermal situation and by the other cracks proceeding from the same margin. When the whole of the magma has solidified, the stress-situation will be entirely different, since the stresses at the tip of any crack will now be determined by the whole region with all its cracks rather than its upper or lower part. This provides a simple explanation of the commonly observed division of a flow into three parts (Tomkeieff, 1940); the upper and lower sets of columns may have formed while the interior was still liquid, and the center region after solidification was complete. The more complicated stress-situation in this latter case could account for the greater complexity of the jointing in this region, as frequently observed. An additional reason for this complexity is that any irregularity of the surface (fig. 9) or of the initial temperature (fig. 5) may result in a system of closed isotherms near the center of the sheet, and it is reasonable to suppose that a complicated system of isotherms will lead to a complicated stress pattern. In the cases of very uniform flows and intrusions, where the boundaries of the region are relatively flat and the initial temperatures uniform, these complexities might well be absent.

If jointing takes place, as suggested above, by propagation inwards of a number of cracks, it is highly probable that in situations involving curved isotherms some particular crack or cracks will be favored and will propagate more rapidly than others; it is reasonable to suggest that the direction PQ, figure 9, along which the isotherms and hence also the surface of solidification move most rapidly, might be so favored. If this were the case, the whole stress-situation would be altered, and other joints that are propagated more slowly, either from above or below, might be expected to be terminated abruptly at PQ. In this way the well-known "basin" structure (Iddings, 1886; McDougall, 1959), in which some joints run almost horizontally and others intersect them, might be explained.

The general mechanism described above is very similar to that envisaged by James (1920), but the present drawings of calculated instead of hypothetical isotherms make it possible to be more precise. It should be stated that an entirely different explanation for the tripartite division of a flow has been given by Tomkeieff (1940). The rate of movement of the isotherms is greatest at the center and boundaries of a flow, and least at two intermediate points (this effect is shown very clearly in all the figures), and Tomkeieff associated the three major divisions of the flow with the three regions in which the isotherms move rapidly. It seems likely that the rate of movement of the isotherms will have some effect on the process of jointing, though possibly not the major one envisaged by Tomkeieff.

As shown in the last section, the present assumption that joints proceed simultaneously from both the upper and lower surfaces towards the hotter region in the center of the flow implies that jointing takes place at a relatively high temperature—above the contact temperature, which, allowing for latent heat, is probably between 600° and 700°C. It implies also that, since at any time in the early stages of cooling the greatest temperature is slightly nearer to the lower contact than to the upper surface, the length of the set of columns terminating at the lower contact should be of the same order as, but if anything rather less than, the length of the upper set of columns, which terminate in the upper surface. Broadly speaking, this seems to be the case (Tomkeieff, 1940; Waters, 1960), but one interesting exception has been noted by Mathews (1951), who reports a sequence of flows with thicknesses of the order of 200 ft in which the length of the lower set of columns is only about 20 ft. On the present hypothesis this could occur if the time-interval between flows was short, so that the heat lost by a partially cooled flow would chill only a small portion of a subsequent flow and the two would then cool together. This case is discussed in the last section, where the time between flows under the conditions mentioned above is shown to be of the order of eight years.

CONTACT TEMPERATURES

On the simple thermal assumptions made here, the maximum contact temperature for a sheet is one-half the magma temperature. If latent heat is taken into account it will be substantially increased, and it may also be increased by pre-heating of the country rock and also if the conductivity of the country rock is less than that of the magma.

If the margin is not plane there may be further increases. On the simple thermal assumptions of this paper, one simple result is that, at a corner where the magma chamber has an internal angle α, the contact temperature rises on emplacement to a value of $\alpha/2\pi$ times the magma temperature. If $\alpha = \pi$ this gives the value ½ mentioned above. If $\alpha = \pi/2$ as at E, figures 3 and 4, the contact temperature is one-quarter of the magma temperature; if $\alpha = 3\pi/2$, as at D, three-quarters, and so on. Figure 4 shows that in corners of the latter type, where the country rock projects into the magma, the country rock is raised to a higher temperature and kept there longer than near a plane surface. Thus such corners should provide the regions in which the country rock should be most metamorphosed. This may explain why, for example, although the contacts of the Tasmanian dolerites are in general very little metamorphosed, regions do occur in which the sediments have been mobilized; such regions might occur as an internal corner of the intrusion is approached.

THE PROGRESS OF THE SURFACE OF SOLIDIFICATION

Figures 1 to 8 may be used to give an indication of the way in which the surface at which solidification is complete moves in an igneous mass cooling by conduction alone. As shown in the next section, the effect of the emission of latent heat is to raise the contact temperature and to slow up the early stages of the process of cooling, but it will not greatly affect the general pattern of the isotherms.

During the process of cooling, differentiation takes place, and its effect is to concentrate the more acid material into the last regions to solidify. The most common assumption about the mechanism of differentiation is that it proceeds by a process of crystal settling.

To see the effect of this in two dimensions, consider a region shaped as in figure 9. Suppose that the range of solidification is from T_c to T_s; then at any time the region outside the T_c isotherm in figure 9 will be solid, while early-formed crystals in the region between the T_c and T_s isotherms will settle through the liquid onto the solidified floor. By this process the region within the T_c isotherm becomes progressively richer in the more acid constituents. This region will finally contract into a small puddle near the geometrical center of the body. Thus, for an intrusion shaped as in figure 9, the most acid material would be concentrated in an area such as the isotherm for $\tau = 0.06$ in figure 8. This puddle of residual liquid would be displaced upwards from its position in figure 8 by the crystals that have fallen to the floor of the intrusion, but the general configuration should not be greatly changed.

NUMERICAL RESULTS

The fundamental numerical information about the cooling of a flow is given in figure 10, which shows the variation of temperature across a flow of

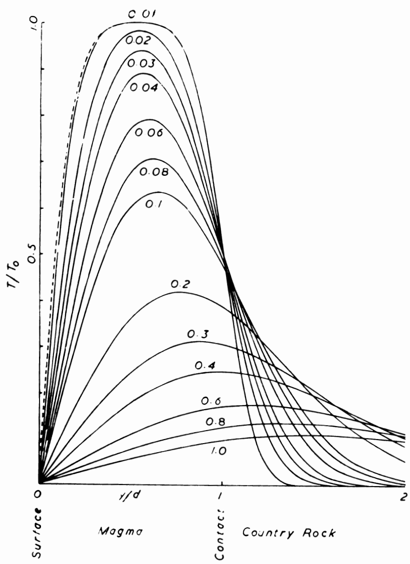

Fig. 10

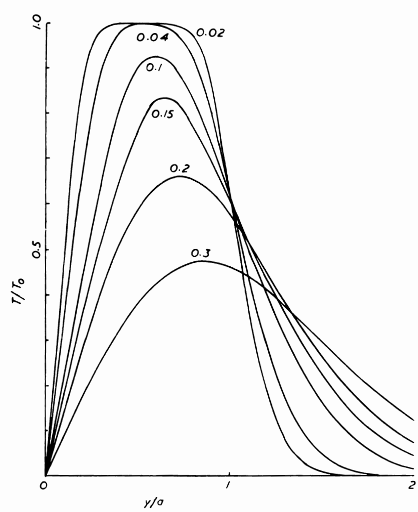

Fig. 11

Fig. 10. The temperature distribution at various times in an extrusive sheet of thickness d with its surface maintained at zero. y is the depth below the surface and the numbers on the curves are values of $\tau = \kappa t/d^2$. The dotted curve is for $\tau = 0.01$ with radiation at the surface.

Fig. 11. The temperature distribution at various times in an extrusive sheet of magma allowing for emission of latent heat of 100 cal/gm over a range of solidification of 1000° to 800°C. The initial temperature of the magma is 1000°C; the thickness of the sheet is d and y is the depth below its surface; the numbers on the curves are values of $\tau = \kappa t/d^2$ with $\kappa = 0.0071$.

thickness d at various times specified by the quantity $\tau = \kappa t/d^2$ defined in (1) and (2). y is the distance below the surface of the flow.

Many important results follow from inspection of figure 10. For example, the uppermost curve shows that if $\tau < 0.01$, or, by (2), $t_y < 0.00045D^2$, virtually no cooling will have taken place at the center of a flow. For a flow of thickness $D = 10$ meters this time is 0.045 years, and for 50 meters it is 1.1 years.

From the lowest curve, $\tau = 1$, in figure 10, it follows that the whole of the flow has cooled to temperatures of less than $T_o/10$ or about 100°C after a time of $0.045D^2$, that is, 4 years for a 10-meter flow or 100 years for a 50-meter flow. After such times a second flow would be quite unaffected by the heat remaining in an earlier one.

One important point mentioned above in connection with columnar jointing remains to be discussed. The graphs of figure 10 show that for $\tau < 0.4$ the temperature has a maximum within the flow, and that if $\tau < 0.1$ this maximum is quite pronounced. This is consistent with the process envisaged earlier by which jointing begins independently at the upper and lower surfaces and propagates towards the hotter region inwards. Clearly, if this is the case, jointing must occur at a higher temperature than the contact temperature. Also, since the maxima of the curves of figure 10 are always nearer to the lower contact than to the upper surface of the flow, the lengths of the columns beginning at the lower contact (colonnade) would be less than though not very different from those extending from the upper surface. On the other hand, there is no a priori reason why jointing should not take place at lower temperatures, say less than $\frac{1}{2}T_o$. Figure 10 shows, however, that the contact does not fall below this temperature until fairly late in the cooling history of the flow, say $\tau > 0.1$, at which time solidification would long have been complete. At such times the temperature distribution is highly asymmetrical and a totally different pattern of thermal stress would result which would not be likely to lead to a tripartite division of the flow.

The formulae from which the temperature distributions of figure 10 have been calculated are derived in Ingersoll and Zobel (1913) ¶86(53) or Carslaw and Jaeger (1959) ¶2.4(14). Mathematically they may be stated as follows:

Suppose that the magma is initially at a uniform temperature T_o and the country rock at zero temperature. Suppose also that for all times $t > 0$ after extrusion the upper surface of the magma is maintained at zero temperature. Then if the magma occupies the region $0 > y > -d$ and country rock the region $y < -d$ the temperature T at depth y at time t is given by

$$T/T_o = \tfrac{1}{2}\left\{ \operatorname{erf}\frac{y+d}{2d\tau^{1/2}} + \operatorname{erf}\frac{y-d}{2d\tau^{1/2}} - 2\operatorname{erf}\frac{y}{2d\tau^{1/2}} \right\}, \qquad (3)$$

where erf x is the tabulated error function defined by

$$(4) \qquad \operatorname{erf} x = \frac{2}{\pi^{1/2}} \int_o^x e^{-u^2}\, du\ ,$$

and τ is the dimensionless parameter defined in (1) or (2).

The extension to the two-dimensional regions discussed earlier leads also to simple formulae involving error functions. For example in Case (i) above, the magma occupies the semi-infinite strip BCDE, $0 > y > -d$, $x > 0$ and the temperature T at (x,y) at time t is given by

$$\frac{T}{T_o} = \frac{1}{4}\left\{1 + \operatorname{erf}\frac{x}{2d\tau^{1/2}}\right\}\left\{\operatorname{erf}\frac{(y+d)}{2d\tau^{1/2}} + \operatorname{erf}\frac{(y-d)}{2d\tau^{1/2}} - 2\operatorname{erf}\frac{y}{2d\tau^{1/2}}\right\}. \quad (5)$$

The effect on the temperature distributions of figure 10 caused by relaxing the assumptions made earlier may now be considered. Firstly, the free surface was assumed to be at zero temperature while, in fact, it cools by radiation. It is well known, however, that the surface cools very rapidly, and it is frequently reported that flows can be walked on after one or two days; assuming, for example, that the surface temperature has fallen to $0.05\ T_o$ after two days, a numerical estimate of the rate of radiation and calculations for a flow of thickness 4 meters (Carslaw and Jaeger, 1959, ¶14.2) gives the dashed curve for $\tau = 0.01$ in figure 10. This differs so little from the corresponding curve for zero surface temperature that the assumption that the free surface is always at zero temperature is perfectly adequate for the present purposes.

The effect of latent heat of solidification may be taken into account as in Jaeger (1957, 1959). Calculations of the same type have been made for an extrusive sheet of thickness d with its upper surface at zero temperature, its thermal diffusivity $\kappa = 0.0071$ that of dolerite, and latent heat of 100 cal/gm liberated over a range of solidification of 1000° to 800°C. These values, which are rather low, are used because they enable comparison to be made with the corresponding calculations for an intrusive sheet given by Jaeger (1957). The initial temperature T_o of the magma is taken to be 1000°C. The temperature distribution for various values of $\tau = \kappa t/d^2$ is shown in figure 11. As always,

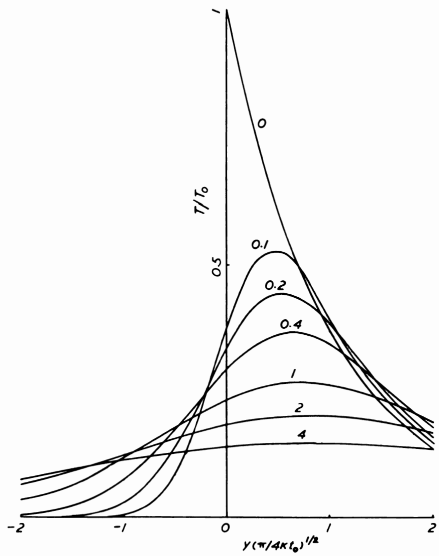

Fig. 12. Temperature distribution at time t in the infinite solid $-\infty < y < \infty$ if the initial temperature is zero in the region $y < 0$, and if the region $y > 0$ is pre-heated by maintaining $y = 0$ at temperature T_o for time t_o. The numbers on the curves are the values of t/t_o.

the effect of latent heat is to raise the contact temperature, which in this case becomes 0.62 T_o in place of 0.5 T_o in figure 10, and to slow up the early stages of cooling. The asymmetry of the temperature distributions, mentioned earlier, is also increased. Comparison of figures 10 and 11 suggests that in two-dimensional cases the pattern of the isotherms will not be greatly altered.

The effects of dissimilar thermal properties of the various materials have been studied by Lovering (1936, 1955) and Jaeger (1959).

Finally, the effect of pre-heating of the country rock may be considered. Assume that magma at temperature T_o flows for time t_o over the country rock before quiescence and the beginning of cooling and solidification, and that the country rock (the region $y > 0$) was at zero temperature before the flow began. The surface of the country rock ($y = 0$) will be maintained at constant temperature T_o for a time t_o after which time the temperature at depth y will be

$$T_o \left\{ 1 - \operatorname{erf} \frac{y}{2(\kappa t_o)^{1/2}} \right\}, \tag{6}$$

which for the present purposes is sufficiently near to $T_o \exp\{-y(\pi/4\kappa t_o)^{1/2}\}$ for which calculations are easily made. The effect of this additional heat in the early stages of the cooling of the flow is simply to add to the temperatures of figure 10 those calculated for the infinite solid with this initial distribution of temperature; these are shown in figure 12 for various values of the ratio t/t_o. In this figure the region $y > 0$ corresponds to country rock and $y < 0$ to magma. It appears, for example, that there is a temporary rise above $T_o/10$ within a distance of approximately $3(\kappa t_o/\pi)^{1/2}$ from the contact; for t_o equal to 2 days this distance is 60 cm. Thus in many cases at least, such effects will be superficial.

The same figure, for cooling instead of heating, may also be used to estimate the effect of a second flow on top of a partially cooled earlier one. If the time interval between the flows is short, the heat lost by the partially cooled flow would chill only a small portion of the second flow, and the two would subsequently cool together. For the example mentioned above (Mathews, 1951), in which there are 6 meters of colonnade in the second flow, let us suppose this involves lowering the temperature by at least $T_o/5$ in this region. It follows from figure 12 that the temperature in the second flow is lowered by an amount greater than $T_o/5$ for a distance y from the contact given by $y(\pi/4\kappa t_o)^{1/2} = 0.4$; taking $y = 6$ meters, $\kappa = 0.0071$, this gives about eight years for the time t_o between the two flows.

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