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# AN EXPERIMENTAL TEST OF VISUAL COMPARISON TECHNIQUE IN ESTIMATING TWO DIMENSIONAL SPHERICITY AND ROUNDNESS OF QUARTZ GRAINS

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ABSTRACT. This experiment attempts to evaluate the visual comparison technique for the estimation of sphericity and roundness of quartz grains in sediments. The experiment was designed to enable the data to be statistically analyzed by means of analysis of variance—a powerful statistical device which has a wide field of application in geology.

The results of the experiment indicate that valid estimates of sphericity and roundness may be obtained by visual comparison with standards; but unless the variation due to differences between operators is suitably evaluated, visually estimated differences in sphericity and roundness cannot be attributed solely to differences between grains but are affected to an unknown degree by operator variation. The sympathetic relationship between sphericity and roundness, for example, appears on the basis of this experiment to arise from psychological bias on the part of operators and not from an inherent relationship between these properties of quartz grains.

## INTRODUCTION

IN attempting to estimate quantitatively the variation in the properties of sediments it is essential to evaluate the limitations of technique. Certain fundamentals must be fulfilled before a technique may be adopted as a routine for estimating variation in some particular property, and in testing the technique, the more restricted the test the more restricted the conclusions to be drawn from it. Ordinarily an experiment which purports to compile a set of measured data is somewhat circumscribed in its implications unless an extensive background of similarly measured reference data is available to which to relate the new experimental determinations. Before attempting to compile the extensive background it is, however, necessary to choose a suitable, efficient technique which will lead to meaningful measurements. Hence painstakingly thorough evaluation of a technique is essential, and if this aspect is neglected it will inexorably lead to very considerable wasted effort.

In the present case we are interested in attempting to evaluate limitations in the estimation of sphericity and roundness. The most rapid technique for both purposes and the only practicable technique for routine estimation of roundness of large numbers of grains is by means of visual comparison charts (Krumbein, 1941*a*, and Rittenhouse, 1943), and if this approach could be safely adopted, a large amount of data on different sediments could be compiled in a reasonably short space of time.

Initially it is clear both from laboratory experiments (Krumbein, 1941*b*) and actual measurements of quartz grains in sediments (Pettijohn, 1949, p. 46-55) that the magnitude of variation measured on the conventional scale of 0-1 is likely to be very much less in the case of sphericity than roundness. On this basis, therefore, it is necessary to use greater precision in the case of estimation of sphericity than roundness and it may well be that two different techniques will be finally necessary. From the many experiments quoted in the literature (see Pettijohn, 1949, for extensive bibliography) it is also clear, and this is very fortunate, that it is much simpler to measure sphericity than roundness.

Again it is reasonably clear from Pettijohn's graphic portrayal (1949, fig. 26, p. 54) that while the trends of relationship between sphericity and roundness are similar in different sediments differences between techniques cause as much displacement in the position of the trends as differences between sediments.

Finally, we have learned by way of extensive experience that differences between operators performing the same tasks may well interfere with, and sometimes exceed, differences between sediments and/or techniques.

We may conclude, therefore, that in order to test a technique, an experiment should be reasonably comprehensive and, as the description of any technique in the literature is presumably published with the intention that other investigators should adopt it, differences between investigators should always be evaluated in testing a technique. It may not be out of place here to emphasize that reproducibility as frequently reported can be very misleading; for example, many chemical analysts will claim quite justifiably that they can achieve duplicate or replicate analyses within enviable margins of precision, yet differences between analysts will far exceed this precision error and lead to very embarrassing results if not realized and evaluated before the compilation of the necessary extensive background of reference data (e.g., Fairbairn, 1951).

The present experiment is one of a series in which various techniques and various measurable properties of sediments are being tested with a view to ultimate adoption of suitable analytical procedures for compiling the necessary extensive background of reference data on the magnitude of variation in the properties of sedimentary rocks.

## PROCEDURE AND DESIGN OF EXPERIMENT

The details of the procedure used in the estimation of variation in sphericity and roundness in quartz grains from sedimentary rocks by visual comparison with standard reference charts are adequately described in the original articles (Krumbein, 1941*a*, and Rittenhouse, 1943). In the present case 21 grains of quartz were randomly selected from a beach sand and mounted in semi-permanent mounts; the 21 grains encompassed a variation of from 0.71 to 0.93 in sphericity and from 0.10 to 0.80 in roundness.

The published charts (see literature cited) were photographically reproduced and brought to a standard size of 4″ x 5″ overall. Seven operators, all graduate students in the Division of Mineralogy of the Pennsylvania State College, took part in the test[1] which was carried out in 1948 under the immediate supervision of one of us (M.A.R.). The experiment was initially designed to evaluate differences between grains and differences between operators but, by arranging the replicate runs of each operator on different days, variation from day to day was also included. In terms of experimental design the differences between grains, differences between operators and differences between days are termed main effects, and subsidiary factors comprise three first order interactions (Youden, 1951), namely, operators and days, operators and grains, and grains and days. Finally, a second order interaction of operators and grains and days completes the list of effects included in the experiment.

[1] The seven operators were C. T. Bressler, E. Cressman, D. B. Doan, R. L. Folk, M. A. Rosenfeld, A. Swineford, and C. E. Weaver.

Table 1. Mathematical model of analysis of variance for testing sphericity and roundness data

| Source of variation | Degrees of freedom | Sum of squares | Mean square | Components of variance |
|---|---|---|---|---|
| Grains G | $g-1$ | $\sum_1^g \left(\sum_1^{dn} x\right)^2 / dn - CT$ | $\frac{SS_G}{(g-1)}$ | $s^2_e + ds^2_{GN} + ns^2_{DG} + nds^2_G$ |
| Operators N | $n-1$ | $\sum_1^n \left(\sum_1^{dg} x\right)^2 / dg - CT$ | $\frac{SS_N}{(n-1)}$ | $s^2_e + ds^2_{GN} + gs^2_{DN} + dgs^2_N$ |
| Days D | $d-1$ | $\sum_1^d \left(\sum_1^{ng} x\right)^2 / ng - CT$ | $\frac{SS_D}{(d-1)}$ | $s^2_e + ns^2_{DG} + gs^2_{DN} + ngs^2_D$ |
| Operators times grains | $(n-1)(g-1)$ | $\sum_1^{ng} \left(\sum_1^{d} x\right)^2 / d - CT - SS_N - SS_G$ | $\frac{SS_{NG}}{(n-1)(g-1)}$ | $s^2_e + ds^2_{NG}$ |
| Operators times days | $(n-1)(d-1)$ | $\sum_1^{nd} \left(\sum_1^{g} x\right)^2 / g - CT - SS_N - SS_D$ | $\frac{SS_{ND}}{(n-1)(d-1)}$ | $s^2_e + gs^2_{ND}$ |
| Days times grains | $(d-1)(g-1)$ | $\sum_1^{dg} \left(\sum_1^{n} x\right)^2 / n - CT - SS_D - SS_G$ | $\frac{SS_{DG}}{(d-1)(g-1)}$ | $s^2_e + ns^2_{DG}$ |
| Days times grains times Operators | $(n-1)(g-1)(d-1)$ | $\sum_1^{ndg} x^2 - CT - SS_N - SS_G - SS_D - SS_{ND} - SS_{NG} - SS_{DG}$ | $\frac{SS_{NDG}}{(n-1)(g-1)(d-1)}$ | $s^2_e$ |
| TOTAL | $ngd-1$ | $\sum_1^{ndg} x^2 - CT$ | | |

Where $CT = \left(\sum_1^{ndg} x\right)^2 / ndg$ ; n = no. of operators ; g = no. of grains ; d = no. of days

The separate sources of variation may now be arranged in tabular summary (table 1) as an analysis of variance table (Fisher, 1948) in which the design of experiment and the resulting statistical analysis is summarized. The following remarks are appended by way of brief explanation of the meaning of each source of variation; for greater details the interested reader may be referred to relevant statistical texts (Fisher, 1948; Snedecor, 1946; Youden, 1951, etc.) and, for examples of similar procedures applied to geological problems, to a growing number of publications (Swineford and Swineford, 1946; Chayes and Fairbairn, 1951; Miller, 1949; Griffiths and Rosenfeld, 1950; Rosenfeld and Griffiths, 1950).

The main effects arising from differences between grains, operators and days respectively may appear to carry fairly obvious implications, but the use of analysis of variance is best understood when it is realized that each main effect is used to test a separate null hypothesis (Fisher, 1949). Thus we may set up a null hypothesis concerning grains as follows:

> There are no differences between grains in sphericity or roundness; as a corollary the entire set of measurements is then a random sample of an infinite population of such measurements in which there are no differences in sphericity or roundness as the case may be.

Our test consists of an attempt to disprove this hypothesis. The setting up of a null hypothesis actually implies that we have defined a population of which our set of measurements is a random sample, and our test comprises computation of statistics, the mean and variance, from the sample and estimation of how likely such statistical values are to arise on the basis of our hypothesis about the population.

In the present case, as implied by the title "analysis of variance," we shall be testing the variance around the means and using the mean and variance but especially the latter as a basis of judgment of whether the sample of measurements has indeed been drawn from a population in which the differences in sphericity are non-existent. Each main effect leads to a separate null hypothesis and, of course, to a separate population for testing. The interactions are, however, somewhat more intricate; we may compare, by way of illustration, the main effect differences between operators and the first order interaction operators times grains. There may be and, indeed, usually are differences between operators in which one operator always estimates, say, a high value of sphericity whereas another measures a low one (see, for example, operators in figure 5). In addition some operators estimate consistently, i.e., always yield high or always low values, whereas others are inconsistent, sometimes measuring high and at other times measuring low values. The differences between operators is estimated as the main effect whereas the inconsistency of operators from grain to grain is measured by the interaction term operators times grains. It is reasonably simple to extend this argument to variations of operators from day to day and so to account for operators times grains and operators times days first order interactions. There appears to be no obvious physical meaning to the first order interaction of grains times days principally because it is difficult to conceive of a grain varying from day to day or being inconsistent in this sense; nevertheless, this term is mathematically separately evaluated here although it is customary to include the variance from this source with "error" in the usual case. The reason for this departure from convention is twofold; firstly, it is simpler to explain the analysis in this way and, secondly, we are in such a dubious position geologically in terms of evaluating what is and what is not important that it behooves us to be extremely cautious in attempts to simplify by jumping to conclusions where we have no prior experience to guide us. Although, therefore, we cannot at present imagine a meaning for such a source of variation, it is as well to be aware of its possible eventuality in future experiments!

Finally, the second order interaction entitled grains times operators times days includes all variation from unassigned sources. This then becomes our basic error term for the experiment. It may be as well to add that this term is capable of physical interpretation in experiments possessing a more complex design and has been of value in evaluating certain agricultural experiments. In the present case, however, we do not attempt to interpret the physical meaning of the term but accept it as a measure of experimental error. The magnitude of this error determines the sensitivity of the entire experiment.

## TREATMENT OF THE DATA

The use of analysis of variance technique carries with it certain implications concerning the variation in the measured variable (Eisenhart, 1947); ideally the frequency distribution of the measurements should follow the normal distribution but considerable departure from this ideal is permissible in practice.

Nevertheless, it is necessary to ensure that the data are suitable for this kind of statistical treatment, and certain standardized tests exist for this purpose. Furthermore, it is always advisable to be aware of the frequency distribution of the measured variable because this information is of considerable importance both in understanding the significance of the measurements and in evaluating the usefulness of the technique.

The scales of measurement of both sphericity and roundness extend from 0 to 1 and are, therefore, "percentage" or proportion type scales, whereas the limits of the ideal normal distribution extend from minus to plus infinity; as a result, the frequency distributions of sphericity and roundness may show cutoffs at zero and unity. The recommended practice is to transform the variable from percentage or proportion to angular values (the inverse or arc sin transformation), and tables to facilitate this transformation may be found in many standard texts (Fisher and Yates, 1948; Snedecor, 1946; etc.).

In using analysis of variance techniques a common error variance is computed as the basis for comparison of all main and subsidiary effects, and for this reason it is necessary to ensure that the variance and especially the error variance is not related to the mean; the independence of mean and variance in the normal distribution is, of course, a fundamental property of such a frequency distribution. In order to ensure as near fulfillment of these prerequisites as possible, it is necessary to examine the frequency distribution of the variable in some detail, and should any doubt concerning the requirements exist, it is advisable to transform the variable to a scale in which as many as possible of the assumptions involved in using analysis of variance technique are fulfilled.[2]

*Sphericity measurements.*—The 441 values of sphericity compiled for this investigation are listed in table 2 and are graphically displayed as a histogram in figure 1, together with the computed normal curve possessing the same mean and standard deviation. The importance of the departures may be estimated in several ways and here the chi-square test was

[2] A more detailed and formal discussion of this and other aspects of analysis of variance is included in a series of articles in Biometrics Bulletin (Eisenhart, 1947; Cochran, 1947; and Bartlett, 1947), and the use of the inverse sin transformation is more fully ventilated in "Selected techniques of statistical analysis," Eisenhart, C., et al., McGraw-Hill Book Co., 1947, part 3, p. 395-416.

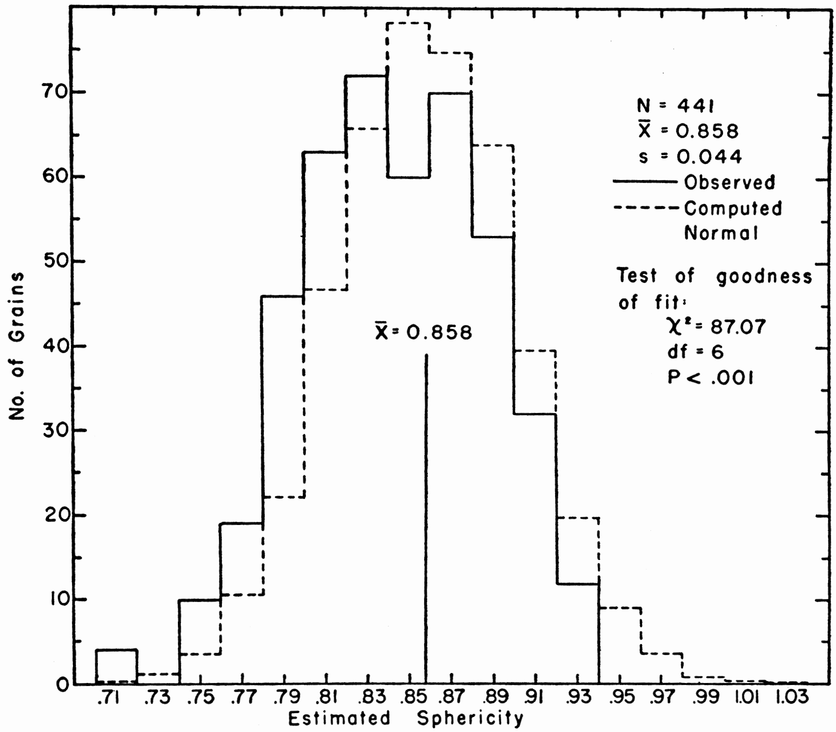


**Fig. 1. Histogram of raw data sphericity values.**

used (Dixon and Massey, 1951, p. 190). The computed chi-square value yields a probability of less than .001; this may be interpreted to signify that if our 441 values were a random sample from a normal population of the same mean and standard deviation, such a chi-square would occur less than once in every 1000 samples. We judge, therefore, that our 441 values depart significantly from a normal distribution.

If we now calculate the mean and variance for each of the 21 grains based on the 21 independent estimates for each grain, we find that the variance shows no obvious relationship to the mean. For our present purpose and based on this limited set of data we may deduce that, although the departure from normalcy is significant, the effect on the levels of significance in the analysis of variance and particularly the F test (Snedecor, 1946) is not serious. Nevertheless, for purposes of exploration, each value was transformed to its arc sin equivalent and

TABLE 2

Raw Data Sphericity Values (x100)

| Operator | A | | | B | | | C | | | D | | | E | | | F | | | G | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| Grain | | | | | | | | | | | | | | | | | | | | | |
| 1 | 91 | 91 | 89 | 81 | 87 | 79 | 87 | 90 | 89 | 79 | 87 | 83 | 83 | 89 | 85 | 87 | 91 | 87 | 89 | 87 | 87 |
| 2 | 81 | 81 | 83 | 79 | 81 | 81 | 83 | 83 | 81 | 83 | 85 | 87 | 85 | 79 | 83 | 83 | 87 | 85 | 85 | 85 | 85 |
| 3 | 81 | 81 | 81 | 75 | 77 | 77 | 79 | 77 | 79 | 83 | 83 | 85 | 81 | 79 | 81 | 83 | 83 | 85 | 79 | 83 | 81 |
| 4 | 87 | 91 | 85 | 71 | 87 | 83 | 87 | 83 | 87 | 83 | 85 | 85 | 83 | 91 | 83 | 87 | 87 | 87 | 87 | 85 | 87 |
| 5 | 77 | 75 | 79 | 85 | 77 | 75 | 75 | 77 | 77 | 79 | 81 | 79 | 71 | 79 | 75 | 83 | 79 | 81 | 71 | 79 | 77 |
| 6 | 87 | 89 | 89 | 87 | 85 | 87 | 93 | 87 | 91 | 83 | 89 | 91 | 81 | 89 | 89 | 91 | 89 | 91 | 87 | 87 | 89 |
| 7 | 85 | 83 | 85 | 81 | 85 | 81 | 83 | 79 | 85 | 91 | 85 | 85 | 79 | 87 | 81 | 87 | 81 | 87 | 87 | 85 | 87 |
| 8 | 89 | 87 | 89 | 83 | 81 | 81 | 85 | 85 | 87 | 87 | 89 | 91 | 83 | 85 | 81 | 89 | 91 | 89 | 83 | 87 | 87 |
| 9 | 83 | 81 | 83 | 77 | 83 | 77 | 79 | 77 | 79 | 83 | 79 | 83 | 81 | 80 | 77 | 81 | 79 | 77 | 81 | 79 | 83 |
| 10 | 85 | 83 | 85 | 79 | 85 | 81 | 75 | 81 | 83 | 81 | 83 | 87 | 83 | 84 | 75 | 89 | 87 | 87 | 85 | 85 | 81 |
| 11 | 87 | 85 | 87 | 77 | 81 | 83 | 83 | 81 | 85 | 81 | 81 | 81 | 85 | 83 | 79 | 89 | 87 | 87 | 79 | 83 | 85 |
| 12 | 85 | 83 | 81 | 81 | 85 | 79 | 77 | 77 | 85 | 83 | 83 | 87 | 83 | 81 | 81 | 83 | 79 | 83 | 79 | 83 | 81 |
| 13 | 81 | 83 | 83 | 79 | 79 | 79 | 79 | 80 | 81 | 77 | 81 | 85 | 87 | 78 | 79 | 81 | 79 | 83 | 75 | 81 | 79 |
| 14 | 85 | 87 | 85 | 83 | 85 | 83 | 83 | 81 | 79 | 87 | 71 | 87 | 81 | 81 | 75 | 85 | 89 | 89 | 79 | 81 | 81 |
| 15 | 83 | 87 | 85 | 79 | 79 | 79 | 85 | 85 | 85 | 83 | 81 | 89 | 77 | 82 | 81 | 81 | 83 | 87 | 81 | 79 | 79 |
| 16 | 89 | 91 | 89 | 87 | 87 | 87 | 91 | 89 | 93 | 91 | 87 | 93 | 87 | 89 | 90 | 93 | 89 | 93 | 89 | 91 | 91 |
| 17 | 89 | 87 | 89 | 87 | 87 | 87 | 89 | 89 | 89 | 91 | 87 | 89 | 91 | 90 | 91 | 89 | 87 | 87 | 91 | 89 | 89 |
| 18 | 89 | 91 | 89 | 85 | 91 | 89 | 85 | 85 | 89 | 93 | 87 | 91 | 89 | 89 | 89 | 91 | 91 | 89 | 87 | 89 | 87 |
| 19 | 87 | 85 | 85 | 79 | 81 | 81 | 81 | 83 | 83 | 83 | 85 | 83 | 83 | 83 | 77 | 81 | 85 | 83 | 83 | 87 | 85 |
| 20 | 85 | 85 | 85 | 75 | 81 | 79 | 83 | 85 | 87 | 83 | 83 | 83 | 79 | 81 | 79 | 87 | 87 | 83 | 83 | 83 | 83 |
| 21 | 93 | 93 | 91 | 81 | 89 | 85 | 89 | 89 | 91 | 89 | 93 | 93 | 87 | 87 | 89 | 93 | 93 | 91 | 89 | 89 | 89 |

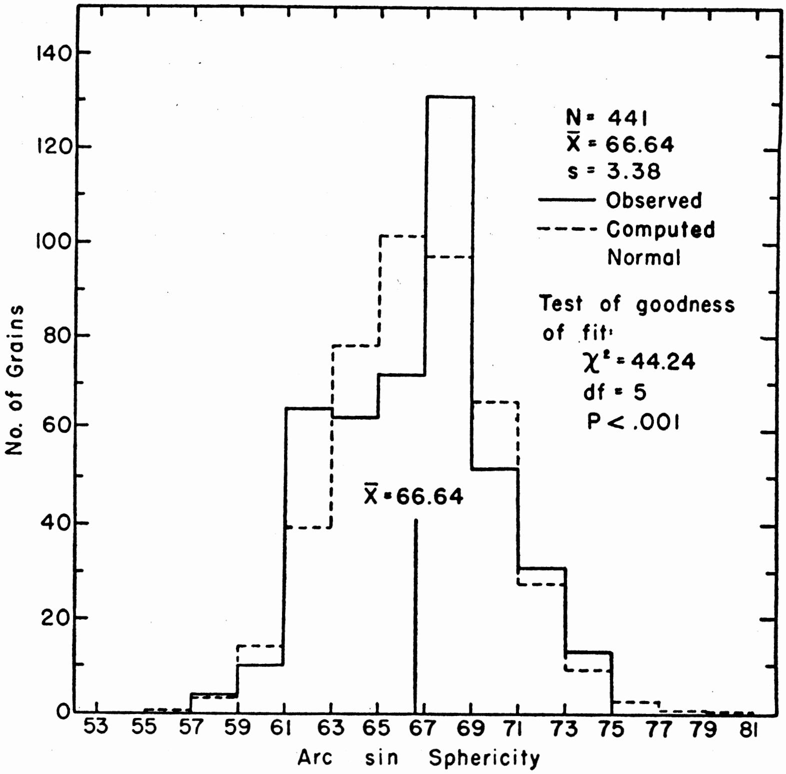


Fig. 2. Histogram of sphericity values—data transformed to arc sin.

the analysis of variance performed on both raw and transformed data. The comparison of the results follows later (table 5).

As a matter of interest the histogram of the transformed data is given in figure 2, together with the normal curve computed for the same mean and variance. The chi-square test yields a probability value of less than .001. It seems clear that the arc sin transformation in this case did not suffice to normalize the data. It should be noted from the frequency distribution of the raw data that the mean value of 0.858, the almost symmetrical histogram, and the lack of cutoffs at either end suggest that no transformation is necessary in this case.

*Roundness measurements.*—The roundness data based on the 441 estimates is given in table 3 and plotted as a histogram in figure 3. This distribution tested against the normal curve

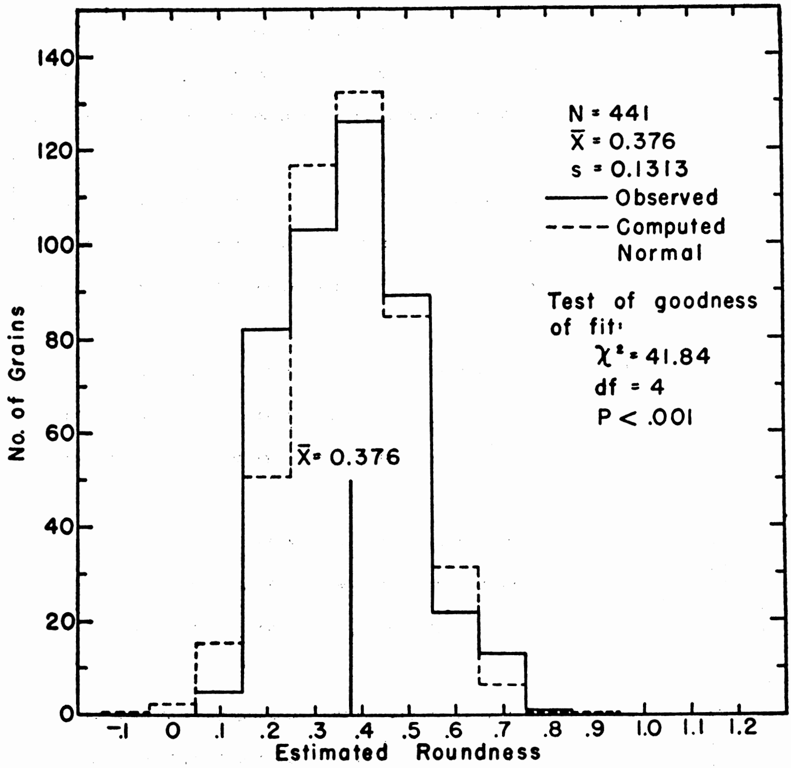


**Fig. 3. Histogram of raw data roundness values.**

computed for the same mean and variance yields a chi-square of 41.84 and P $<0.001$, again indicating a non-normal distribution. Furthermore, a plot of the mean roundness against variance for each of the 21 grains yields a definite trend (fig. 4). In this case the association is estimated by the correlation coefficient r = —0.4272 which, on the basis of 21 sets of values, yields a P of $<0.1>$ 0.05 (Fisher, 1948, table 6, p. 209). Although not significant at the conventional level, such a degree of association is clear indication that the mean and variance are not independent in this set of data. Transformation to inverse sin equivalents failed either to improve the normalcy or to render the mean independent of the variance. It was decided, therefore, to use Bartlett's approach (1947, p. 39) and to calculate a correction from the data themselves. The trend of mean versus variance was assumed linear and com-

TABLE 3

Raw Data Roundness Values (x 10)

| Operator | A | | | B | | | C | | | D | | | E | | | F | | | G | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| Grain | | | | | | | | | | | | | | | | | | | | | |
| 1 | 3 | 4 | 3 | 2 | 2 | 2 | 4 | 5 | 5 | 2 | 4 | 2 | 2 | 2 | 2 | 4 | 5 | 3 | 2 | 2 | 2 |
| 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 5 | 4 | 3 | 3 | 2 | 2 | 5 | 3 | 3 | 3 | 2 |
| 3 | 5 | 4 | 5 | 4 | 5 | 5 | 5 | 5 | 5 | 4 | 5 | 5 | 5 | 4 | 4 | 5 | 6 | 5 | 4 | 4 | 4 |
| 4 | 2 | 3 | 3 | 1 | 3 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 3 | 1 |
| 5 | 3 | 2 | 3 | 3 | 2 | 2 | 4 | 5 | 4 | 2 | 3 | 3 | 1 | 2 | 2 | 2 | 3 | 3 | 2 | 5 | 2 |
| 6 | 3 | 5 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 2 | 2 | 2 | 3 | 2 | 2 | 3 | 2 | 2 |
| 7 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 3 | 5 | 5 | 5 | 5 | 4 | 3 | 3 | 5 | 4 | 3 | 3 | 4 | 4 |
| 8 | 2 | 5 | 2 | 2 | 4 | 3 | 3 | 2 | 2 | 3 | 4 | 5 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 2 | 3 |
| 9 | 4 | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 2 | 2 | 2 | 4 | 3 | 3 | 2 | 1 | 2 |
| 10 | 4 | 4 | 5 | 3 | 5 | 3 | 5 | 5 | 5 | 5 | 5 | 4 | 2 | 4 | 3 | 5 | 4 | 5 | 2 | 3 | 4 |
| 11 | 5 | 5 | 5 | 3 | 3 | 4 | 3 | 4 | 4 | 3 | 4 | 4 | 3 | 4 | 4 | 5 | 5 | 5 | 3 | 3 | 4 |
| 12 | 6 | 6 | 6 | 7 | 8 | 7 | 6 | 6 | 7 | 6 | 7 | 7 | 6 | 5 | 6 | 7 | 7 | 7 | 5 | 5 | 5 |
| 13 | 4 | 4 | 4 | 4 | 3 | 4 | 4 | 4 | 4 | 5 | 3 | 4 | 4 | 2 | 3 | 4 | 4 | 4 | 4 | 4 | 3 |
| 14 | 2 | 4 | 3 | 3 | 2 | 2 | 3 | 3 | 3 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 1 | 3 | 2 |
| 15 | 3 | 4 | 4 | 4 | 4 | 3 | 5 | 2 | 4 | 2 | 3 | 4 | 2 | 3 | 4 | 5 | 4 | 4 | 3 | 4 | 3 |
| 16 | 3 | 4 | 3 | 5 | 4 | 3 | 3 | 3 | 4 | 5 | 3 | 5 | 4 | 4 | 3 | 2 | 2 | 3 | 4 | 2 | 3 |
| 17 | 6 | 6 | 6 | 6 | 6 | 5 | 7 | 7 | 7 | 7 | 6 | 7 | 5 | 6 | 5 | 6 | 5 | 6 | 6 | 6 | 6 |
| 18 | 4 | 5 | 4 | 5 | 4 | 4 | 4 | 5 | 4 | 5 | 5 | 5 | 4 | 4 | 3 | 2 | 3 | 4 | 3 | 4 | 4 |
| 19 | 5 | 4 | 4 | 3 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | 4 | 2 | 4 |
| 20 | 5 | 5 | 5 | 4 | 5 | 5 | 4 | 4 | 4 | 6 | 4 | 5 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 3 | 4 |
| 21 | 5 | 4 | 5 | 4 | 5 | 5 | 5 | 4 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | 4 | 3 |

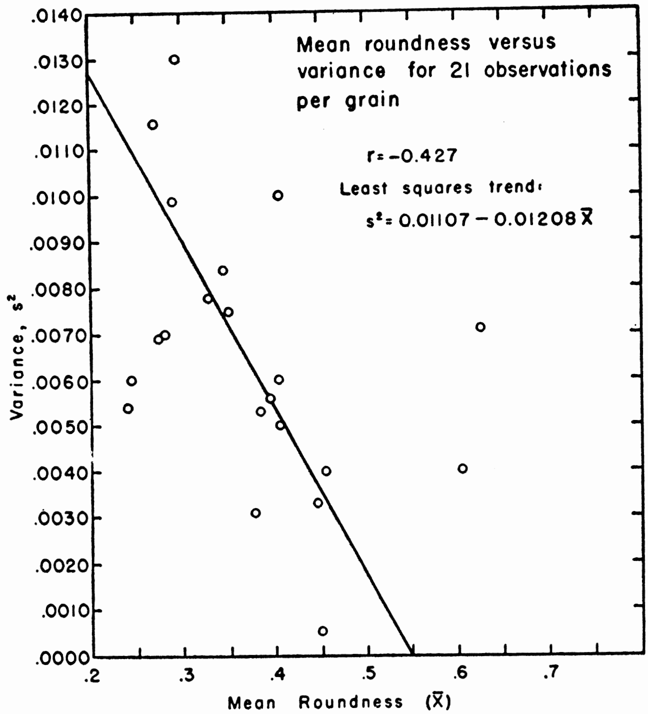


**Fig. 4. Plot of mean roundness versus variance for 21 observations per grain.**

puted by least squares (see fig. 4) yielding $s^2 = 0.01107 - 0.01208\bar{x}$, where $s^2$ = variance, and $\bar{x}$ = mean of the roundness values for each grain respectively. Following Bartlett's recommendations (op. cit. p. 39) the linear relationship was integrated to yield the required transformation:

$$P_A = \sqrt{0.01107 - 0.0121P} \qquad (1)$$

where P = measured roundness value and $P_A$ = adjusted roundness value. Again the analysis of variance was computed on the basis of raw data, inverse sin and adjusted values as a basis for comparison (table 7). It should be noted that, using the computed formula given in equation (1) above, any roundness value greater than 0.7 will yield a negative adjusted value, i.e., this transformation is not suitable for adoption other than with the data herein recorded. Needless to say, the regression adjusted roundness values show a mean independent of the variance.

In conclusion, it should perhaps be emphasized that the requirement of normalcy, while desirable, is not so essential as that the variance should be independent of the mean, and this is particularly true if, as in the present analysis, one is more interested in the estimation of components of variance than in the difference between means (see, in particular, Eisenhart, 1947, p. 19).

## SOURCES OF VARIATION IN SPHERICITY

*The analysis of variance.*—The analysis of variance of the sphericity data in terms of the transformed (arc sin) values is summarized in table 4. The steps in the analysis may be outlined as follows:

From the components of variance (see mathematical model table 1) it can be seen that each source of variance may be tested against its appropriate error term; thus, the first order interactions are tested against the second order interaction. The actual test is performed by comparing the mean squares and computing the variance ratio or F value (Snedecor, 1946, p. 219). For example, if we divide the mean square for the days times grains interaction by the mean square for the second order interaction we are evaluating the variance ratio, $F=(s^2_e+7s^2_{DG})/s^2_e$; returning to our initial argument, if there are no differences, then these two independent estimates are estimates of the same variance. In the ideal case, $s^2_{DG}$ is zero and the variance ratio becomes $(s^2_e/s^2_e)$ which is, of course, unity. Departures from this ideal value which arise from chance causes or from random sampling of a homogeneous population are estimated by the appropriate F distribution. In the present case the F value of 1.04 is a near estimate of unity; in fact, the 20% point of the F distribution with 40 and 200 degrees of freedom is approximately 1.208,[3] and hence we would expect such an F value more than 20 times in every 100 of such tests in sampling from a homogeneous population. It would appear, therefore, that on the basis of the present evidence, variation from inconsistency of grains from day to day adds nothing more to the variation than that contained in the error

[3] Computed using Fisher's formula, Fisher and Yates, 1948, p. 34, footnote to table.

TABLE 4

Analysis of Variance

Sphericity of 21 Grains Determined by 7 Operators on each of 3 Days (arc sin transformation)

| Source of Variation | DF | Sum of Squares | Mean Square | F | $F_{.05}$ | $F_{.01}$ | $F_{.001}$ | DF used for test |
|---|---|---|---|---|---|---|---|---|
| Between Grains | 20 | 3204.397 | 160.2198 | 27.137*** | 1.83 | 2.34 | 3.02 | 20/120 |
| Between Operators | 6 | 532.77 | 88.795 | 6.575** | 3.00 | 4.82 | 8.38 | 6/12 |
| | | | | 15.04*** | 2.17 | 2.96 | 4.04 | 6/120 |
| Between Days | 2 | 25.845 | 12.992 | | | | | 2/12 |
| | | | | 3.88* | 2.99 | 4.60 | 6.91 | 2/280 |
| Operators times Grains | 120 | 708.509 | 5.904 | 1.7735*** | 1.35 | 1.53 | ... | 120/280 |
| Operators times Days | 12 | 162.049 | 13.504 | 4.056*** | 1.83 | 2.34 | 3.02 | 12/280 |
| Days times Grains | 40 | 138.128 | 3.4532 | 1.04 | 1.45 | 1.69 | ... | 40/200 |
| Days times Grains times Operators | 240 | 793.932 | 3.308 | | | | | |
| Total | 440 | 5565.63 | 12.6492 | | | | | |

CT = 1,967,941.36

Pooled Error Term = 3.329; DF 280

* = Significant at 5.0% level
** = Significant at 1.0% level
*** = Significant at 0.1% level

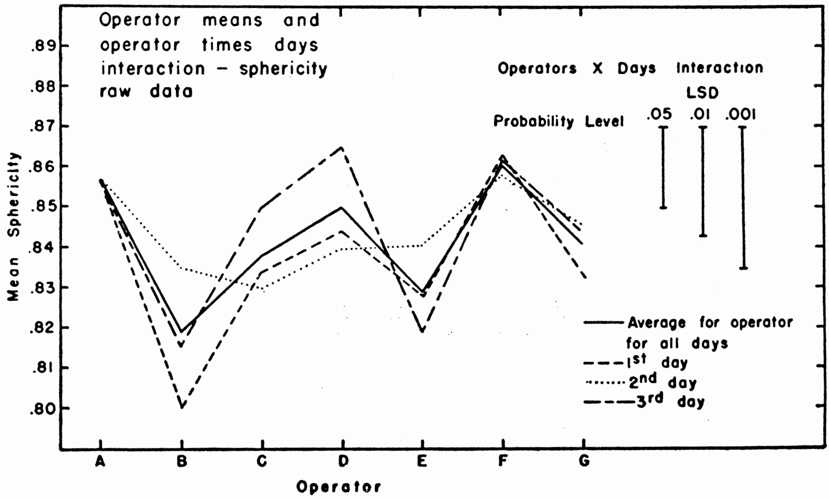


Fig. 5. Variation in sphericity for each operator, and for each operator on each day.

variance. In such a case we are justified in combining these two estimates to obtain a more efficient estimate of the error variance. The pooled estimate is found by adding the sums of squares for each term and dividing by the summed degrees of freedom; we obtain a mean square of 3.329 with 280 degrees of freedom and this value is recorded at the foot of table 4.

The next step is to test the interaction operators times days and this yields an F value of 4.056 when tested against the pooled error. The 0.1% point of the F distribution for 12 and 280 degrees of freedom respectively is 3.02, and hence we would expect such a large F value as that found very much less often than 1 time in 1000. We cannot but conclude that our initial hypothesis of no differences in the population is probably false in this case. Hence this interaction, the inconsistency of operators from day to day, contributes a significant amount of variance to the total. The variation of each operator for each day is illustrated in figure 5. Similarly the operators times grains inconsistency is significant at the 0.1% level.

In attempting to evaluate the main effects we have a choice of two tests and each test supplies somewhat different information. Thus in the case of differences in sphericity from day to day the logical error term might be days times grains or days times operators. Let us assume for the moment that differences between operators are omitted from consideration (e.g., the tests were performed by a single operator); then we may test differences between day means against days times grains or the more efficient estimate of the pooled error. The resultant F value of 3.88 exceeds the 5% F value of the equivalent F distribution ($F_{05} = 2.99$, $F_{01} = 4.60$) but is less than the 1% value. It seems clear that the differences between days significantly exceeds the experimental error and this suggests that it may be as well to include in any subsequent experiment a more extended period to evaluate differences between days before concluding that this source of variation may be neglected.

The second test of differences between the means for days may be made by comparison against the inconsistency of operators from day to day, and this yields an F value of less than unity. In effect, then, the contribution to variation of the inconsistency of operators from day to day reduces the contribution due to differences between days to negligible proportions.

The conclusion to be drawn from these two tests is that with a number of operators the inconsistency of operators from day to day is sufficiently large to overshadow differences due to measurements performed on different days; whereas when operator difference is removed, such as occurs when the tests are performed by a single operator, the day to day differences are sufficiently large to necessitate separate evaluation. These findings need confirmation from a more extended test over a longer period before final acceptance.

Similarly, differences between operators may be tested both against operators times days and operators times grains interaction. In both cases the results are substantially similar, both F values being significant beyond the 1% value of the corresponding F distributions.[4] Differences between operators are clearly established. The variation in operator means is contained in the graph of figure 5 (solid line).

Differences between grains tested against operators times grains interaction are also highly significant, the resultant F value far exceeding the 0.1% value of the F distribution for 20 and 120 degrees of freedom. The differences between grain means are illustrated in figure 6.

[4] In fact, operator differences tested against operator x grains interaction is significant beyond the 0.1% level.

TABLE 5

Comparison of the Analysis of Raw and Transformed Data for Sphericity

| Source of Variation | DF | Arc Sin Data F | Arc Sin Data Sig. Level | Raw Data F | Raw Data Sig. Level |
|---|---|---|---|---|---|
| Between Grains | 20 | 27.137 | $P < 0.001$ | 27.47 | $P < 0.001$ |
| | | 6.575a | $P < 0.01$ | 9.25 | $P < 0.001$ |
| Between Operators | 6 | 15.04b | $P < 0.001$ | 16.90 | $P < 0.001$ |
| | | 0.96c | Not sig. | 1.41 | $P > .20$ Not sig. |
| Between Days | 2 | 3.90d | $P < 0.05$ | 3.99 | $P < 0.05$ |
| Operators x Grains | 120 | 1.774 | $P < 0.001$ | 1.552 | $P < 0.01$ |
| Operators x Days | 12 | 4.056 | $P < 0.001$ | 2.836 | $P < 0.01$ |
| Days x Grains | 40 | 1.04 | $P > 0.20$ Not sig. | 0.875 | Not sig. |
| Error Stand. Dev. | .. | 0.024e | .. ... | 1.82 | .. ... |
| Coeff. Variation | .. | 2.87e | .. ... | 2.73 | .. ... |

a — tested against ops x days 6/12 DF

b — tested against ops x grains 6/120 DF

c — tested against ops x days 2/12 DF

d — tested against pooled error term 2/250 DF

e — the computations leading to analysis of variance were performed on coded sphericity values where $\Psi^1 = (\Psi_{\%} - .70)100$ whereas the error standard deviation and coefficient of variation have been decoded.

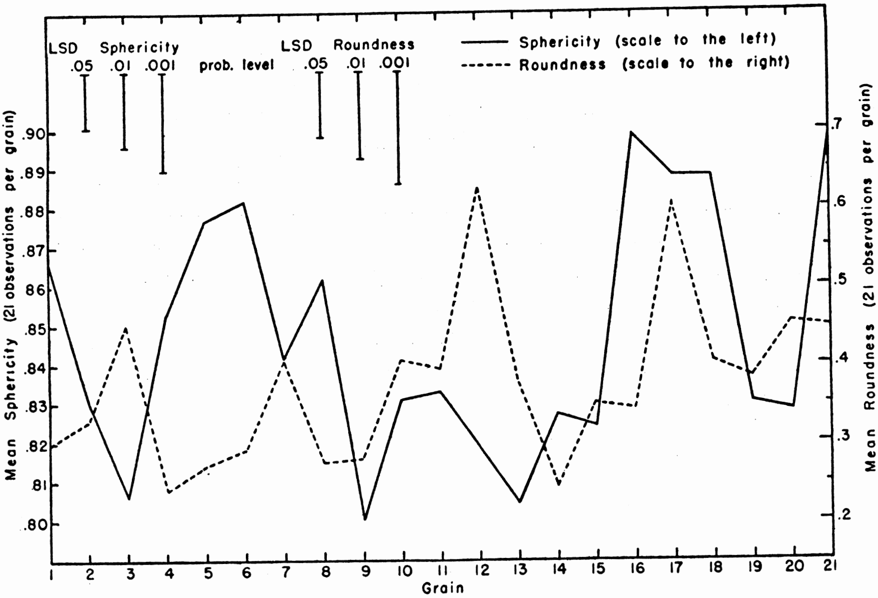


**Fig. 6. Variation in mean sphericity and roundness among grains.**

*Comparison of the analysis of raw and transformed data.*—The analysis of variance performed on the raw data and on the data transformed to its arc sin equivalent is displayed in table 5 in terms of the resulting F values and their corresponding significance levels. It can be seen that only minor changes in levels of significance occur and no differences would arise in the conclusions drawn from the analysis of the data based on either set of values. This is only to be expected in this case where the original frequency distribution does not suffer from the kind of aberration which the inverse sin transformation is used to ameliorate. Nevertheless, it is comforting to know that no great change occurs in using the transformation where it is not essential; if, for example, two sets of data were to be compared, one of which necessitated the arc sin transformation and the other did not, such a property of the transformation is a distinct advantage.

*Estimation of components of variance in sphericity.*—One of the advantages of the use of analysis of variance in the treatment of data arises from the fact that a suitable estimate of the variance contributed by each source may be obtained from the data. The items contributing to the variance of each factor together with their weighting coefficients are listed opposite the respective sources in the mathematical model of table 1. It will be noted, for example, that the variance arising from variation of operators from day to day—operators times days interaction—is composed of a contribution from error ($s^2_e$) and a second contribution from the interacting items ($s^2_{ND}$). The second component in this variance is based on the 21 grain means and the coefficient g is used to make the estimate conform with the error variance.[5] Thus we have an estimate of the operators times days interaction as $s^2_e + gs^2_{ND}$; we also have an estimate $s^2_e$ for the error variance, and so the magnitude of the variance for the interaction ($s^2_{ND}$) may be obtained by simple arithmetic. The variance for each term in the analysis is listed in table 6.

It can be seen that the largest contribution to variance, or the greatest source of variation, arises from differences between grains, and this is almost twice the next largest contribution from unassigned sources—the error term. The third largest contribution is from differences between operators and this is about one-fifth of the variance from differences between grains.

These components of variance clearly indicate that there must be many sources contributing to error, and from the magnitude of this residual variation there are other effects which could be profitably introduced into the analysis. In addition, this analysis of the variance components suggests that should the range of variation in sphericity between grains be reduced then the differences between operators will show a relative increase.

Again it can be seen that the variance arising from inconsistency of operators from grain to grain is about twice as large as that from the inconsistency of operator from day to day. The variance contributed by differences between days is negligible.

[5] This rule arises from the relationship between the variance of means and the variance of the raw items, i.e., $ns^2_{\bar{x}} = s^2_x$ where $s^2_{\bar{x}}$ is the variance between the means, $s^2_x$ is the variance of the original items, and n is the number of items used in calculating each mean (Crump, 1946).

TABLE 6

Components of Variance in Sphericity Estimation

| Source of Variation | Coded Raw Data: Significance Level P | Coded Raw Data: $s^2$ | Coded Raw Data: s | Decoded s | Inverse Sin Data: Significance Level P | Inverse Sin Data: $s^2$ | Inverse Sin Data: s |
|---|---|---|---|---|---|---|---|
| Between Grains ......... | $< 0.001$ | 11.2590 | 3.35 | 0.033 | $< 0.001$ | 7.3415 | 2.71 |
| Between Operators ...... | $< 0.001$ | 2.0877 | 1.44 | 0.014 | $< 0.001$ | 1.1542 | 1.07 |
| Between Days ........... | N. S. | 0.0455 | 0.21 | 0.002 | N. S. | 0 | 0 |
| Operators x Grains ...... | $< 0.01$ | 1.0234 | 1.0117 | 0.0101 | $< 0.001$ | 0.8583 | 0.93 |
| Operators x Days ....... | $< 0.01$ | 0.4981 | 0.7058 | 0.007 | $< 0.001$ | 0.4845 | 0.6961 |
| Grains x Days .......... | N. S. | 0 | 0 | 0 | N. S. | 0.0207 | 0.1440 |
| Error .................. | .... | 5.8601 | 2.4208 | 0.024 | .... | 3.3080 | 1.8188 |
| | | Coded | | Decoded | | | |
| $\overline{X}$ | .... | 14.2290 | ..... | 0.842 | .... | 66.8016 | ..... |
| Cv % | .... | ...... | ..... | 2.87 | .... | 2.73 | ..... |

The standard deviation derived from the variance is also listed in original units in the column headed "decoded s" but it must be remembered that the relationships between the standard deviations are not as simple as those between the variances. The coefficient of variation is computed from the error standard deviation (square root of the error mean square) and the grand mean of the entire experiment[6] and is considered to be a reflection of the experimental control; in general, a coefficient of variation less than 20% is reasonable and less than 10%, good.

The comparison of variance components based on estimation using the raw data and the transformed data are also included in table 6. The relative magnitudes are certainly not the same, but the conclusions based upon them are, so that, in this case, the use of the transformation is evidently not necessary.

It must be emphasized that this simple technique for the evaluation of the variance contributed by each factor in the analysis is based upon assumptions concerning the contributions to variance in the original data. One such assumption, implicit in the manipulations leading to separate evaluation of the variances associated with each factor, is the additivity of the variance from each source. If the original frequency distribution were normal such conditions would hold, but in non-normal distributions it is necessary to demonstrate the independence of mean and variance for such conditions to hold. Transformation of the variable is one approach to this problem and accounts for the attempt to use the inverse sin conversion in this investigation (Bartlett, 1947; Eisenhart, et al., 1947). In the case of sphericity determination, however, although the frequency distribution is non-normal, the mean does not appear to be related to the variance in any systematic manner. The use of the arc sin transformation is not therefore mandatory in this case. In general, however, it is as well to ascertain empirically that additivity is likely before the analysis of components of variance is attempted (Crump, 1946).

[6] According to the formula $Cv\% = \frac{E.M.S.}{\overline{X}_G} \times 100$; where Cv = coefficient of variation, E.M.S. = error mean square, and $\overline{X}_G$ is the grand mean.

TABLE 7

Analysis of Variance

Roundness of 21 Quartz Grains Determined by 7 Operators on each of 3 Days (regression adjusted data)

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Squares x $10^4$ | F | $F_{.05}$ | $F_{.01}$ | $F_{.001}$ | DF used for test |
|---|---|---|---|---|---|---|---|---|
| Between Grains ............ | 20 | 0.029828 | 14.914 | 28.96*** | ... | ... | 2.40 | 24/120 |
| Between Operators ......... | 6 | 0.004612 | 7.686 | 14.92*** | ... | ... | 4.04 | 6/120 |
| Between Days ............. | 2 | 0.000049 | 0.245 | 1.0 | ... | ... | ... | 2/120 |
| Grains times Operators ..... | 120 | 0.006180 | 0.515 | 1.9360*** | 1.32 | 1.48 | ... | 120/240 |
| Operators times Days ...... | 12 | 0.000401 | 0.334 | 1.255 | 1.80 | ... | ... | 12/240 |
| Days times Grains .......... | 40 | 0.000910 | 0.2275 | 0.8553 | ... | ... | ... | 40/240 |
| Days times Grains times Operators (Error) ....... | 240 | 0.006373 | 0.2655 | ..... | ... | ... | ... | ....... |
| Total ..................... | 440 | 0.048353 | 1.0989 | | | | | |

Pooled Error Term = 0.2631; DF 292

## SOURCES OF VARIATION IN THE VISUAL ESTIMATION OF ROUNDNESS

*The analysis of variance.*—The analysis of variance of roundness in terms of the regression adjusted data is illustrated in table 7 and it will be unnecessary to repeat the steps in detail because the same procedure is followed as that described in the analysis of the sphericity data. The results of the tests and minor modifications appropriate to the different data are outlined in the following paragraphs.

Testing the days times grains and operators times grains mean square against that for error yield F values which are not significant. The variance for these terms may then be pooled, and the pooled error is recorded at the foot of table 9. Little is gained by this manipulation, and for subsequent tests the unmodified error term is used for testing the grains times operators interaction. This is significant beyond the 0.1% point of the F distribution for 120 and 240 degrees of freedom, and the F value would only be increased by using the pooled error term. Clearly the inconsistency of operators from grain to grain makes a considerable contribution and becomes the basic error for comparison with main effects.

The difference between days yields an F value less than unity when tested against either the pooled error or the grain times operator interaction. Differences from day to day are therefore not significant.

Differences between operators and between grains are each significant beyond the 0.1% level of the respective F distributions. Hence these differences are highly important; the magnitudes of variation for these two main effects are included in figures 7 and 6 respectively.

*Comparison of the analysis of raw and transformed data.*—The comparison of the results of analysis of variance using raw data, arc sin and regression adjusted values is summarized in table 8. No difference in the conclusions to be drawn from the analysis would arise whichever kind of data were followed. This serves to illustrate the degree of relaxation of the assumptions of normality which may be permitted without affecting the levels of significance in using the analysis of variance procedure for testing the differences between means.

*Estimation of the components of variance in roundness.*—An estimate of the contribution to variation arising from each term in the analysis is recorded in table 9, and the results

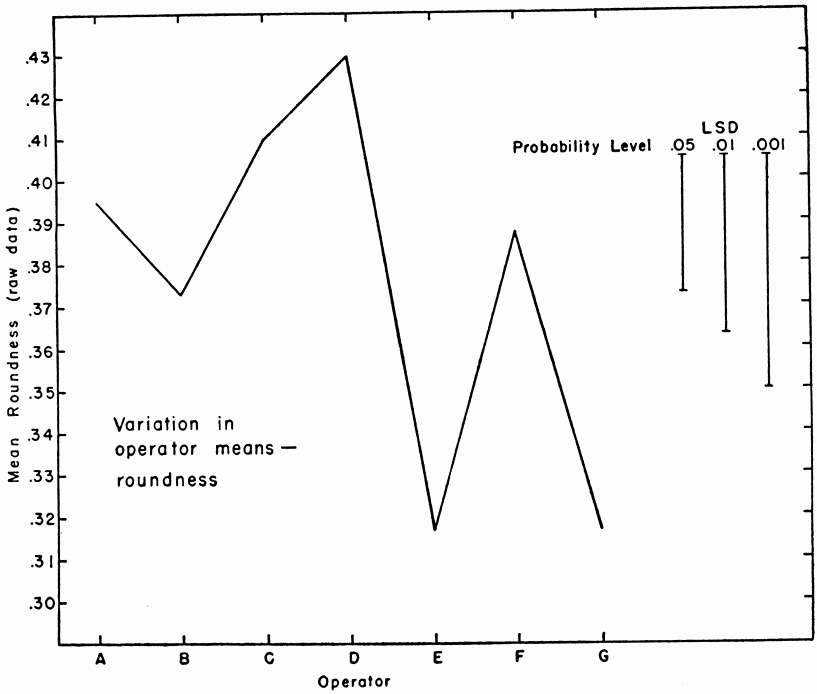


**Fig. 7. Variation in mean roundness among operators.**

of this computation using the raw data, the arc sin transformed data and the regression adjusted data are included for comparison.

The qualitative treatment of these values by comparing their relative magnitudes in terms of each of the respective scales would hardly differ in any important respect, but it should be emphasized that for further manipulations the most reliable estimate is that based on the regression adjusted data. This conclusion is predicated upon the fact that in both raw and arc sin data the mean is related to the variance and conditions upon which the computations of the estimates of variance are based do not hold. In the case of the regression adjusted data the mean is independent of the variance and additive relations between the components of variance are most likely. The subsequent discussion utilizes all three estimates, but attempts to compute the number of items necessary for the reduction of any variance component for future experiments must rely solely on the regression adjusted data.

TABLE 8

Comparison of Analysis of Variance of Roundness Estimation in Terms of Observed, Arc Sin and Regression Data

| Source of Variation | DF | Observed Data | | Arc Sin Data | | Regression Adjusted Data | |
|---|---|---|---|---|---|---|---|
| | | F | Significance Level P | F | Significance Level P | F | Significance Level P |
| Between Grains ............ | 20 | 27.77 | $\ll 0.001$ | 13.33 | $\ll 0.001$ | 28.96 | $\ll 0.001$ |
| Between Operators ........ | 6 | 14.509 | $\ll 0.001$ | 27.01 | $\ll 0.001$ | 14.92 | $\ll 0.001$ |
| Between Days ............ | 2 | 1.257 | N. S. | 1.380 | N.S. | $<1$ | N. S. |
| Operators x Grains ........ | 120 | 2.142 | $\ll 0.001$ | 1.714 | $\ll 0.001$ | 1.936 | $<0.001$ |
| Operators x Days ......... | 12 | 1.168 | N. S. | 0.827 | N. S. | 1.255 | N. S. |
| Days x Grains ............ | 40 | 1.048 | N. S. | 0.876 | N. S. | 0.855 | N. S. |
| Error .................... | 240 | | | | | | |

The main source of variation arises from the differences in roundness between the different grains, and this term is at least twice that of the next largest source of variation, the error term. The variation associated with the error term is unassigned so that we may consider that the experiment has reduced the error variation to less than half of the variation we wish to determine. Evidently differences in roundness between grains may be established by the visual comparison technique.

The third source of variation in order of magnitude arises from differences between operators, and this is approximately one-sixth of the variation arising from differences in roundness between grains. The next source, the interaction term, operators times grains, also contributes an important part of the total variation. In this experiment the basic error term for the comparison of main effects is this interaction term, and hence the inconsistency of operator from grain to grain determines the level of sensitivity of the visual comparison technique. It can be realized from the form of the analysis that differences between operators can be segregated but their inconsistency determines the magnitude of the difference in roundness between grains which can be detected by this technique. It should be emphasized that any reduction in the variation in roundness between grains will result in a relative increase in magnitude of the inconsistency; hence in any cooperative experiment this term should always be included and evaluated as a measure of sensitivity of the experiment.

The other sources, differences between days, and the respective interactions, operators times days, and days times grains do not contribute in any appreciable degree to the variation and may be included in the error term without inflating that item if future experiments confirm the present results.

The recording of the decoded standard deviation in table 9 is included merely as an appropriate guide to the relative importance of the several factors. In these experiments as shown by the coefficients of variation the use of the regression adjusted data has resulted in an improvement in the experimental control.

## DISCUSSION AND CONCLUSIONS

It should be clear from the preceding analysis that valid differences in both sphericity and roundness of quartz grains may be established by the use of visual comparison techniques.

TABLE 9

Comparison of Components of Variance in Roundness Estimation

| Source of Variation | Raw data Mean square ($x10^4$) | Raw data decoded s | Arc Sin Mean square | Regression adjusted Mean square ($x10^4$) |
|---|---|---|---|---|
| Between Grains | 105.73 | 0.103 | 38.633 | 0.686 |
| Between Operators | 17.69 | 0.042 | 6.138 | 0.113 |
| Between Days | 0.015 | 0.001 | .... | .... |
| Operators x Grains | 14.75 | 0.038 | 4.318 | 0.083 |
| Operators x Days | 0.310 | 0.006 | .... | 0.003 |
| Days x Grains | 0.270 | 0.005 | .... | .... |
| Error | 38.75 | 0.062 | 18.152 | 0.265 |
| Cv % | 16.6 | .... | 11.35 | 6.41 |

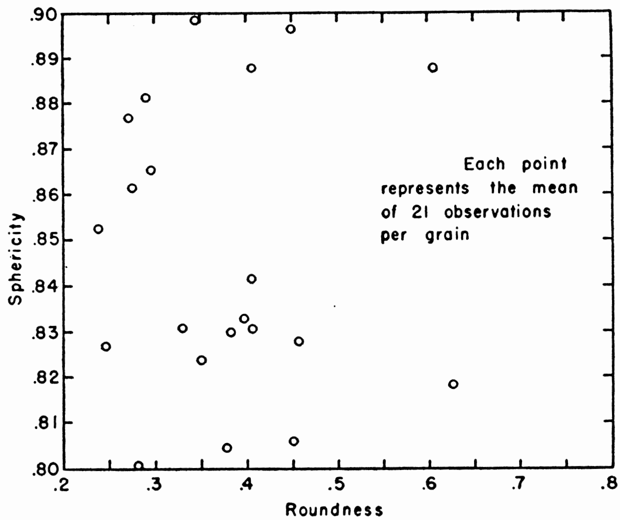


**Fig. 8. Plot of mean sphericity versus mean roundness for 21 observations per grain.**

It is, furthermore, confirmed that variation in roundness is larger than that in sphericity.

The magnitudes of the variation may be compared by means of figure 6, and the statistic "least significant difference" (LSD) may be considered as a very approximate measure of the least differences which can be detected in each case.[7] It would appear to be prudent to measure sphericity rather than estimate it, both because measuring techniques available are simple and rapid and because the entire range of variation in sphericity is not likely to be much larger than that included in this experiment.

In any future experiments some measure of operator variation should be included, and in cooperative experiments including more than one operator the inconsistency of operators from grain to grain should also be evaluated. It must be emphasized that where this has not been carried out the differences in either sphericity or roundness may not be solely ascribable to geological factors but include experimental or technique error.

[7] This measure should not be taken too literally because it achieves precise meaning only under very restricted conditions and particularly where the frequency distributions of the measured variable are homogeneous and closely approximate normality.

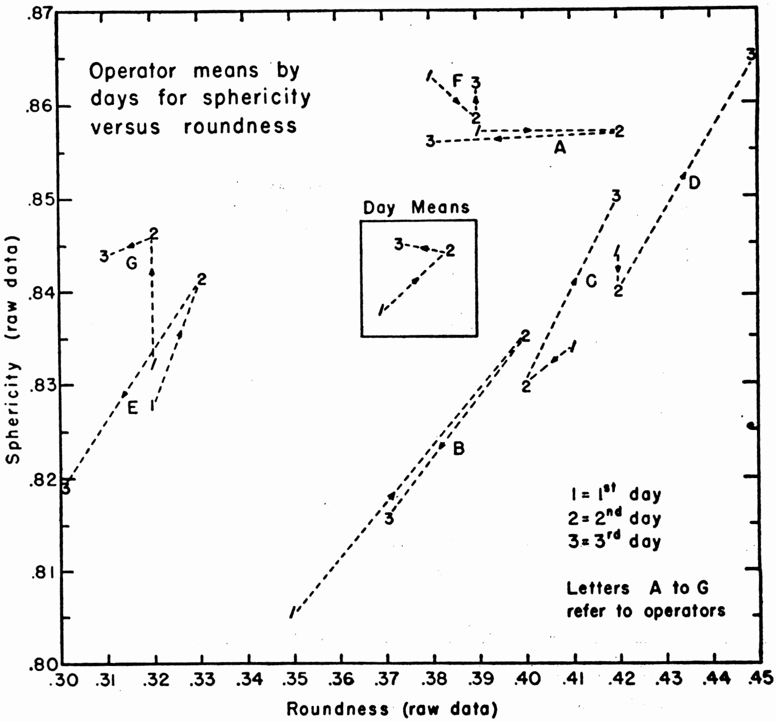


**Fig. 9. Plot of mean sphericity versus mean roundness for 21 observations per operator.**

It is often considered that sphericity and roundness tend to vary sympathetically. The means of the 21 independent estimates of sphericity and roundness for each of the 21 grains are plotted in figure 8. No obvious trend appears.

As a matter of interest, if we plot the mean sphericity and roundness for each operator, however (fig. 9), a distinct trend is shown. Firstly, the approximate slope of the trends suggests that operators tend to react in sympathy to visual estimates of roundness and sphericity. In other words, when an operator estimates high sphericity, he also tends to estimate high roundness and vice versa.

Secondly, there is a reversal in the case of five of the seven operators which shows that on the first and third occasions (days 1 and 3) three of the operators measured high sphericity and roundness, whereas on the second day these three estimated relatively lower roundness and sphericity on the same grains. Two other operators showed the reverse tendency. This is presumably connected with an arrangement of the technique; some of the operators remarked that there was a tendency to compare the first grain with the comparison chart and thereafter to compare each grain with the preceding one. To counteract this effect it was suggested (by M.A.R.) that the operators run through the grains from numbers 1 to 21 the first day, reverse the order of grains the second, and return to the original order the third. The reversal of the estimates is doubtless connected with this reversal in arrangement. The inconsistency in the reaction of the three operators contrasted with that of the two operators reduced the effect of differences due to days—or more correctly, due to order of the grains.

As was noted during the analysis, the differences between day means in sphericity were significant at the 5% level (table 4), but the differences between day means for roundness were not significant. Similarly, variation of grains from day to day as estimated by the interaction term grains times days was not significant; nevertheless, plotting the day means (inset, fig. 9) shows again the sympathetic variation of sphericity with roundness.

The moral is obvious, a more extended test of day to day or order of treatment of grains is necessary before these items can be considered relatively unimportant. In practice it would be advisable to randomize the order of treatment of grains on each occasion. Furthermore, the tendency among operators to record high sphericity along with high roundness and vice versa when using visual comparison techniques suggests some care in accepting sympathetic variation in sphericity and roundness as an attribute of grains.

An interesting item in the estimation of roundness is emphasized by the relationship between the mean and variance of the 21 grains (fig. 4). The regression indicates that the variance increases with decreasing mean roundness, which in turn implies that operators can estimate with less ambiguity differences in roundness in well-rounded grains and are increasingly inconsistent as the grain becomes more angular. In other words smaller differences can be detected in grains of high roundness than in grains of low roundness. This is in complete contradiction to the conclusion stated by Pettijohn (1949, p.

51), " . . . because the eye can readily distinguish slight differences in roundness when the roundness values are low, but cannot make such distinctions when the values are large." If future experiments confirm our findings, then the roundness scale suggested by Pettijohn should be reversed and more classes included in the higher range of roundness than the lower. Theoretical analysis of these problems is always insidiously deceptive, but it does appear reasonable to conclude that small departures from a smooth rounded surface are easier to detect than on a serrated one! It seems abundantly clear to the writers that subjective classification, particularly by visual estimates, even with comparison scales such as the charts used in this experiment, are no guarantee of valid conclusions from the observations unless the experiment is suitably designed; it is essential to evaluate the different sources of variation and establish that the differences observed are associated with real differences in the variable estimated and not introduced by lack of control of the psychological bias present in every observer.

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