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EQUILIBRIUM THEORY OF EROSIONAL SLOPES APPROACHED BY FREQUENCY DISTRIBUTION ANALYSIS

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PART I

ABSTRACT. In the quantitative analysis of erosional landforms one of several form-elements requiring measurement is slope angle of valley walls. Field sampling of slope angles in several mature regions differing widely in lithology, relief, vegetation, climate and soils has yielded data suited to frequency distribution analysis. Determination of means, estimated standard deviations, skewness and normal distribution fitness have provided a quantitative basis for slope description and significance tests.

Distributions show a wide range of means, but are symmetrical and have low dispersions. This is taken as evidence of a prevailing condition of form-equilibrium accompanying a steady state in an open system of erosion and transportation. Comparison of valley-wall slopes with adjacent channel gradients reveals a strong, positive correlation, indicating a high degree of adjustment among component parts of a drainage system.

Tests for significance of differences in slope means of three localities in the Verdugo and San Rafael Hills showed that (1) factors other than lithology exert major control over observed differences in slope angles, (2) directional exposure has no significant effect upon slope angles, and (3) slopes left to weathering, sheet wash and creep without stream corrasion at the base have reclined in angle.

INTRODUCTION

A LARGE proportion of the earth's land surface is composed of erosional landforms produced by channel erosion in drainage networks operating in conjunction with raindrop and sheet-runoff erosion and mass gravity wasting on the contributing slopes. In a mature stage of development the landscape consists of an intricate combination of channels, slopes and divides. Although the same unit form is not exactly repeated throughout this type of topography, an observer cannot fail to recognize that the forms are approximately the same throughout an area having a uniform lithology, geologic history, climate, soil and vegetation. Examples may be cited from the intricately dissected California Coast Ranges; from the monotonously repetitious landforms of the dissected Appalachian Plateaus; or from the marl hills of badlands in northern Arizona.

Although our understanding of erosional landforms is still in a stage of generalized verbal description, some attempts have been made to separate the various elements of form from the topographic complex. Robert E. Horton (1945) has analyzed drainage network composition in terms of drainage density, stream frequency, stream number, stream order, and bifurcation and length ratio.

Despite the apparent complexity of a quantitative analysis, it is possible to define and measure the various form-elements which comprise a fluvially dissected landscape. This is the first step in an over-all investigation aimed at relating quantitatively the influence of causative factors to form characteristics. Slope is one of the form-elements and is the principal topic of this paper.

In considering quantitatively the element of slope in an erosional landmass it is possible to measure and compare (1) forms of slope profiles as mathematical curves, (2) lengths of slopes from divide to base (scale-aspect of the topography), and (3) angles of slope with respect to the horizontal (gradient-aspect of the topography). Because the principal concern of this paper is with the third measure, the first two are discussed only briefly.

In a maturely dissected region of homogeneous rock the common slope profile, when taken along an orthogonal line with respect to the contours, tends to be straight in its middle and lower parts (fig. 1). At the upper end the profile becomes convex-up, passing into the rounded divide. Where streams are effectively corrading a channel at the slope base the profile continues to the base as a nearly straight line (plate 1). Concavity at the base, so commonly shown in elementary textbooks, seems to occur only where accumulations of slopewash or slide-rock lie at the slope base, or where structural control is present. Straightness of profile has been noted by Lawson (1933) in the California Coast Ranges and by the writer in that same province as well as in many other places in the United States. It may be seen in the bold mountains of the Great Smoky mass as well as in small-scale badland forms in weak clays. The common illusion of generally concave-up slopes comes from viewing profiles of lateral spurs or other lines which are not true orthogonals.

Length of slope, a function of the general scale of the drainage network, is related to the infiltration capacity and resistivity of the surface to corrasion and other factors and is therefore left to be treated elsewhere in quantitative considerations of drainage density and its variations.

Slope steepness is a particular form-element which lends itself to analysis by the standard methods of frequency distribution statistics. Even the most casual geological observer has noted that slope steepness varies widely from one region to another, but that it seems fairly constant within the limits of a small area of uniform relief and rock composition. Thus slopes may be said to reflect a steady state in the rate of removal of debris and the rate of supply of debris. Slopes seem to tend toward a constant angle which is related to the particular conditions of

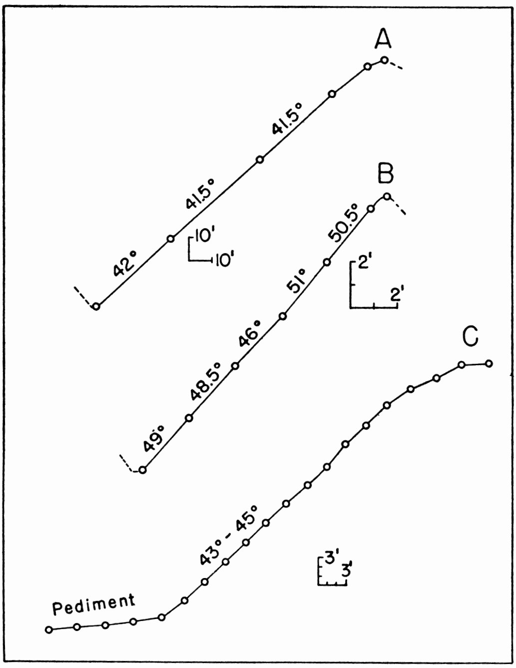

Fig. 1. Profiles of three valley-wall slopes surveyed in the field. (A) Verdugo Hills, Los Angeles Co., California. (B) Dissected clay dump, Perth Amboy, N. J. (C) Petrified Forest National Monument, Ariz.

soil, bedrock, vegetation, climate and relief, and which is integrally adjusted with conditions of channel gradient and stream regimen.

The concept of equilibrium has long been applied to graded streams and their associated slopes but the nature of this equilibrium and its basic similarities with other systems of equilibrium in nature seem not to have been fully examined. A graded drainage system is perhaps best described as an open system in a steady state (von Bertalanffy, 1950) which differs from a closed system in equilibrium in that the open system has import and export of components. An example of an open system in physics might be the flow of heat through a conducting solid. When a steady state is attained the temperature at a given point remains constant while the flow of heat continues. In biology the metabolism of a stable organism is best described as operating in a steady state requiring an exchange of materials with the environment and a continuous building up and breaking down of materials within the organism. Open systems consume energy to maintain a steady state, whereas closed systems require no energy for the maintenance of equilibrium. Still another property of the open system is that a disturbance in the flow of materials or energy will cause a readjustment to take place until a time-independent steady state is reestablished.

In a graded drainage system the steady state manifests itself in the development of certain topographic form characteristics which achieve a time-independent condition. (The forms may be described as "equilibrium forms".) Erosional and transportational processes meanwhile produce a steady flow (averaged over periods of years or tens of years) of water and waste from and through the landform system. Potential energy of position is transformed into kinetic energy of water and debris movement or heat. Over the long span of the erosion cycle continual readjustment of the components in the steady state is required as relief lowers and available energy diminishes. The forms will likewise show a slow evolution.

Applied to erosion processes and forms, the concept of the steady state in an open system focuses attention upon the relationships between dynamics and morphology. With this concept in mind, the first steps in measurement and description of slope forms typical of the steady state can be undertaken.

ACKNOWLEDGMENTS

The writer wishes to thank Dr. Howard Levene of the Department of Mathematical Statistics, Columbia University, for critically reading the paper and suggesting improvements. Field work in 1947 was supported by the Kemp Memorial Fund of Columbia University and in 1948 and 1949 by Grant 525-48 from the Penrose Bequest of the Geological Society of America.

GENERAL PRINCIPLES

Method of Obtaining Field Data.—Slope sampling requires decision as to (1) type of data wanted, (2) rules of sampling to be followed in the field, and (3) size of sample required.

In the present investigation it was required that only the steepest part of a profile line of slope be measured; furthermore, that the profile line must follow the true down-slope line from divide to base of slope (the orthogonal line with respect to contours). Following this rule the observer is prevented from having a choice as to what part of a profile to select; the use of apparent, rather than true maximum slopes is avoided. Thus any two or more observers should obtain the same angle of slope from any given point in the field, subject only to errors of measurement.

There is a more important and fundamental reason for using the steepest part of a slope profile. It is postulated that in mature topography a slope is the morphological manifestation of a steady state in which the forces which remove material are so adjusted to the resistive properties of the surface as to provide a steady supply of debris to the streams. The steepest part of a slope will then reflect the maximum angle which can be maintained, and is an indicator of the relative effectiveness of the opposed forces in the equilibrium relationship. Because the slope at its upper part declines in angle to become horizontal on the divide, angles less than the maximum have little significance and their use would bring in a broad element of subjective sampling. As the ultimate aim in geomorphology must be a dynamic understanding, it is essential that those form-elements be measured which can be most easily related to causative forces.

In field sampling, only valley-wall slopes leading down into the sides of a channel were used. Slopes leading into the extreme head of a ravine were not used because of the element of strong convergence present. Slopes down the axes of spurs, where lateral divides descend to a main stream, were not used because these are not true orthogonals. Drainage basins of the first and second, sometimes third orders were used. Avoidance of higher order master streams eliminates the reading of slopes abnormally steepened by strong lateral corrasion of a large stream.

Within each area an attempt was made to take slope readings at approximately equally spaced positions along the contour. In the case of fine-textured badlands these were spaced only 5 to 10 feet apart, or about one-fourth to one-half of the prevailing slope length. In medium-textured topography with slope lengths of 100 to 300 feet the spacing was 100 to 150 feet. This raises the question of how close together any two profile lines can lie before they can be considered as one statistical item in a frequency distribution, rather than as two separate items. The problem exists because the sampling is from a continuous surface which varies in orientation from place to place, rather than from a population of discrete pieces or objects, as for example, a clastic sediment. Because all movement of detritus on a slope takes place along the true down-slope profile lines, one would not have to move far laterally along a slope to encounter relatively independent threads of debris movement. With the spacing used by the writer successive profiles were separated by what seemed to be a relatively broad belt of slope, equivalent to one-half to one-third of the slope length itself; hence no mutual interference seemed likely.

Slope angles were read to the nearest degree or half degree with Abney hand level or Brunton compass. A class interval of 2° or more is desirable in the frequency distribution analysis, because the range of error for any item is almost a half degree. On mountain or hill slopes the steepest down-slope line was determined by rolling boulders or pebbles down-slope, or by selecting the maximum of a range of readings aimed down-slope through a lateral arc. When a two-man team is used, each man takes a position on the slope line. The instrument man then reads the slope to the eye-level of his partner. In this way minor ground irregularities and brush can be overcome. When one is working alone, he can put up a perpendicular rod with the eye-level marked, or make a mark at eye-level on a convenient tree trunk or bush. Slopes were read over lengths equivalent to one-fourth or one-fifth of the slope length so as to avoid minor irregularities made by boulders or clumps of vegetation. As most slopes are relatively straight or steepen toward the base the measurement is taken near the base. Some slopes have a basal concavity due to local accumulations of slopewash or talus; the slope is then read above the zone of accumulation. In the case of small-scale badlands a 3-foot board was laid on the slope and the measurement taken as with dip of a bedding plane.

It is desirable to read the azimuth of each slope measurement so as to permit study of the effects of direction of exposure on slope angle. Readings should be located on the field map by a station number corresponding to the notebook record. This permits statistical analysis between individual basins or groups of basins.

Sample sizes taken by the writer ranged from 20 to 200 readings. Subsequent statistical analysis has shown that a sample of 50 slopes is adequate to reveal the characteristics of the distribution in a homogeneous region of 6 to 12 first-order drainage basins; while a sample of 25 will closely fix the mean.

The use of topographic maps as sources of slope data has been investigated and tests run between field and map readings of the same slopes. The statistical analysis of this procedure is treated in a later section of this paper. Regarding collection of data from maps the following conclusions have been reached: Maps on a scale of 1:24,000 or 1:25,000 published by the U. S. Geological Survey and the Army Map Service in recent years will give reasonably accurate results where topography is coarse-textured and the relief strong. These maps are prepared by photogrammetric methods and comply with National Standard map accuracy requirements. It was found that maps of a scale of 1:31,680 in Los Angeles County are unsuited to slope sampling because the topographic texture is too fine for the map scale and contour interval; *i.e.*, slopes are too short to be represented accurately under the cartographic conditions. Maps of the scale 1:62,500 are generally beyond the limits of reasonable accuracy and should be entirely avoided for slope sampling.

General Characteristics of Slope Frequency Distributions.—The methods of slope sampling described above were applied to several areas differing widely in lithology, soil, relief, vegetation, and climate in order to obtain some idea of the range of characteristics present and the degree of homogeneity within each area. Figure 2 shows several histograms drawn on a common abscissa to show comparative positions on the slope scale 0° to 90°. For each distribution the following measures are given:

N Number of items in the sample.

$\overline{X}$ Arithmetic mean.

s Estimated standard deviation.

Cumulative frequency percentage distributions for the same samples are shown in figure 3. Both step-graphs, showing individual classes, and smoothed curves are shown.

With regard to general distribution properties, two points are especially noteworthy: (1) Dispersions are low, considering the range of angle possible, giving evidence of a high degree of homogeneity in the populations sampled. (2) Distributions are generally symmetrical, the principal exception being sample D. Inasmuch as sampling was carried out according to previously set rules and no readings were discarded

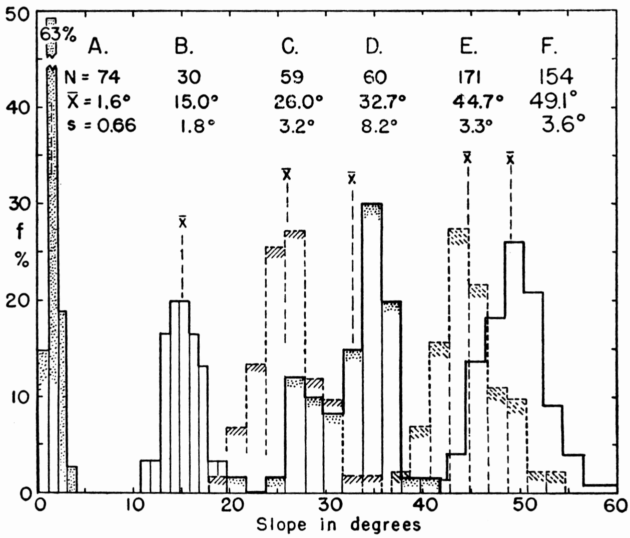

Fig. 2. Histograms of six slope frequency distributions. (A) Steenvoorde, France, (B) Rose Well gravels, Arizona, (C) Bernalillo, N. M., Santa Fe formation, (D) Hunter-Shandaken area, Catskill Mts., N. Y., (E) Kline Canyon area, Verdugo Hills, Calif., (F) Dissected clay fill, Perth Amboy, N. J. All data except (A) from field readings.

arbitrarily, either during the field sampling or during the processing of the data, the low dispersions and symmetry are interpreted as an indication that within a given small region where conditions of lithology, climate, soil, vegetation and relief are uniform, slopes tend to approach a certain equilibrium angle, appropriate to those controlling factors.

Slope Distributions and the Normal Curve of Error.—The problem of whether the distribution of maximum slopes in a selected region follows the normal curve of error was considered worthy of analysis in view of the possibility that the distribution is inherently skewed and is better represented by a logarithmic-normal curve, or some other variety of normal curve, such as one in which the abscissa is on a scale of tangents or log-tangents.

Three methods were used in analyzing the form of the distributions: (1) A plot on arithmetic probability paper. (2) A fitted normal curve and χ^2 test of goodness of fit. (3) Determination of significant skewness and kurtosis based on the third and fourth moments of the distribution.

A rough test of normal distribution may be had by plotting cumulative percentage frequencies on arithmetic probability paper. This paper is constructed to show the normal curve of

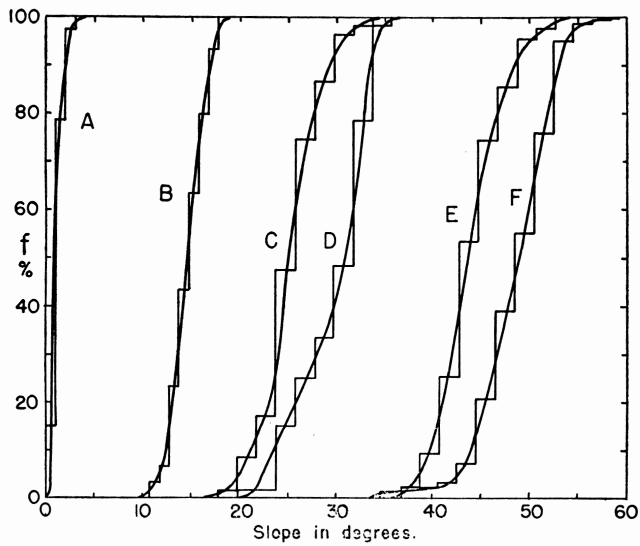

Fig. 3. Cumulative percentage frequency distributions. Same data as in figure 2.

error (cumulative) as a straight line. If the distribution appears reasonably straight one can proceed to fit a normal curve to the data. An additional feature of the cumulative curve on probability paper is that the slope is proportional to dispersion, hence the differences in dispersions between two normal distributions can be noted at a glance.

Figure 4 shows the data of figure 3 plotted on arithmetic probability paper. End classes containing only one item have been combined to compensate for the influence of scarcity. With the exception of sample D and the lower tail of sample F a reasonable straightness prevails, suggesting that the distributions are well described by normal curves, which may now be fitted to the data.

Fitting of the normal curve to the slope data of sample C, an area of dissected Santa Fe formation near Bernalillo, New Mexico, is illustrated in figure 5.

Goodness of fit has been determined by use of a χ^2 test in which theoretical frequencies and observed frequencies are compared as follows: (Croxton and Cowden, 1946, p. 286)

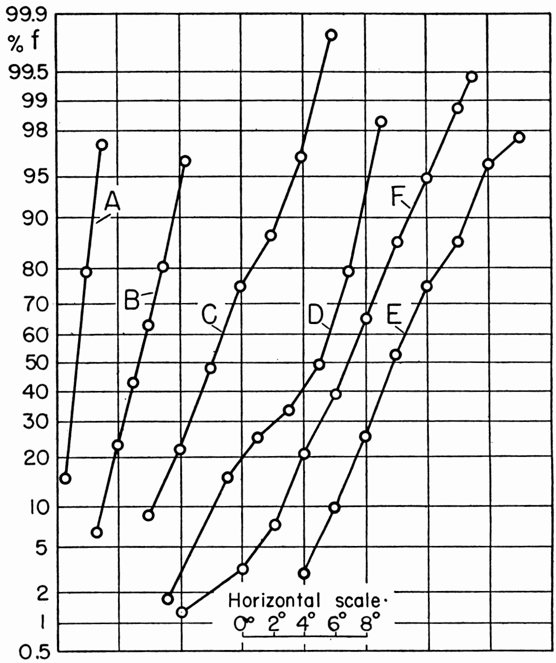

Fig. 4. Cumulative percentage frequency distributions on arithmetic probability paper. Same data as figures 2 and 3.

$$\chi^2 = \Sigma \frac{(f - f_c)^2}{f_c} = 0.631 \qquad \text{where}$$

$f \;= $ observed frequency
$f_c = $ expected frequency

The value n, representing the number of degrees of freedom, is obtained by subtracting the three degrees of freedom lost in the curve-fitting process from the original number of classes, seven, and is therefore equal to four. By consulting a prepared table we find for these values of χ^2 and n a probability, P, of approximately 0.97. Thus a normal curve of error is an excellent description of the distribution and, if the distribution is in fact normal, we should expect to find this good a fit or better in only 6 times out of one hundred such samples because of chance variations due to sampling only.

Tests of significant skewness and kurtosis were carried out on the raw data of three frequency distributions of slopes in the Verdugo and San Rafael Hills, Los Angeles County, California[1].

The purpose of the investigation was to determine if the various amounts of skewness and kurtosis visible in the three distributions are of such an order as to be commonly ob-

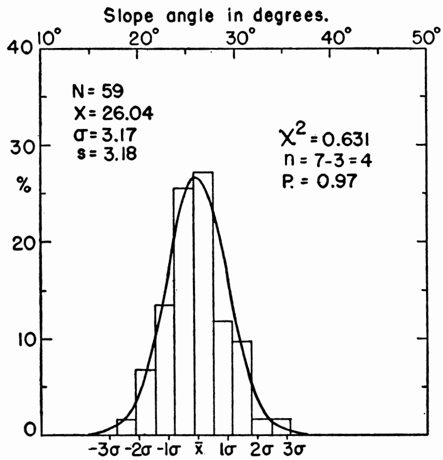

Fig. 5. Normal curve of error fitted to a frequency distribution of slope angles from the dissected Santa Fe formation near Bernalillo, New Mexico. Slopes measured in field.

[1] The analysis was carried out under the direction of Mr. Richard Ostheimer, Instructor in Statistics, Columbia University. Moments were computed from the raw data in order to remove possible influences of grouping.

tained by chance variations in sampling of a normal population, or whether they are significant of actual departures from normal in the population (fig. 6).

Skewness was determined from the value of β_1, a measure of skewness based upon the third moment of the frequency distribution:

$$\beta_1 = \frac{N\ (\Sigma x^3)^2}{(\Sigma x^2)^3}$$

Table 1 shows the results and conclusions for samples A, B, and C of figure 6. Samples B and C showed no significant skewness; sample A is doubtful of interpretation. Kurtosis was determined by the value of β_2, a measure of kurtosis based upon the fourth moment of the frequency distribution:

$$\beta_2 = \frac{N\ \Sigma x^4}{(\Sigma x^2)^2}$$

TABLE 1

Summary of analysis of skewness in three sample slope frequency distributions.

DETERMINATIONS MADE	Area A	Area B	Area C
β_1 (Distribution symmetrical when β_1 is 0; Right skewness when β_1 is +; left skewness when β_1 is −.)	+0.153	—0.005	—0.103
Probability of sample β_1 at least this large, if β_1 of universe is 0.	0.02 to 0.10	>0.10	>0.10
g_1 (Normally distributed measure of skewness. Symmetry when $g_1 = 0$.)	0.394	0.070	0.340
$\frac{g_1}{\sigma_{g_1}}\left(\frac{x}{\sigma}\right.$ of a normal curve $\left.\right)$...........	2.12	0.27	0.75
Probability of sample g_1 at least this large if g_1 of universe is 0.	0.02	0.39	0.23
Is the skewness significant?	?	No	No

Table 2 shows the results and conclusions for samples A, B and C. None of the samples shows significant kurtosis although the probability shows a considerable range.

The general conclusion which may be stated from the analysis of β_1 and β_2 is as follows: There is no reason to doubt that frequencies of each of the three areas are normally distributed. A possible exception is area A, in which the sample has right skewness, but the skewness is of questionable significance.

Law of Constancy of Slopes.—The tendency of slopes to group closely about a mean value may be taken as an expression of a morphologic law relating slope to other form factors. This law may be stated as follows: Within an area of essentially uniform lithology, soils, vegetation, climate and stage of development, maximum slope angles tend to be normally distributed with low dispersion about a mean value determined by the combined factors of drainage density, relief and slope-profile curvature.

TABLE 2

Summary of analysis of kurtosis in three sample slope frequency distributions.

DETERMINATIONS MADE	Area A	Area B	Area C
β_2 (Distribution normal, mesokurtic, when $\beta_2 = 3.0$. Leptokurtic, or peaked, when $\beta_2 > 3$. Platykurtic, or flattened, when $\beta_2 < 3$.)	2.992	3.707	2.381
Probability of sample β_2 differing by at least this much from a population β_2 of 3.0	0.05	0.05	0.05
g_2 (Normally distributed measure of kurtosis. Mesokurtic when $g_2 = 0$.)	0.027	0.819	0.485
$\frac{g_2}{\sigma_{g_2}}\left(\frac{x}{\sigma} \text{ of a normal curve}\right)$............	0.07	1.62	0.55
Probability of sample g_2 at least this large, if g_2 of universe is 0.	0.47	0.05	0.29
Is there significant departure from normal, mesokurtic form?	No	No	No

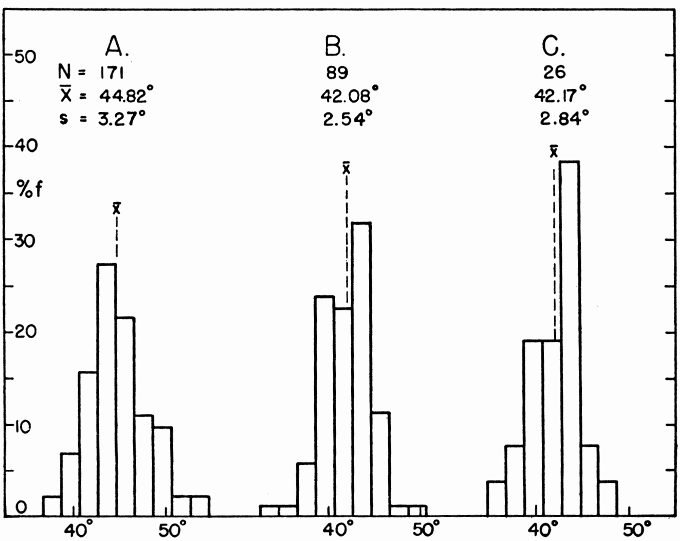

Fig. 6. Histograms of three slope frequency distributions from the Verdugo and San Rafael Hills, California. Location of areas A, B, and C is shown in figure 10. See tables 1, 2, 3, 4, 5, and 6 for summaries of data analysis.

If drainage density (Horton, 1945, p. 248),

$$D_d = \frac{\Sigma L}{A} \quad \text{where } \Sigma L = \text{sum of stream lengths (projected onto a horizontal plane)}$$
$$A = \text{area.}$$

then the mean horizontal distance from divide to channel is roughly one-half of the reciprocal of drainage density:

$$H = \frac{1}{2D_d} \quad \text{where } H = \text{horizontal distance from stream to divide.}$$

Now, if the spacing of streams is uniform throughout and the length of contributing slopes is similarly uniform, slope steepness can vary only as relief or profile curvature are varied (fig. 7).

Given two areas of the same drainage density, the area having the lower relief must have lower slopes, assuming profile curvature characteristics to be similar (fig. 7A). Given two areas of similar relief, but of different drainage density, slope must be less steep where drainage density is less (fig. 7B). If divides are broadly convex and only the basal fraction of the slope is straight, a steeper maximum angle is required than in a region of narrow divides and long, straight slopes, even though relief and drainage density are the same in both areas (fig. 7C). Because divide curvature is a factor readily influenced by stage of development or by structural conditions, it can be minimized only by the use of mature areas which behave as if lithology and structure were homogeneous.

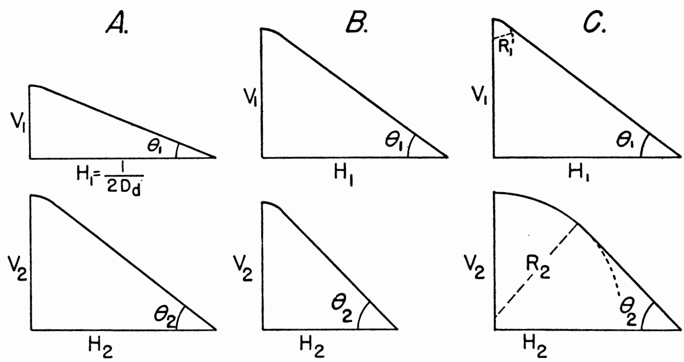

Fig. 7. Relationship of relief (V), slope (θ) and horizontal distance between divide and stream (H). Radius of divide curvature, R, is a variable diagram *C*.

Slope is thus seen to be proportional to the ratio between relief and one-half of the reciprocal of drainage density, assuming constant profile curvature characteristics. From figure 1 it is apparent that profiles are similar in form for both the fine-textured badlands on dissected clay fill near Perth Amboy (A) and for the medium-textured Verdugo Hills in Los Angeles County (B). Both have long, straight slopes and only a small curature radius at the divides (plates 1 and 2). On the basis of detailed profile plotting of slopes in the field and plane table mapping it appears that relief in the first area is about 20 times the second, but drainage density is only 1/20 as great, so that the slope angles are of the same general order of magnitude (fig. 2, E, F). If land slopes were perfectly straight from divide to base, the relationship of relief to drainage density would be approximately

$$\tan\theta = \frac{V}{\frac{1}{2D_d}} = 2VD_d$$

where θ = slope angle

V = relief (average vertical distance between divide and channel)

D_d = drainage density

For example, it was found in the Perth Amboy locality that the mean of a large number of measurements of vertical distance between channel and divide was approximately 6.0 feet, or 0.00114 miles. Drainage density sampled from several basins was 500 (that is, 500 miles of channel per square mile of area, projected upon a horizontal plane). Then

$$\tan\theta = \frac{V}{\frac{1}{2D_d}} = \frac{0.00114}{0.00100} = 1.14$$

$$\text{and } \theta = 48.7^\circ$$

Note that the mean of slope readings in this area as shown in figure 2, sample F, was 49.1°. One would not normally expect such good agreement between calculated and observed slope angles, and because the range of error has not been investigated, the coincidence may be misleading.

Relations of Slopes to Channel Gradients.—Davis (1909, p. 266-268) clearly stated the concept that the slopes of a mature landmass are graded, as are the streams. We may restate this principle by saying that slope profiles are in equilibrium with the channel profiles to which the slopes contribute their debris.

For a given area free of systematic structural control, but subject to uniformly controlling factors of climate, vegetation, soil and stage of development, all morphological characteristics tend to approach a time-independent form. Ground and channel gradients, as well as drainage density achieve a form best adapted to maintaining a steady state in the removal of debris. This is simply an extension of Playfair's Law to include all form-aspects of the topography.

If the type of adjustment described above actually exists, one would expect the angle of slope of valley walls to vary systematically with gradient of channel at the slope base. Steep ground slopes would be expected to correspond with steep channel gradients; low ground slopes with low stream channel gradients. This relationship has been stated by R. E. Horton (1945, p. 285) who uses the definition

$$\text{slope ratio} = \frac{S_c}{S_g} \quad \begin{array}{l} \text{where } S_c = \text{channel slope} \\ \text{and } \quad S_g = \text{ground slope} \end{array}$$

Ground slope can be determined by the field methods previously described in this paper. For each slope reading the channel gradient at the base of the slope would need to be measured. This was not done in the field by the writer, but the general trend of the slope ratio was obtained from large-scale topographic maps, including two which were specially mapped for the present study. Figure 8 shows the means of ground slopes and channel slopes plotted on log-log paper. Each point represents means of a large number of measurements of ground slopes and channel slopes. Because samples were taken from maps and the correlation is believed to have wide inherent spread of values, a curve was fitted by inspection. We might say that, if these data are representative of general conditions, ground slope varies as somewhat less than the first power of channel slope, and that the slope ratio tends to range from 1/5 or 1/4 for low values of slopes, up to about 1/2 in the higher values.

There are several uncertain elements in the foregoing analysis. Data from a greater number of regions would be expected to modify the estimating equation considerably. Substitution of careful field measurements for map measurements would increase the confidence in the data. It is further obvious that channel slope steepens rapidly toward the head of a valley, whereas ground slope tends to remain nearly constant along the

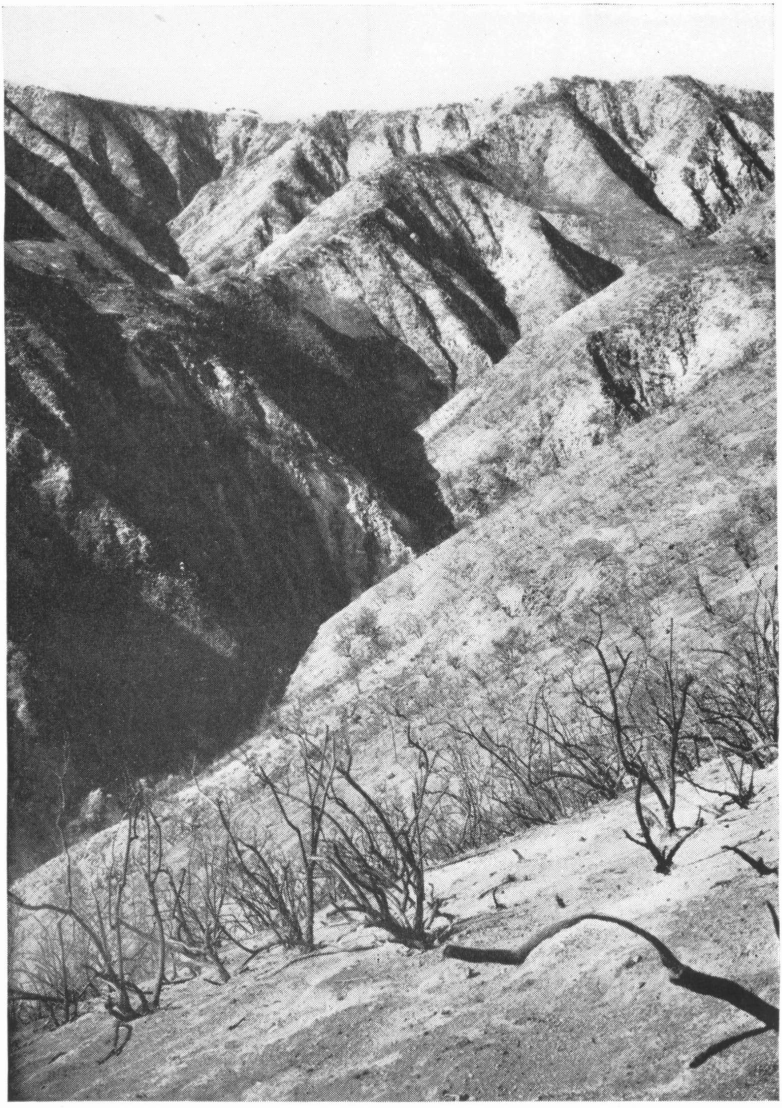

Plate 1. Topography of Verdugo Hills. Cabrini Canyon, south side of Verdugo Hills, Los Angeles Co., Calif. Slope in foreground faces northwest, had chamise-chaparral cover prior to burning. Canyon is about 200 feet deep.

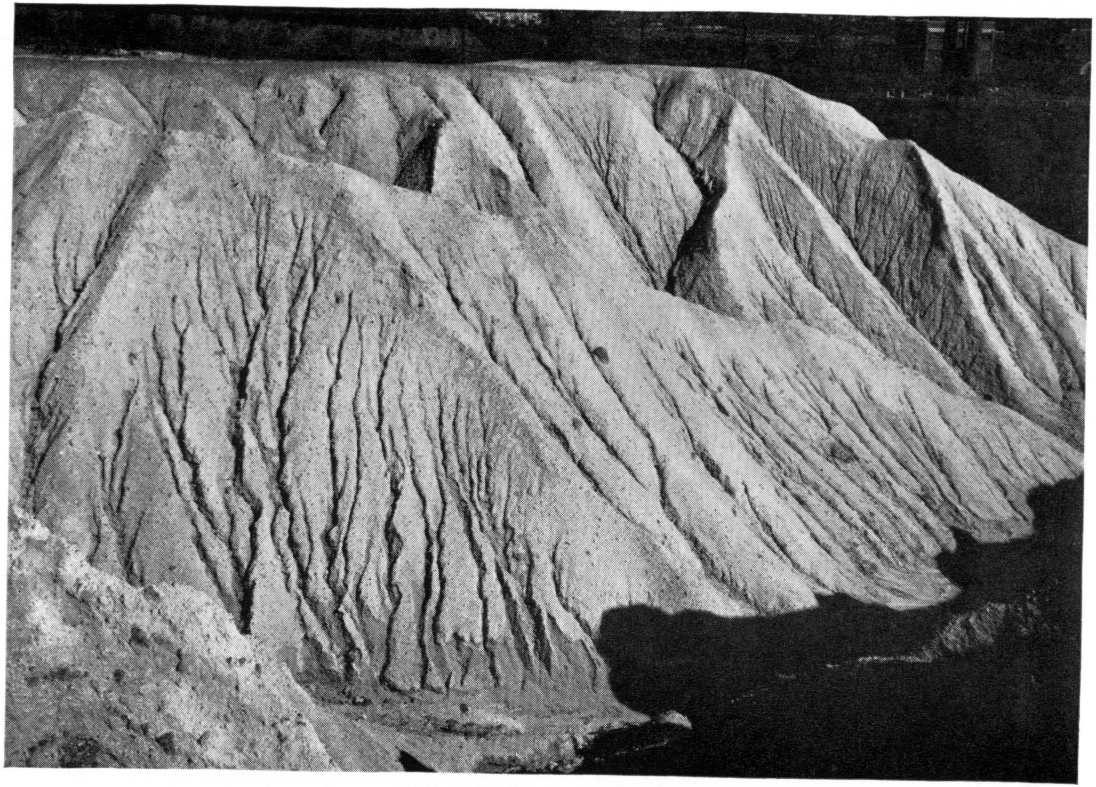

Plate 2. Topography of Perth Amboy badlands. Badlands in clay-sand backfiill, Perth Amboy, N. J. Relief in this photograph is 20 to 30 feet. This topography is similar in form-characteristics to that shown in plate 1.

sides of the valley, as the frequency distribution studies show. To minimize this variable, which can easily vitiate the results, readings were made along second-order stream basins, near the point where they are formed by the junction of two first-order streams (Horton, 1945, p. 281). This assures that all data come from the same relative position in the drainage system. Further standardization is desirable.

There is a suggestion from the data of figure 8 that climate has a profound influence on the slope ratio. Areas 7 and 9, which lie to the right of the line of the estimating equation, and area 8 which is on the line, have typical bare ground surfaces of badlands, or are lightly covered by chaparral, as in the Mt. Gleason area (7). Here one might expect a given slope of valley side to supply proportionately more detritus to the channel than a comparable slope in vegetation-clothed regions

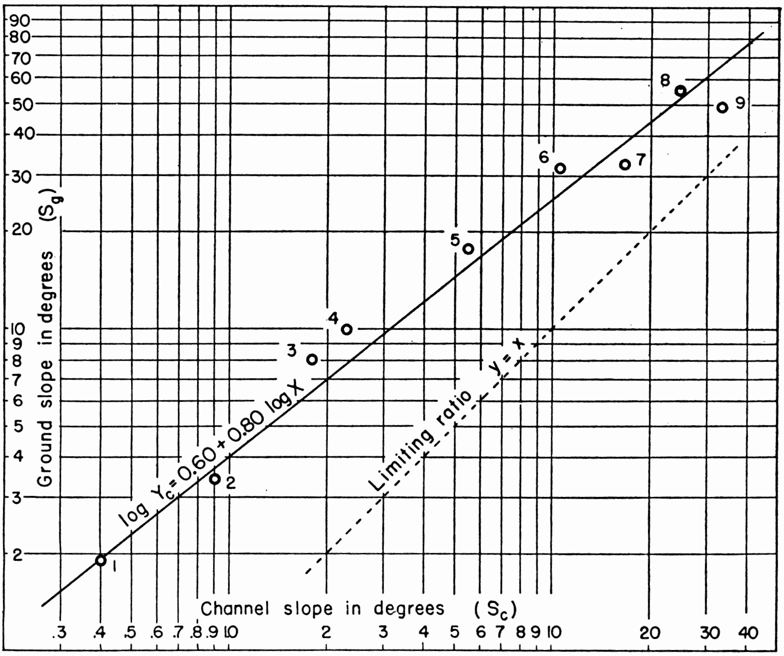

Fig. 8. Slope ratio, $\frac{S_c}{S_g}$, for nine maturely dissected regions. 1. and 2. Grant, La., 3. Rappahannock Academy, Va., 4. Belmont, Va., 5. Allen's Creek, Ind. 6. Hunter-Shandaken, N. Y. 7. Mt. Gleason, Calif. 8. Petrified Forest, Ariz. 9. Perth Amboy (clay fill), N. J. All data from U. S. G. S., A. M. S. or special field maps.

in the humid climates. The thinly vegetated areas would require relatively steeper channel gradients for the transportation of the greater quantity of detritus. In the two cases lying to the right of the line the channel slope is high in proportion to ground slope. While this could easily be merely the result of chance sampling, it is in agreement with the deduced behavior. On the other hand, well-vegetated slopes in the humid climate would tend to supply less detritus because of the increased surface resistivity and infiltration capacity, and would be associated with relatively lower channel slopes. As if to verify this deduction, points 3 through 6, representing humid regions, lie to the left of the line of the estimating equation. As this could readily be the result of chance, the observation merely makes more intriguing the prospect for a more adequate investigation of this phase of equilibrium relationships.

Comparison of Slope Data of Maps with those of Field Observations.—The problem of reliability of slope readings taken from maps in comparison with readings made at the same locations in the field is an important one, inasmuch as the labor and cost of the first method is only a fraction of the second. The writer was inclined to suppose that the most recent maps of the Army Map Service on a scale of 1:25,000 and of the U. S. Geological Survey on a scale of 1:24,000 made by photogrammetric methods would give results equal to ground observations.

For test purposes the Shandaken and Hunter, N. Y. quadrangles of the Army Map Service were selected. In the field the writer walked along the side slopes of five drainage basins, following trails marked on the maps. The maps have been prepared from air photographs. Planimetric detail seemed remarkably good when checked in field use. The drainage basins are large, the slopes long and of relatively simple, smooth character. A total of 60 field readings was taken, or about 12 in each basin. These readings were spaced from 50 to 150 yards apart. A frequency distribution diagram of these data is shown in figure 9, lower half.

In the laboratory slopes were measured from the contours along the same stretches of the trails. Approximately the same number of readings, 58, was taken, and while these are sampled from zones corresponding to those in the field, they are equidistantly spaced and do not coincide exactly with the field readings. A frequency distribution diagram of these readings is shown in figure 9, upper half. A comparison of statistical measures of the two distributions is given in table 3.

TABLE 3

Summary of field and map data of five drainage basins, Hunter and Shandaken Quadrangles, N. Y.

DRAINAGE BASIN	FIELD DATA $\bar{X}$		MAP DATA $\bar{X}$	
Hunter Brook	27.7°		26.9°	
Becker Hollow	30.0°	$N=60$, $\bar{X}=32.69°$	30.7°	$N=58$, $\bar{X}=30.72°$
Taylor Hollow	32.8°		28.3°	
Notch Lake Hollow	33.4°	$s=8.24°$, $s_{\bar{x}}=1.07°$	31.9°	$s=6.20°$, $s_{\bar{x}}=0.81°$
Shanty Hollow	35.5°		34.4°	

Examination of the histograms and summary table shows that the field distribution has a higher degree of irregularity

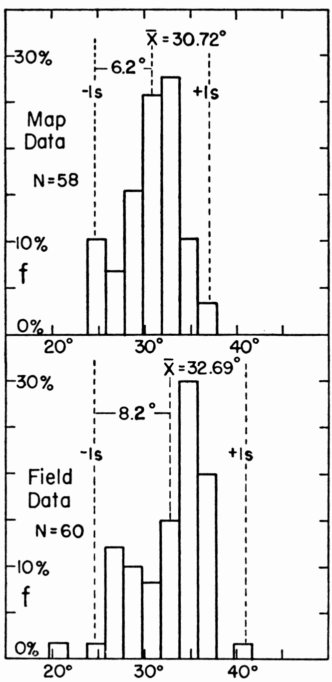

Fig. 9. Histograms comparing field and map samples of slope angles from the same drainage basins. Shandaken and Hunter, N. Y. quadrangles, A. M. S. 1:25,000.

than the map distribution. Dispersion is greater in the field data, with an estimated standard deviation a full 2° greater and with two end classes which are not present in the map data. It appears that use of the map tends to reduce the dispersion and remove irregularities. This might be expected if the drawing of contours in preparation for reproduction results in increasing the degree of parallelism present in the original contours.

The field data are strongly bi-modal because, of the five drainage basins used, two have low but similar means; the other three are relatively high. Note that while the map data also show a bi-modal tendency, it is greatly reduced; the sequence of means is not in quite the same order as in the field data. The greater homogeneity of the map data is again apparent.

Of greatest interest is the difference in means, a whole 2°. We might at first suppose that the processes of mapping and map drafting have caused a reduction in slopes, through the process of drawing the contours evenly spaced on the map, even where steeper and less-steeper portions of the slope are actually present. This would tend to lower the angle read from contours alone.

Are the field and map means significantly different? To test the difference, the ratio of the observed difference to the standard error of the difference (Croxton and Cowden, 1946, p. 319) was computed as follows:

$$\frac{x}{\sigma} = \frac{\bar{X}_1 - \bar{X}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = 1.462$$

On the normal curve this value of $\frac{x}{\sigma}$ lies at such a position that the area under the tail of the curve is 0.07. The difference is therefore not significant but the level of significance is not convincing. We can make the following statement: Even if the map is an exact replica of the field topography, we should expect a difference in means of any 2 samples of this size to be two degrees (1.97° exact) or more in 15 out of 100 times because of chance variations in sampling alone. Hence we cannot say that the map is inaccurate. Even if a very significant difference were present, it would not necessarily mean that the map were at fault. Differences resulting from the map and field methods of reading slopes might cause the observed differences in means.

In conclusion, we may say that while a fairly close agreement exists between field slope data and those taken from the best available topographic maps, the inconsistencies are of too great an order to permit significance studies between small groups of field and map data. It is advisable to use only the one type of data, preferably the field observations where these can be had.

A Tentative Classification of Graded Erosional Slopes.—As a result of frequency distribution analysis and field study of slopes in a wide range of localities, the classification of erosional slopes into three general groups seems feasible:

(1) *High-cohesion slopes.* Slopes underlain by cohesive, fine-textured materials such as clays or clay-rich soil or by strong, massive bedrock, such as granite, schist or gneiss tend to have means in the range 40 to 50° and sometimes higher in regions of strong relief. These slopes are higher than the angle of repose of the loose, dry fragments of the same materials, so that taluses of lower surface angle may accumulate at the slope base where material rolls or slides down the slope and is not swept away by streams. These steep slopes are subject to occasional rapid flowage and sliding movements of the soil or surficial layer following torrential rains. Where the slope has a thick soil of weathered, clay-rich material resting on a resistant, massive crystalline rock, a debris-avalanche may strip off the entire layer, exposing a slope of bare rock (Bryan, 1940, p. 262-263; Freise, 1938).

Examples of high-cohesion slopes are found in badlands, such as the Petrified Forest, New Mexico, or in eroded clay dumps along the Raritan River, New Jersey. In the Verdugo and San Rafael Hills of the California Coast Ranges are examples of slopes of similar steepness where a soil layer with moderate vegetation rests on deeply decomposed diorite or metasediments. Bailey (1948, p. 304) has described stable vegetated 60-degree slopes underlain by fine-textured soils with well-developed profile. He attributed the high angle to binding and protective action of vegetation, principally grass. Examples where bare, hard, massive crystalline rock forms the slope were seen in the Blue Ridge in North Carolina.

Thus steepness of slope, alone, without regard to scale of slope length or drainage density, can exist under a wide variety of lithologic conditions. There is the common property of maintaining a slope angle above that of the angle of repose of coarse fragmental materials because the slopes possess a high degree of cohesion or strength. All seem to require also that a steep-gradient stream system in process of vigorous corrasion be present.

Bryan has pointed out (1940, p. 263) that for steep slopes of the tropical regions there is nothing to indicate that the slopes should flatten with time. We might reason that, so long as a steady state prevails in which the streams are corrading their channels and constantly removing the detritus from the slope base, conditions of vegetation, climate and lithology remain constant, the slopes would maintain a fairly constant angle in equilibrium with the other factors. However, just as graded streams gradually regrade their profiles to lower gradients with generally decreasing land relief, the contributing slopes might be expected to be gradually reduced in gradient, the system maintaining a steady state through any given short period of time.

(2) *Repose slopes.* Slopes which shed loose, coarse rock particles may be controlled by the angle of repose of those materials provided that vigorous stream corrasion into the slope base promotes the maintenance of the maximum or near maximum repose angle. Meyerhoff (1940, p. 251) seems to have included this type of slope in the term *gravity slope.* Bryan (1922, p. 43) terms certain desert mountain slopes *boulder-controlled slopes.* He states that they are commonly in the range 30-35° and that the grade of the slope is the angle of repose of the average-sized joint fragment.

Repose slopes are illustrated by certain drainage basins in the Catskill Mountains (see figure 9) where a coarse layer of angular sandstone and conglomerate mantles slopes whose mean was found to be 32° in five first-order drainage basins.

The term *talus-slope*, referring to slopes of slide rock accumulation made by accretion of particles rolled or dropped from a higher source, should not be confused with repose slopes here described. The latter are essentially erosional, having only a thin veneer of loose particles over the parent bedrock.

Erosional repose slopes are observable in gravel pits where glacial-fluvial sands and gravels are being excavated. Left undisturbed, a fresh vertical cut into a gravel bank will recline to a straight slope equivalent to the angle of repose of the loosened particles, but composed of the almost bare, undisturbed, bedded sands and gravels. On this slope, which merges into a true talus at the base, particles will just slide or roll when dislodged from the parent mass. Laboratory analysis of angles of repose in fragmental materials (Van Burkalow, 1945) shows that repose slopes have distribution characteristics similar to the valley slopes described in the present paper.

Slide-rock slopes might be expected to maintain a constant angle as a region is progressively denuded, provided that the streams maintain a vigorous corrasion which saps the valley walls and maintains the maximum angle of the slide-rock fragments.

(3) *Slopes reduced by wash and creep.* Where channel corrasion is greatly reduced and channel gradients are low, due to onset of a late-mature stage, slopes are not maintained at the maximum angle by basal corrasion. Here sheet-erosion, rain beat and creep reduce the slope to angles well below the angle of repose. Bryan (1922, p. 46) terms these slopes *rain-washed slopes* and states that they are developed on fine-grained materials, readily subject to removal by rain-wash. Meyerhoff (1940, p. 251) terms these *wash slopes* and states that they seldom exceed 15°. He regards them as secondary forms which replace the steeper varieties by encroachment at the base. In the W. Penck system these would be classed as *Haldenhänge.*

Examples of slopes reduced by wash and creep were found to have means ranging from 20° to as low as 1½° (see fig. 2, A, B). It is postulated here that the slopes were reduced gradually in angle as the relief of the region diminished and stream gradients declined. Some evidence favoring a gradual decline of slope angle rather than replacement of a steep slope by a distinctly lower slope is discussed in a later section.

Summary Statement.—The application of standard methods of frequency distribution statistics to study of slope angles is of value in two principal respects: (1) It provides a standardized quantitative method of describing the characteristics of maximum slope angles within a small area, although it does not rectify fundamental errors of judgment made in the sampling process. (2) It provides a tool, through the method of significance of difference of sample means, of evaluating the results of field measurements and thereby prevents the drawing of unwarranted conclusions from numerical data. At the same time it may give confidence in certain conclusions based upon truly significant differences in sample means.

The measurement and analysis of real landforms is an inductive method, contrasting strongly with the purely deductive projective-geometric treatment of slopes used by Lawson (1915) and Putnam (1917) and recently revived by Bakker and Le Heux (1946). The latter method lays great stress on precision attained by setting up initial assumptions and deducing the slope profile to be expected. Whether the initial assumptions or deduced profile development bear any resemblance to real landforms is dubious at best, because only one process, out of several acting together, is selected for analysis.

(*To be continued*)