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EQUILIBRIUM THEORY OF EROSIONAL SLOPES APPROACHED BY FREQUENCY DISTRIBUTION ANALYSIS

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PART II

SIGNIFICANCE TESTS APPLIED TO SLOPE PROBLEMS IN THE VERDUGO AND SAN RAFAEL HILLS, CALIFORNIA

Introduction—Studies of the significance of differences in sample means have applications in the attack upon a number of fundamental geomorphic problems involving factors which affect equilibrium of graded slopes. Data collected in the Verdugo and San Rafael Hills, Los Angeles County, California, have been examined with a view to determining (1) if differences in underlying rock type are associated with differences in slope angles, (2) if differences in directional exposure to sunlight and other meteorological factors produce differences in slope angles, and (3) if slopes decline in angle when left to weathering and erosion processes not accompanied by basal erosion and removal.

Description of the Area.—For initial research three small areas in the Verdugo and San Rafael Hills were studied (figure 10). These lie in the southern side of a single NW-SE trending

Figure 10.

Coast Range block of moderate relief (Miller, 1934). Conditions of climate, vegetation, relief and tectonic history are essentially the same throughout, but lithologic factors are not. The easternmost area is underlain by a relatively homogeneous mass of Wilson diorite; the western two areas by the metasediments and small intrusive bodies comprising the San Gabriel formation. Despite marked variations in bedrock composition and structure in the latter formation, the entire southern side of this range appears homogeneous in topographic aspect.

The field work was concentrated upon measuring slope angles in several categories, in measuring and plotting detailed topographic profiles from ravine floors to divides, and in determining the number and location of the smallest discrete channels of the drainage network. Information was plotted on photostatically enlarged copies of the U. S. Geological Survey topographic maps on a scale of 1:24,000 corrected on the basis of air photograph study and field inspection (fig. 11).

Topography.—Mountain topography of the Verdugo and San Rafael Hills is in a mature stage of development. The drainage pattern is roughly dendritic, although the predominant trend of streams is southwesterly, from the major divide in which the streams head, to the sloping fan surfaces which encroach from the south upon the ragged foothill belt of the mountain base. Although there are many irregularities of bedrock composition and structure, these seem to have rather minor influences upon the placement of minor streams and divides.

Slopes are steep (plate 1), with a tendency to straightness of profile (figs. 1 and 12). Divides tend to be smoothly rounded throughout, a form emphasized by the fact that firebreaks are made and maintained by bulldozer along the major divides. Serrate crests and craggy peaks, often seen in desert moun-

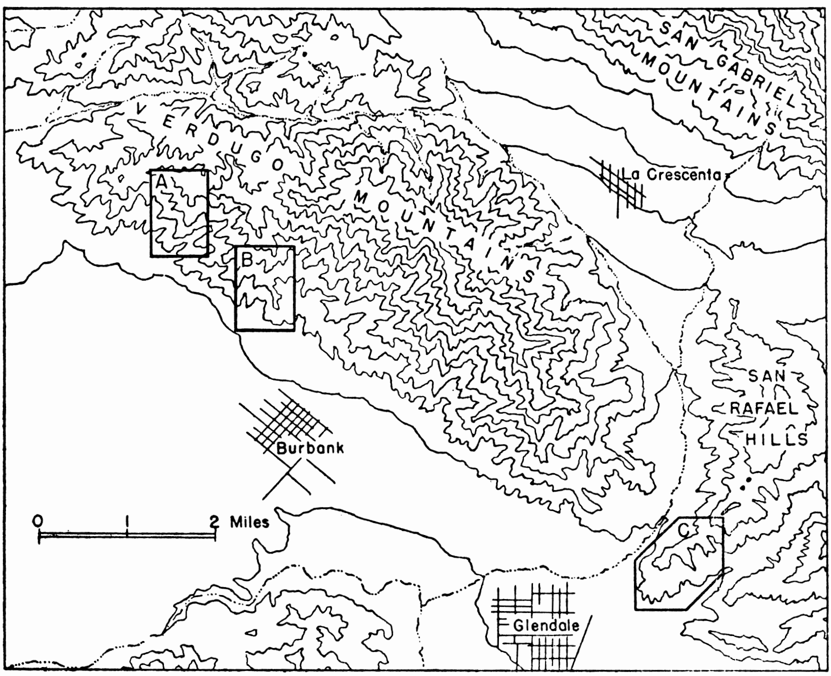

Fig. 10. Outline map of Verdugo and San Rafael Hills, Los Angeles County, California.

tain ranges and glaciated mountains are absent here. Relief is not great, despite the steep slopes. The areas studied have less than 1000 feet total relief while most slope profiles measured in the field have less than 100 feet of difference in elevation from divide to channel.

Ravines may be subdivided into two classes: (1) The smaller ravines and the upper ends of the larger ones are V-shaped in transverse profile. The enclosing walls have slopes in the range 40° to 45° for the most part. The stream channel, dry most of the time, is narrow and is cut in bedrock. Locally, however,

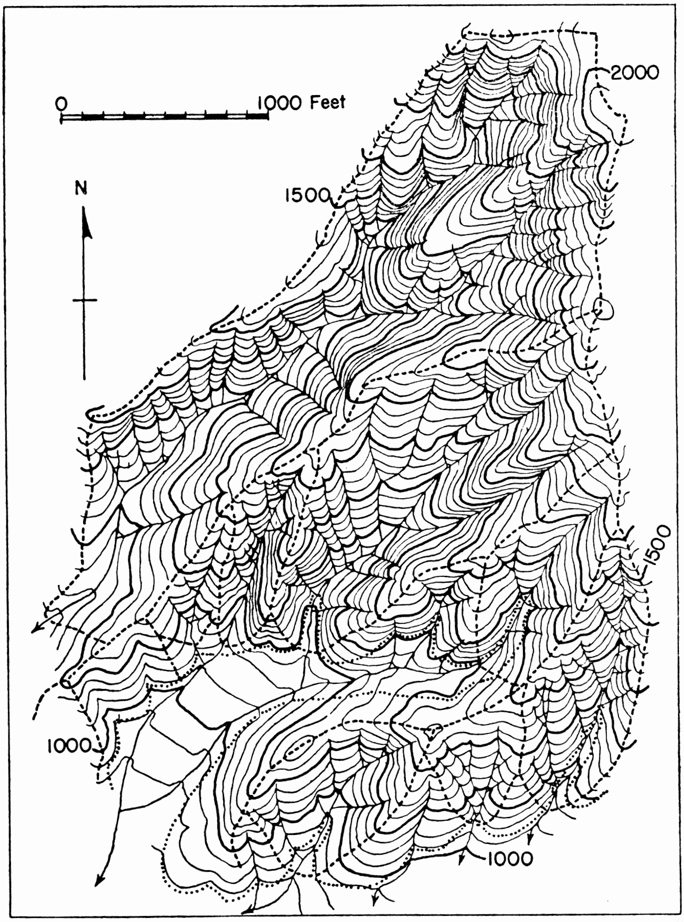

Fig. 11. Detailed map of Kline Canyon area, Verdugo Hills, Calif. (area A of figure 10.) Enlargement of U. S. G. S. map, 1:31,680, corrected in field.

the channel is choked with debris which has slid, rolled or flowed from the steep walls. (2) The lower courses of most of the larger canyons are graded and have a flat alluvial floor ranging from a few feet to one or two hundred feet wide. To what extent the flat is due to lateral corrasion in bedrock, rather than to alluviation by flood- and mud-flow of a formerly V-shaped valley is not easily determined. In the areas studied, a period of recent channel trenching showed alluvial and mud-flow fill. Streams strongly impinge on the base of the steep canyon walls, undercutting the bedrock. Where the stream has temporarily moved away from the slope base steep cones of talus have been built, causing a smoothed, concave-up basal slope of accumulation to be present.

Ravine heads are steep-walled, funnel-like amphitheaters which may be termed "hoppers" because their converging slopes feed detritus into the uppermost end of the stream channel. Hopper walls tend to be nearly straight in profile and to make the form of an inverted part-cone. Most hopper walls are very steep, above 45° in angle. Evidences of rolling and sliding of weathered rock are conspicuous. Bedrock exposure comprises from one-third to three-fourths of the hopper walls. Locally a layer or lens of resistant rock may produce a nearly vertical cliff, but for the most part the rock surfaces conform closely with the straight hopper walls.

Some hopper walls intersect the broadly rounded divide with a sharp, clearly defined break in slope, indicating the rapid expansion of the steep hopper slopes by rapid mass movement into a more stable profile determined by slower processes of creep and rain-wash. Elsewhere the hopper walls merge smoothly into the rounded divide profile and suggest a more nearly balanced slope condition.

The lateral slopes which extend from divide to ravine bottom are characteristically straight in profile for a considerable part of their length (fig. 12). At the upper end the profile merges smoothly into the curve of the divide; at the lower end the straight profile reaches to the ravine floor. In some ravines a composite profile was found, having a lower, or inner, segment steeper than that above it. This was interpreted as the result of a recent epicycle of accelerated erosion locally affecting some of the canyons.

Between-Area Differences in Slope.—Significance of differences in means of three areas was examined for the purpose of determining a possible influence of bedrock on slope. Areas A (Kline Canyon area) and B (Stough Park area) are both underlain by the San Gabriel formation whereas area C (San Rafael Hills) is underlain by the Wilson diorite. Should the means of the first two areas prove to be similar, but to differ significantly from the third, the difference may possibly be due to rock type. Should the results fail to show any significant differences, one might conclude that these particular varieties in bedrock have no noticeable influence. Should the first two areas, both in the same type of rock, differ significantly, but one of them to be the same as the third, we might conclude that factors other than rock type influence slope more strongly.

Histograms of three frequency distributions, one from each area are shown in figure 6. In an earlier section on general distribution characteristics of slopes an analysis of significant skewness and kurtosis was discussed (tables 1, 2). Differences

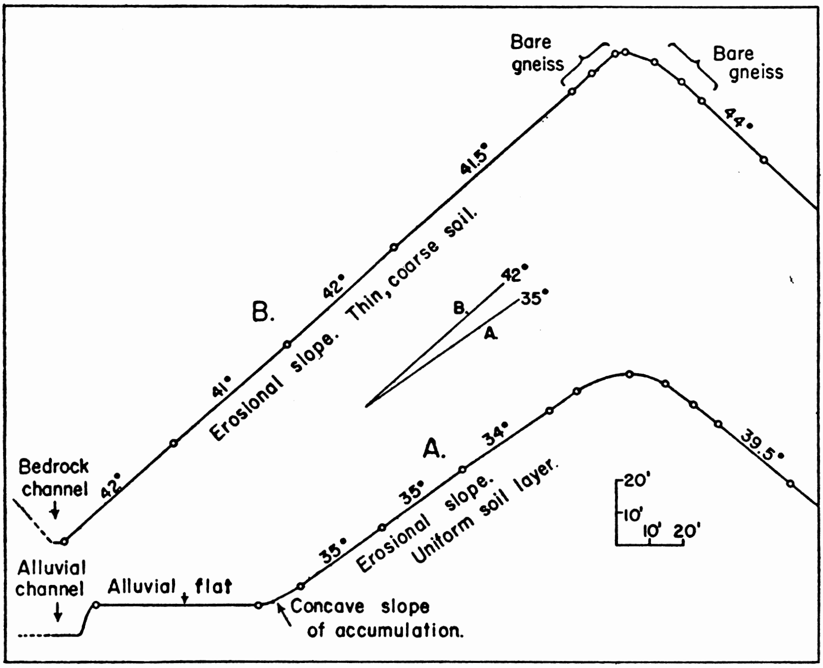

Fig. 12. Surveyed profiles in Verdugo Hills, California. Profile A, representing the type in sample A of figure 14, has escaped basal cutting for a long time. Profile B, representing the type in sample B of figure 14, is being actively corraded at the base.

in sample size are largely the consequence of the prevailing conditions of vegetation at the time of the field work. Area A had been recently burned over and presented an ideal condition for slope work. Area B had a light cover of shrubs following recent burns. Area C was densely overgrown with a chaparral of many years' development.

Tables 4, 5, and 6 give the essential data for the between-area analysis. Tests were made for significance of differences in estimated standard deviations and means.[1] Tests of sig-

TABLE 4

Data used in determinations of significance of differences in standard deviations and arithmetic means (tables 5 and 6.).

DETERMINATIONS	Area A	Area B	Area C
N Number of observations in sample	171	89	26
$\overline{X}$ Arithmetic mean of sample	44.825°	42.084°	42.173°
σ Standard deviation of sample	3.257°	2.522°	2.780°
s^2 Unbiased estimate of population variance	10.669°	6.433°	8.039°

$$s^2 = \frac{N}{N-1}\sigma^2$$

TABLE 5

Tests for significance of differences in standard deviations.

DETERMINATIONS	A and B	A and C	B and C
F (Ratio of larger s^2 to smaller s^2.) .... $F = \frac{s_1^2}{s_2^2}$	1.658	1.327	1.250
n_1 (Degrees of freedom in Sample 1.)	170	170	25
n_2 (Degrees of freedom in Sample 2.)	88	25	88
P. (Ratio of area in two tails of F-distribution to area under entire curve.)	0.02	$>$.40	$>$.40
Is there a significant difference in standard deviations?	Yes (?)	No	No

TABLE 6

Tests for significance of differences in arithmetic means.

DETERMINATIONS	A and B	A and C	B and C
t Where $t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{x}_1 - \overline{x}_2}}$	6.90	3.92	0.15
n Degrees of freedom	258	195	113
P Ratio of area in two tails of t distribution to area under entire curve	$<$.001	$<$.001	.80 to .90
Are the differences in means significant?	Yes	Yes	No

[1] Analysis was performed by Mr. Richard Ostheimer, Instructor in Statistics, Columbia University.

nificance of differences in estimated standard deviations were required by the differences between area A and the other two areas. As indicated in table 5 there is probably a significant difference between deviations of areas A and B, but not between A and C, or between B and C. Inasmuch as the level of significance in the first case is not minute, we cannot rule out the possibility that they are really not significantly different. Certainly any influence of difference in bedrock fails to show strongly in the estimated standard deviations.

Differences in means are treated in table 6. The results are clearcut in terms of probabilities and satisfactory significance levels. Whereas areas A and C, and areas A and B, are significantly different, the differences in B and C are definitely not significant, with a probability of 80 to 90 per cent that this difference or greater may be due to chance sampling variations alone. Because the two areas of similar rock differ significantly in slope, whereas two of the unlike areas do not differ significantly in slope, we may conclude that some factor (or factors) other than bedrock has a controlling influence on slope differences· These factors might be undetected differences in climate, vegetation, soils, geologic (tectonic) history, or sampling differences due to unintentional changes in sampling method. The conclusions are thus negative, even though clear, and serve to emphasize the complexity of the type of problem being treated.

A possible explanation for a lack of any strong bedrock influence may be the deep weathering of bedrock in this region. Decomposition due to alteration of feldspars and ferromagnesian minerals has proceeded to depths of tens of feet, so that highway grading in bedrock may be done with power shovels. Landforms are thus little influenced by differences in physical characteristics of the unaltered rock.

Influence of Differences in Exposure of Slopes.—Throughout the areas studied there is a marked difference in the appearance of slopes facing south through southeast to east and those facing north through northwest to west. This difference is obvious both in vegetation and soil. In the field one is apt to conclude that systematic differences in slope angle are also present because of the fact that one set of slopes affords better footing for climbing than the other. Whether or not a slope angle difference actually exists is here investigated by the method of differences in sample means.

All of the areas studied are clothed in a chaparral vegetation representing the natural flora of the region as it develops in various stages following fire. Two distinct types of vegetation are recognizable; that on the drier south-facing slopes, and that on the relatively protected north-facing slopes.

On the south-facing slopes is a light growth, 3 to 5 feet high, termed *chamise-sage* by J. S. Horton (1941). On the south flank of the Verdugo and San Rafael Hills this consists largely of *Adenostema fasciculatum* (Chamise). There are in addition *Eriogonum fasciculatum* (Buckwheat), *Salvia mellifera* (Black sage), *Artemesia californica*, and *Yucca whipplei*. Chamise seems to have excluded almost all other plants except the yucca on foothill slopes which have escaped burning for many years. Slopes burned over recently (3 to 5 years ?) may consist of almost pure stands of *Eriogonum fasciculatum* or *Salvia mellifera* or a mixture of the two, interspersed with some yucca. *Rhus laurina* occurs throughout the chamise-sage areas, making conspicuous dense clumps 8 to 10 feet high and most commonly attaching to rocky parts of the steeper slopes. According to measurements by J. S. Horton (1941) in the San Dimas Experimental Forest, the average percentage of ground occupied by plants is only about 24 per cent, while 76 per cent is open ground. Although no measurements were taken within the areas studied by the writer, it seems likely that there is a similar proportion of open ground. Viewed from opposite hillsides a chamise-sage slope shows a considerable amount of exposed soil, weathered rock mantle, or bedrock.

In contrast with the chamise-sage association is the *chamise-chaparral* association (J. S. Horton, 1941) of north-facing slopes. This is a denser, higher growth, some 8 to 15 or 20 feet high, composed largely of stunted trees. Horton's measurements show that only about 52 per cent of open ground area is present. Viewed from a distance little or no bare ground may be visible. A considerable amount of litter, made up of dead leaves and twigs, is present. In the area studied by the writer, the chamise-chaparral contained such species as *Prunus ilicifolia*, *Rhamnus crocea*, *Quercus dumosa* and *Rhus integrifolia*, intermixed with chamise.

Protection of ground by both of these chaparral associations is considerable, but would appear to be much greater where the chamise-chaparral association is present. On the chamise-sage slopes individual plants catch considerable quantities of twigs and leaves on the up-slope side of the plant base. This in turn holds soil and rock particles, making a miniature terrace. When the slope is swept by fire, much of the debris is released to slide and roll downslope into the ravine bottoms where it accumulates in small talus cones. The bare ground between individual plants or clumps of plants is readily attacked by raindrop beat (splash erosion) and sheet-runoff, which in places has cut small rill channels several inches into the weathered rock mantle. In dry weather these grooves serve as chutes down which loosened particles roll or slide. The activities of rabbits, coyotes, deer and other animals which live in large numbers on chaparral slopes serve to release much fragmental rock and soil material throughout the long, dry Mediterranean summer.

Nature of the soil cover is likewise contrasted on the two types of slopes. The N-NW-W chamise-chaparral slopes commonly have a thick, dark-brown soil through which no bedrock crops out. A maze of fine rootlets binds the surficial part of the soil together so that free sliding and rolling of particles is minimized. The thick soil is given readily to flowage movements; the scars typical of such movements are conspicuous on burned-over slopes. On the S-SE-E slopes with a light chamise-sage cover, soil is thin. Outcrops are numerous and make up a large portion of the surface area of slopes over 40° or 42°. In the absence of soil held by rootlets there is much free rolling and sliding of weathered rock particles. Both because of the greater proportion of outcrops and because of the greater amount of sliding of detrital fragments, these slopes give the impression of being the steeper. They are more difficult to ascend or descend on foot than the soil-covered slopes (where both are freshly burned over) because footing is poorer in the loose unstable detritus on S-SE-E slopes.

In order to determine if a true and systematic asymmetry does exist between the two sets of slopes, a test was set up using slope angles previously gathered without regard for the particular problem of asymmetry. All ravines having simple, opposed valley walls were used; eight in all. These ranged in trend from N-S to ENE-WSW and showed contrasting slope conditions as previously described. In all cases the ravine was sharply V-shaped with a narrow stream channel occupying the bottom. Thus the deepening of the ravine would cause approximately equal basal corrasion of both valley walls. Due to the complexity of bedrock, which included metasediments and small intrusive bodies, it is not certain that control by rock structure and composition differences is absent. No such relationship suggested itself, however.

Fifty-two readings were analyzed from slopes facing in a westerly or northwesterly direction; 53 from slopes facing southerly or southeasterly. The equality in number of readings was wholly fortuitous, because all available readings were used and none arbitrarily discarded. In general the distribution of readings was even throughout and opposite sides of the same ravine received almost equal sampling.

Of the first group, facing W, NW or N, the arithmetic mean was 44.54°; for the second group, facing S, SE or E, the arithmetic mean was 44.56° (fig. 13). Note that standard deviations differ markedly even though means are almost identical.

A test of significance of difference in means shows

$$\sigma_{\bar{x}_1 - \bar{x}_2} = 0.631$$

$$\frac{x}{\sigma} = 0.03168$$

The area under the two tails of the normal curve is 97.2 per cent. Hence the probability of obtaining this great a difference in means or greater due to sampling variations alone would be about 97 out of 100, if samples were repeatedly drawn from a slope population uniform throughout. It is nevertheless possible that a true difference exists due to exposure but that the two samples happened to have almost identical means. If we take the observed difference, plus or minus 2.6 times the standard error of the difference of the means ($.02 \pm 2.6 \times 0.631$) and we state that the true difference lies somewhere in this range of values, then, if we regularly proceed according to such a rule, in the long run 99 per cent of the statements will be true.

This example serves as an illustration of the value of systematic quantitative measurement and testing in investigating what seems upon casual field observation to be a difference in slope angles due to exposure to meterological elements. The eye alone cannot detect significant small differences and may, on the other hand, be deceived into registering a slope difference where none exists.

Parallel versus Declining Slope Retreat.—A fundamental problem in geomorphology is whether slopes of a fluvially dissected landmass retreat in parallel planes accompanied by replacement with a set of lower basal slopes, as Penck (1924) has held, or whether they decline to progressively lower angles as the landmass relief is lowered, as Davis (1909, p. 268-269) has held.

Under the equilibrium theory, outlined in earlier pages, slopes maintain an equilibrium angle proportional to the channel gradients of the drainage system and are so adjusted as to permit a steady state to be maintained by the process of erosion and transportation under prevailing conditions of climate, vegetation, soils, bedrock and initial relief or stage. Thus both slopes and streams are graded. In any one small area where homogeneity prevails slopes tend to cluster about a mean value with relatively low dispersion. As the landmass is reduced

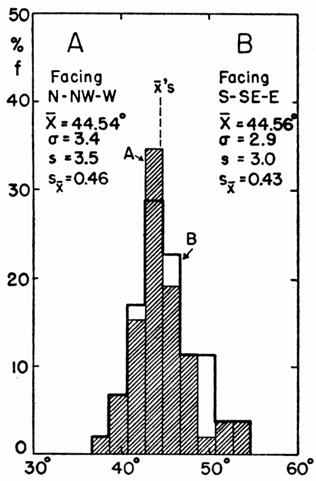

Fig. 13. Histograms of two samples superimposed for comparison. Sample A (cross-hatched) represents the more densely vegetated, sheltered northerly-facing slopes. Sample B (heavy line) represents thinly covered chamise slopes with southerly exposure. Means are almost identical, hence are indicated by one line.

both slopes and stream gradients are reduced, being slowly and continuously regraded to maintain approximate equilibrium. As the correlation of stream gradients with slopes (fig. 8) suggests, the decline of stream gradient is accompanied by slope reduction. This concept of maintenance of a steady state by slow readjustment is essentially Davis' concept of landmass development in the normal cycle.

Under the Penck (1924) theory of landmass development an initially steep slope or wall (*Böschung*) such as the valley wall of a youthful canyon, might be expected to retreat in successive parallel planes. So long as the stream is down-cutting at a constant rate, no other elements of slope would be present and uniform development would prevail. With diminishing rate of down-cutting by the stream there will begin to develop at the base of the steep slope a gentle slope (*Haldenhang*) which expands in area as the steep slope retreats from the valley axis. The *Haldenhang* is, in tùrn, replaced by still lower slopes (*Subhaldenhänge*). This concept fails to correlate stream gradients with slopes, as the theory of equilibrium forms in a steady state demands. Once formed, a steep slope (*Böschung*) would retreat at a constant angle regardless of changes of gradient of the associated stream.

In the Verdugo and San Rafael Hills the prevailing conditions are those of steep-walled, V-shaped canyons, or canyons with narrow alluvial valley flats. If the topography is developing by parallel slope retreat, as under the Penck theory, there might not yet be any development of *Haldenhänge* because of the rapid valley deepening and widening. It would thus be difficult to draw conclusions as to the correctness of these theories on the basis of general topographic aspect. There is, however, a possible means of determining the validity of Penck's assumption that a steep slope retreats in parallel planes once it has been formed and is no longer subject to direct basal stream action. At various places along the narrow alluvial flats in some of the canyons the stream is vigorously undercutting the bedrock at the slope base on one side, but has not recently been active on the opposite side, where there is an accumulation of soil and rock fragments in the form of talus or basal sheet-wash apron. The slope above this zone of accumulation has been free to waste back by processes of mass movement and sheet erosion, but without any removal or undercutting at the slope base. If retreat has been parallel, slopes of this type should maintain a mean angle the same as for the slopes vigorously corraded at the base; if retreat has been of the declining type, these slopes should be significantly lower than the latter group.

During the field work measurements were made of erosional slopes which had basal accumulations of slopewash or talus and had, therefore, for a greater or lesser period of time, no vigorous corrasion at the base. In all 33 measurements of this class of slopes in the Kline Canyon area were made. These slopes were located among the slopes being actively corraded, of which the sample of 171 readings has already been described (sample E in figures 2, 3 and 4). The larger sample includes none of the smaller, but the two are intermixed as far as geographic distribution is concerned. Figure 14 shows the histograms of the two frequency distributions plotted on the same coordinates. The mean of the protected slopes is 38.2°; that of the basally corraded slopes 44.8°, a difference of 6.6°. Estimated standard deviation of the first group (2.70°) is somewhat lower than the second (3.27°) but both are of the same general order of magnitude. It is obvious, in view of the significance levels obtained in previously described tests, that a difference in means of 6.6° is significant beyond doubt. A test shows that

$$\frac{x}{\sigma} = 11.83$$

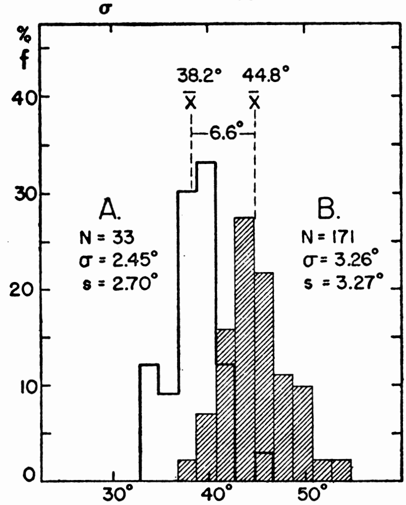

Fig. 14. Histograms of two samples superimposed for comparison. Sample A, protected slopes. Sample B, basally corraded slopes.

Tables of areas under parts of the normal curve fail to include this high a value. If the value were 5.0 the probabilities of obtaining this great difference or greater by chance variations in sampling from the same population would be on the order of one in one million.

We thus conclude that the slopes which have been protected from recent basal cutting have significantly lower angles. The statistical analysis does not explain this difference, but in view of the fact that conditions of climate, vegetation, soils, bedrock, and tectonic history seem to be essentially the same for both samples it is concluded that the one group of slopes declined in angle during the period when only sheet-runoff, creep and other mass-wasting processes operated on the slope. Some suspicion, at least, is thus cast upon the Penck assumption of parallel retreat, while some support is given Davis' scheme of declining slopes.

Summary Statement.—The method of evaluating the significance of differences in sample means appears to have applications in the solution of fundamental problems of geomorphology. The method serves as a check upon the reliability of conclusions drawn from samples. In the three examples described above, the second and third illustrate opposite extremes of significance level which give considerable confidence to the investigator in drawing conclusions. The first case illustrates an inconsistent relationship among three sample means relative to a possible causative factor and points to control by other factors not under consideration.

Although the specific results of this investigation may prove of little consequence in the light of future studies, some value may come through increasing the range of techniques available in the attack upon basic geomorphic problems.

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