GLACIER THINNING DURING DEGLACIATION.

RICHARD FOSTER FLINT and MAX DEMOREST.

CONTENTS.

ABSTRACT. *Part I.* The mechanics of ice flow under various conditions are analyzed for the purpose of showing how variations in accumulation and ablation may affect the regimens of different sorts of glaciers. It is shown that there are four types of movement, here called (1) *extrusion flow,* (2) *obstructed extrusion flow,* (3) *obstructed gravity flow,* and (4) *gravity flow.* The four types form a gradational series that cannot be sharply separated, yet each is characteristic of distinct glacial conditions. The first two are grouped together as *pressure-controlled* types of flow because they result from differential pressures. They are characteristic of glaciers lying on nearly horizontal floors and having surfaces of very slight slope (ice sheets, ice caps, piedmonts, and some valley glaciers). The second two are grouped together as *drainage-controlled* types of flow because they occur only where glaciers lie on floors of sufficient slope to cause relatively rapid drainage from higher-level sources of supply (outlet glaciers from ice sheets and many valley glaciers). Shear components of direct gravitative forces are principally responsible for movement.

It is demonstrated that thinning must accompany glacial recession, especially during late stages of deglaciation.

GENERAL INTRODUCTION.

THE purpose of this paper is to show the extent to which ice thinning may be characteristic of the deglaciation process. To accomplish this the authors have made two separate studies:

(1) A deductive study in which the purpose is to analyze all aspects of glacier regimens and thereby to show how ice motion, accumulation, ablation, and topographic controls may affect both the growth and the shrinkage of glaciers.

(2) An inductive study in which the purpose is to show (a) the extent to which thinned and thinning ice actually occurs within existing glaciers, and (b) the extent to which ice thinning may be inferred from geological features left by glaciers of the past.

The first of these studies is the work of Max Demorest who has benefitted by the advice and criticisms of R. F. Flint. He is also indebted to members of Professor Flint's seminar in Geomorphology at Yale University (1940-1941) who gave numerous and valuable criticisms on the organization of the study. Prof. A. T. Waterman of the Physics Department at Yale is thanked for his kindness in reading the manuscript and in offering valuable suggestions and discussions on the mechanism of flow. The National Research Council is thanked for a fellowship under which the study was carried out.

The second study is the work of R. F. Flint who has received advice and criticism from Max Demorest and has also benefitted from the criticisms of members of the seminar.

It is hoped that these studies may present an accurate concept of the manner in which deglaciation proceeds, and may thereby correct some former concepts that have pictured deglaciation as occurring by one or the other of two extreme methods: either by rapid terminal retreat in which the rôle of ice thinning is completely neglected, or by broad-scale regional stagnation in which thinning is greatly overemphasized.[1]

[1] Since preparation of this paper Prof. Douglas Johnson (1941) has published an excellent comment with a similar objective.

PART I. GLACIER REGIMENS AND ICE MOVEMENT WITHIN GLACIERS.

MAX DEMOREST.

INTRODUCTION.

The study of glacier regimens (this term being used in the hydrologists' sense) is the study of the configuration and "economy" of glaciers. By analysis of data on (1) sources and distribution of snow and ice accumulation (the glacier's income), (2) distribution and manner of ablation (the glacier's expenses), and (3) character and rates of ice flow (the internal transfer of ice from the areas of income to the areas of expense) it is possible to account for the general configuration and economic condition of a glacier. We can show in these terms whether a glacier has a balanced economy, whether it is expanding to a condition of greater affluence, or whether it is shrinking to a condition of bankruptcy. The relative influences of ablation and accumulation on these economic conditions are obvious and easily understood; however, the importance of ice movements within a glacier is not so obvious, and it is therefore necessary that we give it detailed consideration. We shall do this by means of a deductive study. It is hoped that any similarity between the ideal glaciers deduced and other glaciers, either living or dead, will not be entirely coincidental.

The near-surface part of a glacier, usually corresponding to the zone of crevassing, is relatively rigid and is characterized by discontinuous fracture and shearing in which differential movements occur along discrete planes, whereas at depth the ice is plastic and motion is by plastic flow of a streamline nature. Actually, slowly accumulated stresses may cause flow even within the rigid upper zone and suddenly applied high stresses may cause occasional discontinuous shearing at depth. Yet in general plastic, streamline flow is decidedly dominant at depth, and since the major deforming stresses responsible for glacier motion occur at depth, we will here be concerned with flow movements and the mechanisms that control them.

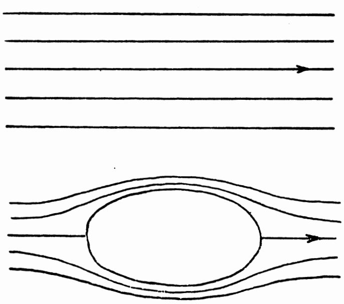
Streamline flow is a process of differential shearing in which infinitesimal "layers" slide past one another on planes that are theoretically infinite in number. Traces of these planes are the lines of flow or streamlines (Text Fig. 1) that mark paths along which infinitesimal particles of the flowing substance are moved. Where flow is undeflected the lines of flow are everywhere straight and parallel; however, deflection is very common, for it results not only from physical obstructions (Text Fig. 1B) but also, in accordance with Bernoulli's theorem, from any differences of velocity that may occur in the flowing substance. The importance of this fact will be made clear after further discussion. For the moment we must emphasize the fact that streamline flow is a process of shearing—that it results from shear stresses.

In all glaciers the shear stresses that cause flow are fundamentally a result of gravity; however, they are not always a direct result. This fact provides a convenient basis on which to classify types of ice movement, for it will be demonstrated that there are four types forming a gradational series, the two end members of which differ in being respectively direct and indirect results of gravitative forces. The type that results directly is called *gravity flow*; the other is called *extrusion flow*. These types are related by two intermediate transitional types which will be discussed later. For the present we will consider only the end members.

Gravity Flow.

Gravity flow results directly from gravitative forces and occurs within glaciers where the ice is of uniform thickness and lies on a surface of uniform slope (Text Fig. 3). In such circumstances one infinitesimal layer of ice slides over a next lower layer in the way that cards slide one over the next when a pack of them is tilted. The forces responsible for such movement are shear components of gravity. It can be shown that gravitative shear stresses exist in all bodies acted upon by gravity. At every point within a body these stresses are infinite in number and exist in all directions and at all possible angles of slope except the vertical and the horizontal. The strength of such stresses is proportional both to the weight of overlying material, that is, the direct gravitative force, and to the sine of the angle of slope; and where they are strong enough they may cause fracture or internal deformation (i. e., flow) within the body.

This may be understood by analogy with the sliding of a book on the surface of a tilted table (Text Fig. 2). We may regard the book and the table as parts of a single body within which the surface between them constitutes a plane of weakness. The total gravitative force involved in movement is measured by the weight of the book. This force is directed vertically downward; however, the book cannot move in that direction because the table stands in the way. Of the infinite number of shear stresses let us consider those in the directions x, y, and z (Text Fig. 2). Obviously x is the strongest, for the sine of its slope is largest (that is, the more nearly vertical a component is, the stronger it must be). However, like the vertical force itself this large component, x, cannot cause movement because the table stands in the way. The same is true of component y



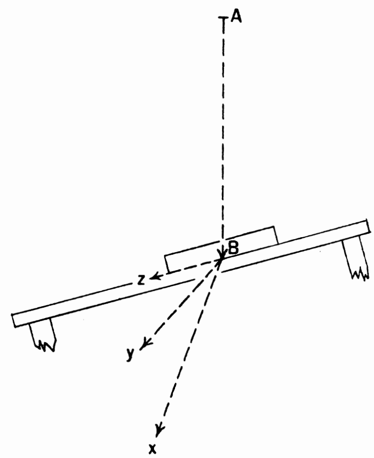


Fig. 1. Flow lines of streamline flow. (A) undeflected, (B) deflected by obstruction.

Fig. 2. Sliding of a book on tilted table. *Bx*, *By*, and *Bz* are representative of shear components of gravitative force AB which is equal to weight of book. Only the component *Bz*, lying in the plane of table top, causes motion.

and all others that lie at angles steeper than the slope of the table. The component z, lying in the plane of weakness, is thus the strongest component capable of causing motion—and it does cause motion. If the table sloped at a higher angle it is obvious that a stronger component would lie in the plane of weakness and would therefore be operative, and the book would slide more rapidly. Likewise, were the slope less, a smaller component would be operative and the motion would be slower.

A similar line of reasoning applies to the motion of our ideal glacier. At point *a* (Text Fig. 3), the total gravitative force involved is the weight of overlying ice. It acts vertically downward but can cause no motion because the glacier floor stands in the way. The floor also stands in the way of the steeply directed shear components *ax* and *ay*. Thus component *az* is the strongest one capable of causing motion.

The glacier being of uniform thickness and resting on a surface of uniform slope, a shear stress of the same magnitude as *az* operates at point *b* and at all other points on the plane represented by *ab*. The mass of ice above the plane therefore slides downstream over the underlying ice. However, such sliding, or shearing as it is better called, is not confined to the

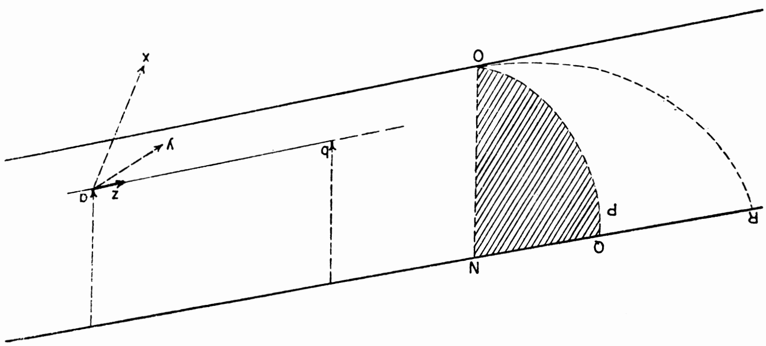


Fig. 3. Long section of glacier moving by gravity flow. Shear component *az* is strongest component capable of causing motion (on plane represented by *ab*). Differential motion on an infinite number of planes parallel to *ab* results in surface of deformation OPQ. Maximum differential motion is at bottom and minimum at top; however the top moves farthest, for it is carried downstream by the sum of all underlying differential motions. Line OR represents surface of deformation such as would occur in a substance of uniform viscosity.

plane *ab*, for similar stresses and shearing occur along the infinite number of parallel planes throughout the glacier. The only difference along other planes is that higher within the glacier, where the weight of overlying ice is less, the shear stresses are weaker; and lower within the glacier, where the weight is greater, shear stresses are stronger. As a result differential movements between layers must also be greater at depth. Actually the greatest differential movement must exist just above the bottom-most infinitesimal layer, which, being in contact with the floor, is theoretically motionless.[2] Above the next layer there must be slightly less overlying weight and consequently slightly less differential motion, and so on toward the upper surface of the glacier where stresses decrease to zero.

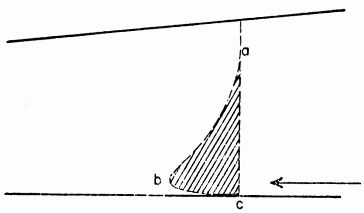
The trace of a surface of deformation resulting from such streamline flow in a true fluid is parabolic, shown by line OR (Text Fig. 3). However, a true fluid has practically the same viscosity at all depths and therefore differs from ice, the plasticity of which is a function of overlying weight, that is, of confining pressure. Thus the trace of a surface of deformation within a glacier must be more nearly similar to the line OPQ. (An exact determination of the shape of the surface must await quantitative information on the variation of plasticity with pressure.) The upper part of the line OPQ—the part between P and Q—is practically straight and parallel to the undeformed trace ON. This is because the upper ice is relatively nonplastic and therefore capable of resisting the weak near-surface shear stresses. Thus it is not subject to internal differential shearing; yet it does move through a greater downstream distance than any other part of the glacier, for its total movement is the sum of all underlying differential movements.

Extrusion Flow.

The shear stresses responsible for extrusion flow result from differential pressures. Differential pressures, in turn, are primarily a result of gravity, so that the shear stresses are only an indirect result of gravity. If we imagine a glacier with a sloping upper surface and a horizontal floor (Text Fig. 4), it is obvious that all particles along a vertical section (line *ac*) must be overlain by slightly more ice (and therefore they are under slightly greater pressure) at their upstream sides than at their downstream sides. Thus there is a differential pressure directed horizontally downstream along the entire section, and this differential is the same on all particles from bottom to top.

[2] The argument has been made that there could be no glacial erosion if the bottom-most ice were motionless. It is conceded that under some nonideal circumstances the bottom ice may actually be in motion over the glacier floor. However, such circumstances do not alter the general concepts of flow as here developed; nor are such circumstances required in order to explain erosion at the base of a glacier. It must be kept in mind that the motionless ice is theoretically of only infinitesimal thickness and that differential movement between this bottom ice and the immediately overlying ice is greater than elsewhere within the glacier.

The result is movement of the ice, but the amount of movement, unlike the differential pressure, is not uniform from bottom to top. This is because ice plasticity increases with depth so that the given pressure causes greater deformation near the bottom than near the top. Consequently the surface of deformation is as shown by its trace, *abc* (Text Fig. 4). Within the upper part of the glacier the ice is sufficiently non-plastic to resist deformation and at the base frictional retardation occurs, but between these two parts the rate of flow is a function of plasticity; that is, a function of the weight of overlying ice. The significance of the term extrusion flow thus becomes apparent, for the effect of such differential velocities of flow is to cause



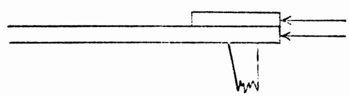


Fig. 4. Long section of glacier moving by extrusion flow. Line *abc* represents surface of deformation. Differential pressure, due to slope of the upper surface, is responsible for flow. Basal ice is retarded by friction, but near-basal ice above limit of retardation is moved farthest—it is extruded from below the overlying ice.

Fig. 5. Sliding of a book on a horizontal table where equal pressures are exerted against both book and table. Book moves relative to the table because it offers least resistance to pressure.

the more plastic ice at depth to be extruded from below the overlying relatively non-plastic ice.

The curved shape of the surface of deformation results from two factors: (1) The plasticity of ice is probably not a straight-line function of overlying weight and depth, but rather an exponential function. (2) In accordance with Bernoulli's theorem, a secondary differential pressure is directed toward the more rapidly flowing ice and therefore a component of movement is also directed toward it throughout the relatively slowly moving ice above and below the zone of maximum velocity. (Note the deflected lines of flow in the radial section, Text Fig. 6.) For a fuller explanation of this phenomenon and a demonstration of the fact that it occurs in flowing ice, see Demorest (1938).

If ice were of the same plasticity at all depths it is obvious that the primary differential pressure, being uniform from top to bottom, could cause no shear stresses and consequently could cause no motion. Fundamentally, therefore, the effective shear stresses depend upon differences of plasticity. It is as if a book were lying on a horizontal table (Text Fig. 5) and pressures of equal magnitude were directed horizontally against both the book and the table. It is apparent that the book would move, for it offers less resistance to the given pressure; in short, a shear stress is set up between the book and the table.

Within a true fluid where there is no significant change of viscosity with depth, flow would not be as simple as that just described. Rather there would be a more complex internal movement involving both differential pressures and direct gravitative shear stresses of the sort previously discussed. To some extent gravitative shear stresses must also play a part in the flow of such glaciers as are illustrated by Text Fig. 4. These stresses, however, are chiefly significant only near the surface where the ice is relatively non-plastic. Therefore, within glaciers of slight surface slope where stresses are small, gravitative shearing must be practically if not entirely absent. It will be shown that it is only within such glaciers that ideal extrusion flow is significantly operative. The principal examples are found in the accumulation area of ice sheets and ice caps where surface slopes are generally less than one or two degrees. Obviously, rates of flow under the small differential pressures accompanying such slopes are extremely low.

Obstructed Flow.

As a further qualification, it must be pointed out that extrusion flow can operate only where motion is in no way obstructed. As will be shown, this fact is important because obstruction often results from poor adjustment of the surface profile of a glacier as well as from physical barriers. At present, however, it is sufficient to point out that obstruction of extrusion flow must result in forward and upward movements which are also controlled by differential pressures. Such obstructed flow will here be called *obstructed extrusion*; and the two types, extrusion flow and obstructed extrusion flow will be grouped together as *pressure controlled* types of movement.

In places where the upper surface of an obstructed glacier is steepened so that gravitative shear stresses as well as differential pressures become operative, there may be a transition, either from extrusion flow or from obstructed extrusion flow, to gravity flow. Actually, however, the transition from extrusion flow to gravity flow is not common in nature, though the other transition, from obstructed extrusion flow to gravity flow, is very common. In fact this transitional type of flow is more common than ideal gravity flow. It results not only from the steepening of the surface of a glacier of obstructed extrusion flow but also from obstruction of a glacier of ideal gravity flow. We shall therefore call this transitional type ***obstructed gravity flow***; and the two types, gravity flow and obstructed gravity flow, will be grouped together as ***drainage-controlled*** types of movement. The appropriateness of this last term will be apparent after further discussion. For the moment, however, we may note that drainage-controlled movements are primarily dependent upon the slope of the glacier floor whereas pressure-controlled movements are dependent upon the slope of the glacier surface.

In summary the various types of glacier movement and their controls are presented in tabular form.

Pressure Controlled (dependent on slope of glacier surface).	Extrusion Flow	Shear stresses due to differential pressure and differential plasticity.
	Obstructed Extrusion Flow	
Drainage Controlled (dependent on slope of glacier floor).	Obstructed Gravity Flow	Shear stresses due to components of gravity.
	Gravity Flow	

PRESSURE-CONTROLLED GLACIERS.

Movement controlled by differential pressures is essentially a mechanism of leveling in which both the surface slope and differential pressures are reduced toward a value of zero. This is understood if we imagine a great mass of ice occupying a topographic basin from which there is no outlet, or imagine it as occupying an enormous tub. We postulate that the ice is thick enough to be plastic at depth and that it has a sloping upper surface. This slope may extend radially outward from a high center as is common on ice sheets, or it may be constant from one side of the tub to the other, or it may be of any other form. It does not matter what the form is, for so long as there is a slope there must be differential pressures and, as a result, there must be movement. If no ice is added by snowfall and if no ablation takes place, pressure-controlled movements must cause a transfer of ice from the higher areas to the lower areas until ultimately the surface is leveled; that is, until the surface slope and accompanying differential pressures are reduced to zero. Thus a great tub of ice, like a tub of water, must ultimately acquire a level surface.

If now we consider a great continental ice sheet lying on an essentially horizontal floor, we have something similar to the tub of water, but with the tub removed. In such a case water would spread in all directions, attempting to flatten out to a zero surface slope; and so would the ice, but its spreading would be extremely slow. However, if sufficient time is allowed and it is again postulated that the ice sheet is affected by neither snowfall nor ablation, there is no reason why it would not spread until the whole mass were finally attenuated and thinned to the extent that movement would cease. The amount of spreading required for such cessation of movement is determined by the rate of ice deformation which is proportional to both the plasticity and the differential pressure, that is, to the weight of overlying ice and to the slope of the surface. Therefore, as the ice spreads, becoming increasingly thinner, the rate of movement will be reduced. At the same time the surface slope, in its approach to zero, is also reduced, and this constitutes a second factor causing a decreased rate of movement. As a result the ice moves at a constantly decreasing rate until ultimately it becomes completely motionless.

Glacier Equilibrium.

An exact understanding of the way in which flow is controlled within an ice sheet is best obtained from a consideration of equilibrium conditions. Glacier equilibrium demands that (1) the net amount of snowfall supplying additions of ice in unit time must be exactly equal to the net amount of loss due to ablation, and (2) internal movements must cause a continuous transfer of ice from the areas of supply to the areas of ablation (Text Fig. 6) so that an unchanging surface profile is maintained. In less exact terms, the spreading glacier is trimmed back by ablation at the margins as rapidly as it extends itself, and the interior is kept from thinning by constant additions of snowfall. In actual glaciers ideal equilibrium is theoretically impossible because the seasons of maximum accumulation and ablation do not coincide and because the absolute amounts of accumulation and ablation vary from year to year. However, the slow movements of ice within a glacier cause such sluggish response to these variations that average values taken over a period of years are generally significant in determining the maintenance or loss of equilibrium.

Accumulation Area.

Assuming an ice sheet at equilibrium, we will first consider ice movement under the zone of accumulation. There, since an unchanging profile must be maintained, the amount of ice moving outward by extrusion from under any area in unit time must be equal to the amount of ice accumulated. Were accumulation and outflow not equal, the glacier would become either thinner or thicker under the area considered, and therefore, would no longer be in equilibrium. This means that the discharge (that is, the volume of ice extruded in unit time) from under the area of circle 1 (Text Fig. 6) must equal the amount accumulated in unit time within that area. In other words, the discharge through vertical section 1 equals the accumulation per unit time within the circle. The rate of movement at section 1 required to allow this discharge is dependent upon both the height of the section and the amount of ice discharged. Considering a given discharge through a section of given height (and assuming for the sake of simplicity that the velocity is uniform from top to bottom of the section) the ice would flow outward at some particular velocity, whereas the velocity of the same discharge through a section of twice the given height would be only half as much. In short, the velocity for a given discharge is inversely proportional to the height of the section. But the surface slope (which controls the primary pressure differential) must be proportional to the velocity, and it therefore follows that the slope required to maintain a given discharge is inversely proportional to the thickness of the glacier. However, an additional factor is involved, for as the section height and glacier thickness increase, the average ice plasticity also increases; and since the surface slope required to maintain a given velocity is inversely proportional to the plasticity, the surface slope may be even further decreased while still producing the same discharge. In summary, the surface slope required to maintain a given discharge decreases by a dispro-

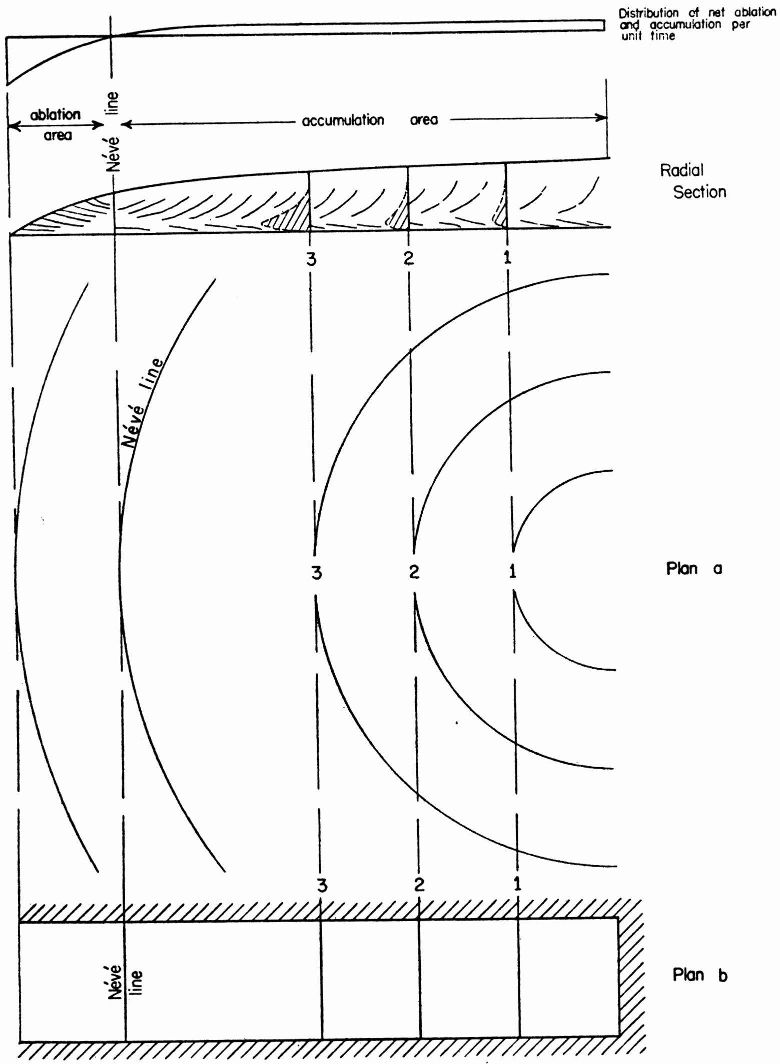


Fig. 6. Radial section and plan of pressure-controlled ice sheet. Radial section shows lines of flow. Greater bulges in downstream surfaces of deformation show increased discharge with distance from locus of maximum accumulation (in this hypothetical case the center of the ice sheet). Plan *b*, hypothetical rectangular pressure-controlled glacier with frictionless walls. Relationships between successive vertical cross sections and surface areas upstream from them are the same as for circular ice sheet, plan *a*.

portionately large amount as the glacier thickens, or increases by a disproportionately large amount as the glacier thins.

This relationship makes it obvious that an area of high land underlying an ice sheet must be reflected in the relief of the upper surface, for any topographic projection into the base of an ice sheet creates a smaller thickness of ice, and therefore, if a given discharge is to be maintained through a section over the high land, an increased surface slope must be established. This increase might be so slight as to require measurement for its detection, but it should nevertheless be present.

A somewhat different and more immediately significant approach to the relationship between thinning ice and steepened surface slope is found in the treatment of other sections taken at greater distances from the center of the ice sheet.

We may treat section 2 (Text Fig. 6) in the same manner as section 1. The only difference is that the volume of accumulation within circle 2 is larger. If we arbitrarily make the radius of circle 2 twice that of circle 1, the included area is four times as great, because the area increases with the square of the radius. Thus, if the depth of accumulation over the entire area, including circle 1, is uniform, the volume of accumulation within circle 2 is four times that within circle 1; and, since the volume of outflowing ice through the vertical section must equal the volume of accumulation, it must be that four times as much ice passes the section in unit time. However, as the circumference of circle 2 is only twice that of circle 1 (assuming for the moment that the heights of the two sections are equal), we find a volume multiplied by 4 passing a section multiplied by 2; in other words, the volume per unit vertical section is twice as great. In consequence the rate of flow through section 2 must be twice as much as through section 1. If we assume the depth of accumulation to be uniform over the entire area and further assume all sections to be of equal height, it would follow, for similar reasons, that the volume of ice passing a section of threefold radius would have to be three times as great as that passing section 1, or in general, the volume passing any section in unit time (i. e., the velocity) would be proportional to the radius of the section.

It is apparent that our assumptions do not represent actual conditions, for in an ice sheet neither the depth of accumulation over the interior nor the heights of successive sections would be uniform. In our ideal ice sheet we will consider that both the accumulation and the section heights decrease toward the margin, as shown in Text Fig. 6. As a result of the decreasing accumulation toward the margin the volume of ice having to pass section 2 would be somewhat less than four times the amount passing section 1, and it therefore follows that the required rate of flow through section 2 would also be slightly less. However, this effect may be offset by the decreased height of section 2, for with decreased height the area of the vertical section also decreases. Consequently the area of section 2 is less than twice that of section 1, so that the fourfold (or nearly fourfold) volume of ice that passes the section in unit time must flow more rapidly than as if the two sections were of equal height. A determination of the extent to which these effects actually cancel each other must depend upon quantitative information that is now lacking. In general, however, it is probable that these effects come close enough to offsetting each other so that the velocity of ice flow across any section may be considered to be approximately proportional to the radius of the section.

Returning now to surface slopes and the accompanying differential pressures required to maintain flow through successive sections, we find that (1) the velocity of flow across successive sections is approximately proportional to the radius of the sections, and therefore the slope must also be approximately proportional to the radius; (2) as discussed above, decreasing thickness of ice through successive sections causes a decreasing average ice mobility, thus giving an additional and independent requirement for a steepened slope. In summary, the surface slope in the accumulation area of our ideal ice sheet must steepen outward at an increasing rate from the locus of maximum accumulation.

The actual rate of slope increase for any particular ice sheet would depend upon the size of the accumulation area, the distribution of accumulation within that area, and the thickness of ice from place to place (account being taken of the subglacial topography). Thus the equilibrium slopes for different ice sheets would not be the same; in fact they would not be the same for all radial sections of any one ice sheet, for ice sheets are not ideally circular. However, accurate profile analysis could be made by treating each radial section as if it were part of an independent circular sheet having a radius equal to the length of the section analyzed. Or, more simply, a radial section may be treated as a long section through an ideal rectangular ice sheet bounded on three sides by frictionless constraining walls (Text Fig. 6, plan *b*). The flow conditions within such a rectangular sheet would be exactly the same as those along a radial section of a circular sheet.

Thus far we have assumed the flow to be of uniform velocity from top to bottom of the vertical sections. Actually, as demonstrated by Text Fig. 4 and as shown in the discussion of extrusion flow, this is not the case, for with increasing depth and plasticity there is an increasing rate of extrusion. Text Fig. 4 may be considered to show the movement in unit time across a given vertical section of an ice sheet in equilibrium. Thus the shaded area between the trace of the surface of deformation (line *abc*) and the vertical section must represent the volume of ice passing the vertical section. It then follows, since successive sections pass increasing volumes of ice in unit time, that there will be successively greater bulges of the surfaces of deformation. This is shown in the radial section of Text Fig. 6 where traces of the successive surfaces express movements occurring in a single unit of time.

Ablation Area.

Obstructed extrusion flow as a result of thinning. Within the ablation zone of an ice sheet at equilibrium the ice movements are different from those described above, and will be more

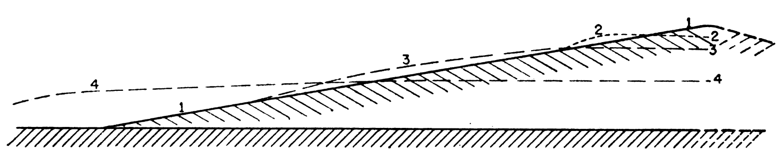


Fig. 7. Hypothetical glacier of uniform surface slope in which flow is obstructed by downstream thinning. Successive profiles 2, 3, and 4, show gradual establishment of extrusion flow (first affecting only the head of glacier) and the manner in which spreading occurs.

readily understood after consideration of a hypothetical glacier having a surface of constant slope that is slight enough to eliminate the possibility of gravitative shear (Text Fig. 7, profile 1). The figure may be thought of either as a long section through a valley glacier or as a radial section through an ice sheet. We shall assume that the glacier floor is horizontal and neglect the effects of ablation and accumulation. The surface slope being constant, the same differential pressure must be exerted throughout the ice at all sections, but the ice plasticity at any given height above the glacier floor must vary, decreasing continuously in the direction of thinning ice. Thus, though differential pressure is uniform throughout the glacier, the ice is not capable of such rapid movements downstream as upstream. In general this means that flow across any section is obstructed in the downstream direction by more slowly moving ice. Near the end of the glacier, at the point where the upper zone of no plasticity makes contact with the glacier floor, the rate of movement is diminished to zero. However, such an obstruction would completely block flow in the ice immediately upstream, and this in turn would block flow through the next section, and so on, with the result that the entire mass would have to be stagnant. Theoretically this condition should exist in a body of perfectly homogenous ice having a surface of perfectly uniform slope, for the stresses would be at equilibrium in all parts of the glacier. But such equilibrium would be very unstable. Even a slight departure from ideal conditions would result in a local departure from equilibrium, and therefore in movement; and movement, once started, would lead to even further departures from equilibrium, so that ultimately all stresses within the glacier would become unbalanced and flow would take place throughout the glacier. Movement under these circumstances would be forward and upward, the ice from upstream at any place overriding the thinner, slower moving ice that causes the downstream obstruction. Such movements must continue to the surface, breaking through the upper zone of relatively non-plastic ice. It is this special sort of pressure-controlled movement that we have called obstructed extrusion flow.

The effect of this flow at any point on the surface of the glacier is obviously to build the surface higher, but that cannot occur without removal of ice from the interior. For the sake of simplicity we shall consider the initial movements to occur near the head of the hypothetical glacier, or near the center of a hypothetical ice sheet. As a consequence the surface just downstream from the glacier head (or the ice sheet center) is built up while the headward portion is lowered by surfaceward extrusion of internal ice (Text Fig. 7, profile 2). As this process continues the headward gradient is reduced and consequently the obstructing effect is also reduced. The result must be the local approach toward an equilibrium profile, and a complete though local elimination of the obstructing effect. When this occurs there is no longer obstructed extrusion flow at the glacier head, but rather true extrusion flow that is capable of supplying internal ice for surfaceward movements in the adjacent downstream part of the glacier. As a result there is further lowering of the headward surface, now affecting the entire area of extrusion flow, and as before, building up of the adjacent downstream surface. There the area of extrusion flow is extended and the process continues downstream until all except the terminal part of the glacier is relieved of the obstructing effect. (Reasons why an obstructing effect remains at the terminus are discussed below.)

This process not only is continuous but it continues at an accelerating rate. This is because the first adjustment at the head of the glacier occurs with only a limited supply of internal ice for surfaceward movements. Therefore such movements occur conspicuously within only a short distance from the head, and the slope over the immediately adjacent downstream surface area is oversteepened, creating a wave-like bulge in the surface (Text Fig. 7, profile 2). However, as the area of extrusion flow grows, the supply of internal ice for downstream surfaceward movements must also increase, for, as was shown above, the rate of extrusion flow at any cross section is approximately proportional to the radius of the section, or in this case, to the distance from the head of the glacier. Thus, as the process continues, the increasing rate of extrusion flow brings an increasingly greater supply of internal ice for surfaceward movements, and this in turn affects an increasingly longer downstream segment of the glacier. As a result the downstream limit of surfaceward movement must progress at an increasing rate, and the oversteepened front of the wave-like bulge becomes less and less steep so that the bulge appears to die out (compare profiles 2 and 3, Text Fig. 7).

Such wave-like bulges dying out as they progress downstream have been observed on existing valley glaciers and are known to be a result of suddenly but temporarily increased accumulation at the head of the glacier. It is possible that the mechanics of their downstream progress may be similar to that of the bulges just discussed.

Conditions at the glacier terminus. A true profile of equilibrium cannot be developed on the hypothetical glacier (Text

Fig. 7) which is affected neither by ablation nor accumulation, for equilibrium requires that ablation and accumulation aid in the establishment of an unchanging profile. The glacier is rather an example of spreading ice on which the profile, though constantly changing, becomes adjusted for the maintenance of continuous unobstructed extrusion flow (except at the terminus). Obviously such spreading involves a continuous advance of the terminus and a general thinning throughout the glacier.

Terminal advance must be by obstructed movements rather than by extrusion flow, for there the ice must be thin to the extent of non-plasticity, so that an obstructing effect always persists. Also, because the thin, non-plastic ice is in contact with the glacier floor, there is no underlying supply of internal ice. The only source of internal ice is thus upstream in a horizontal direction, and therefore the lowermost terminal planes of movement must approach the horizontal (Text Fig. 8). But such movements require a steeper surface gradient than is required for less nearly horizontal movements, or for

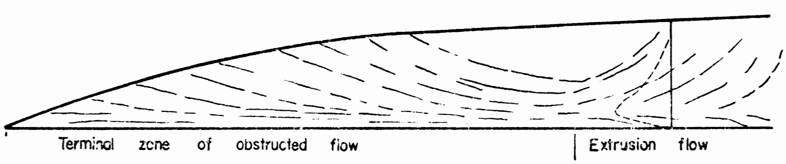


Fig. 8. Radial or long section through ablation area and terminus of pressure-controlled glacier. Lines of flow show surfaceward movements of obstructed extrusion flow and horizontal basal flow at the terminus,

extrusion flow upstream. As a result the surface slope of the terminal zone steepens somewhat, producing a slightly more convex profile.

If we now consider the effects of accumulation (momentarily neglecting ablation) on the hypothetical glacier, it is obvious that accumulation will add to the total volume of ice. If the amount accumulated in unit time exceeds the amount extruded from beneath the area of accumulation, the glacier must increase in thickness under that area. But as thickness increases, the average mobility of the moving ice increases. Consequently the rate of flow and the volume of ice extruded in unit time must also increase. These factors can continue to increase only until the volume extruded in unit time becomes equal to the volume accumulated, and then the part of the glacier underlying the area of accumulation must be at equilibrium, maintaining an unchanging thickness and surface profile. By similar reasoning we can understand that a unit-time-amount of accumulation that is less than the amount of ice extruded would result in equilibrium only after the glacier had become somewhat thinner.

With equilibrium established in the part of the glacier that underlies the area of accumulation, the rest of the glacier must continue to spread; and the rate of such spreading must be determined by the rate of accumulation, for (as demanded by equilibrium conditions) in unit time an amount of ice equal to the volume accumulated will be added by extrusion to the marginal parts which lack accumulation. However, if ablation occurs in the marginal areas, the spreading will be modified because the effect of ablation is to trim back the margins as they advance. In places where the volume of ice ablated equals the volume accumulated, further spreading of the glacier will be prohibited, and a profile of equilibrium will be established for the whole of the glacier. On the other hand, should the amount of ablation be less than the amount of accumulation, the glacier must continue to spread until, as is sometimes the case with actual glaciers, the margins have advanced into warmer regions where increased ablation finally equals accumulation and equilibrium is again established. Similarly, an excess of ablation must cause the margins to retreat into cooler regions where ablation is less and equilibrium can be established.

Ideally, ablation extends over a broad marginal area of a glacier, decreasing from a maximum at the terminus to a net amount equal to zero at the névé line (Text Fig. 6). The effect of such ablation is to cause a continuous removal of ice from the glacier surface and so to prohibit the attainment of a profile of unobstructed extrusion flow. Consequently obstructed flow must occur not only at the terminus but throughout the part of the glacier that underlies the area of ablation; and if the area of ablation should be increased by a rise of the névé line, ice that earlier moved by extrusion flow must suffer a transition to obstructed flow.

Effects of superglacial debris. Equilibrium of the ablation area is established only when the ice extruded at each point on the surface is trimmed back by ablation exactly as fast as it emerges. For this reason the profile of equilibrium must depend upon the distribution of ablation. At the terminus, where ablation is usually greatest, the emerging ice is trimmed back more rapidly than elsewhere. Consequently the surface slope becomes steeper, producing an increased differential pressure that in turn produces more rapid movements toward the surface. Ultimately movement must become rapid enough to balance the excessive ablation so that equilibrium is established. Thus on our ideal glacier (Text Fig. 6) where ablation increased rapidly toward the terminus, there is an added factor creating a more highly convex terminal profile.

On actual glaciers superglacial debris is very important in controlling the local distribution of ablation, and so in modifying the ideal surface profile. It is generally known that sparsely distributed superglacial debris greatly increases ablation, whereas thick covers of debris reduce ablation. Parts of the margin of the Greenland ice sheet offer an extreme example. There the englacial load is very small, emerging with greatest concentration in the basal ice near the extreme terminus. However, the distribution is sparse enough to cause increased ablation, and, as a consequence, there is an undersapping effect and the production of a vertical or overhanging face on which the relatively unablated higher ice overrides the base to such an extent as to fall by gravity (Text Fig. 9,B).

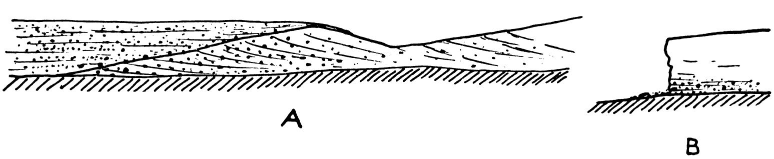


Fig. 9. (A) Glacier terminus becoming separated as result of burial under excessive accumulations of drift. (B) Detail of cliff-like glacier terminus produced by excessive basal ablation.

Examples of the other extreme are found in several Alaskan glaciers (Tarr, 1909A, p. 59-62) where terminal and near-terminal movements bring accumulations of debris to the surface in such large quantities that a protective cover is produced, and terminal ablation is reduced to the extent that it is less than ablation further upstream. Since these glaciers are in a state of recession, the upstream ice is thinning, and is thinning more rapidly than the debris-covered terminal ice. The end result is separation of the debris-covered ice from the rest of the glacier (Text Fig. 9,A). Where upstream thinning prior to separation has gone sufficiently far, the debris-covered ice must cease to move, since there can be no source of supply from upstream. The remainder of the glacier must then continue to retreat until equilibrium is re-established, or until the glacier disappears completely. Were such glaciers not retreating, flow from upstream would continue in sufficient volume to prohibit separation, and equilibrium would be maintained. However, because of the irregular distribution of ablation, the profile would not resemble that of our ideal glacier.

It must be emphasized that debris-controlled ablation usually occurs only at the terminus of a glacier, whereas our ideal distribution of accumulation and ablation, and the resulting profile, represent, in a general way, the conditions on most ice sheets. The reason why debris-controlled ablation usually occurs only at the terminus is obvious from Text Figs. 6 and 8 in which it is seen that the basal ice, which would contain the debris, is not brought to the surface except near the terminus. Exception to this generalization is found in the surface debris of lateral and medial moraines that are characteristic of valley glaciers.

Adjustments to Changes of Ablation and Accumulation.

We have already discussed the manner in which changes of accumulation may affect an ice sheet. It is to be noted, however, that changes of accumulation are almost always accompanied by changes of ablation. In the ideal examples given above increased accumulation caused an increased thickness and at the same time an increased spread due to an excess of accumulation over ablation. On the other hand, reduced accumulation caused a reduced thickness and a marginal retreat due to excess of ablation over accumulation. It may happen, however, that reduced accumulation would be accompanied by reduced ablation in such a way that an ice sheet would become thinner and more widespread at the same time. Actually this has occurred in the Antarctic, where a climatic change toward an extreme of coldness has resulted both in greatly reduced marginal ablation and in greatly reduced snowfall (Gould, 1940, p. 867). The ice sheet therefore approaches the condition of our ideal spreading glacier which is affected by neither accumulation nor ablation. As was noted, such spreading and thinning are accompanied by continually decreased rates of flow which, if carried to an extreme, would ultimately result in stagnation of the whole ice sheet.

It is paradoxical, as pointed out by Gould, that amelioration of the Antarctic climate should first cause increased snowfall and therefore an increase in the thickness of the ice sheet. At the same time there would be accelerated spreading until amelioration had gone far enough to produce greatly augmented marginal ablation. In a late stage of amelioration the area as well as the rate of ablation would be increased. This would involve a reduction in both the area and the volume of accumulation, and consequently the ice sheet would become thinner at the same time that its margins were retreating. Local circumstances would determine whether or not the ice would thin to the point of stagnation before terminal retreat had caused its total disappearance. However, in view of the very slow rates of movement in even thick ice sheets (discussed in the following section of this paper), and in view of conditions observed on many existing mountain glaciers and on the ice sheet in North-East Land it seems that rather extensive remnants of ice sheets might stagnate during late stages of deglaciation.

In areas of mountain glaciers the late-glacial recessions seem to have resulted largely from rising névé lines which have increased the areas of ablation at the expense of the areas of accumulation. Increased intensity of ablation near the termini, causing terminal retreat, has also taken place, but not so rapidly as to prohibit extensive thinning of the ice. Comparative photographs taken over a period of 45 to 50 years on some of the extensive but retreating glaciers of Mount Rainier show that ice in some of the high-level cirques has thinned by more than 200 feet. These glaciers, lying on steep slopes, are drainage controlled and are subject to comparatively strong deformational stresses, so stagnation is less likely to occur than within ice sheets. The important point, however, is that thinning does occur and that it plays a very significant rôle in the deglaciation process. Similar observations were made by Ahlmann and Thorarinsson (1937) on the Höffelsjökull in Iceland, where, over a period of 46 years, photographs showed thinning to have been extensive (Text Fig. 10). In this region, as on Mount Rainier, thinning and deglaciation appear to be due as much to reduced accumulation (resulting from rising névé lines) as to increased ablation.

In North-East Land similar conditions are found, there affecting an ice sheet. The névé line is raised to such an extent that the accumulation area now occupies less than 25 per cent of the ice sheet's surface area and the thickness of annual net accumulation within that area is equivalent to less than three inches of water (Ahlmann, 1933, p. 228). Excessive ablation occurs over the rest of the surface, and, according to Glen (1937, p. 308) and Ahlmann, most of the ice is stagnant. Perhaps some undetectably feeble movement does occur, but the difference between such feeble movement and stagnation is hardly to be distinguished.

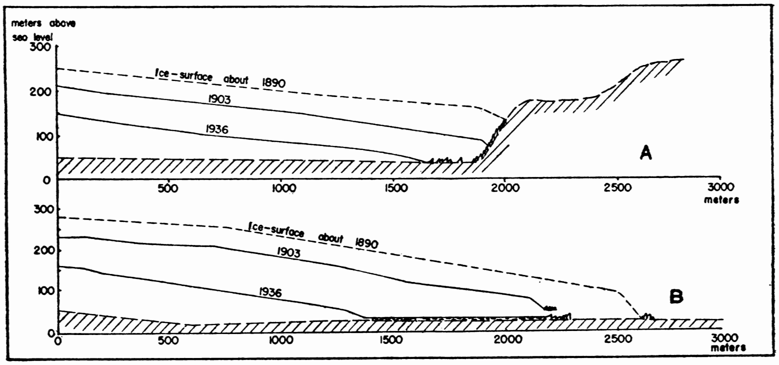


Fig. 10. (A) successive cross sections of the Höffelsjökull, Iceland, showing dominance of thinning. (B) long sections of same glacier. (After Ahlmann and Thorarinsson).

A more remarkable aspect of the North-East Land ice sheet is the fact pointed out by Ahlmann (1933, p. 165) that in general its margins have retreated no more than 100 yards or so from moraines that must have been built by actively moving ice. This must indicate that thinning rather than marginal retreat is decidedly dominant in the local deglaciation process. That similar thinning may have been dominant in the deglaciation of the larger Pleistocene ice sheets is suggested in Part II of this paper.

Rates of Flow.

Movement within ice sheets[3] has been discussed thus far without reference to quantitative values of rates of flow. This is because quantitative values are unknown. Extrusion flow

[3] Though the present discussion has been largely confined to ice sheets, it should be pointed out that valley glaciers (where their surface slopes are very slight) and expanded-foot and piedmont glaciers are also to be classified as pressure controlled.

cannot be measured directly because it occurs only at depth, and though surface movements within the border regions of ice sheets can be and have been measured, all such measurements are from localities where drainage control rather than pressure control prevails. Yet for purposes of making crude estimates of rates of flow within an ice sheet these measurements are useful.

With but one exception the measurements were made in western Greenland where the inland ice abuts against a barrier

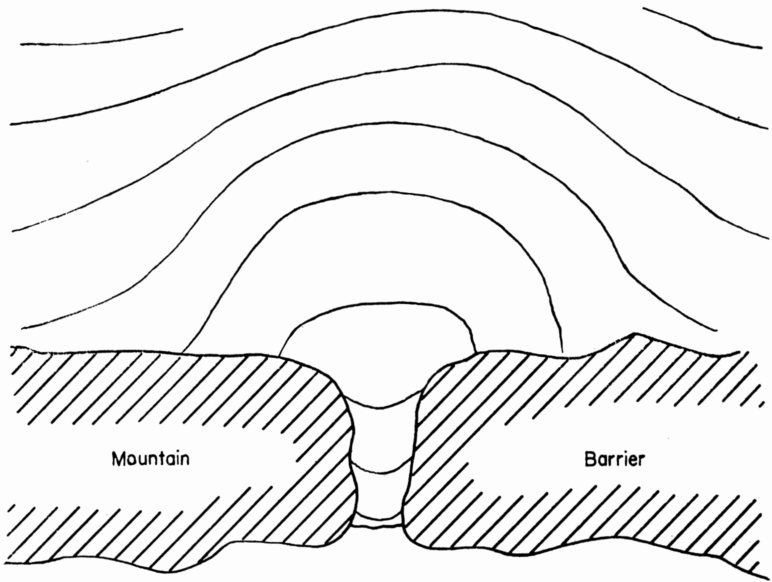


Fig. 11. Schematic plan showing contours on a Greenland outlet glacier and drainage basin.

of coastal mountains. In such places the only escape of ice is in outlet glaciers which flow at relatively high velocities through low but steep valleys transecting the mountains. The headward parts of these valleys, on the inland side of the mountains, are presumably expanded in dendritic fashion with small tributaries rising against interior divides to form typical drainage basins. The inland ice, however, completely fills these tributaries and covers both the headward divide and the minor interstream divides (Text Figs. 11 and 12). Yet within the basins the ice moves more rapidly and is thinner than within surrounding parts of the ice sheet, so depressions are formed in the ice-sheet surface (Text Fig. 11). As is explained more fully in the section on drainage-controlled glaciers, the flow of ice within these depressions is almost entirely by obstructed gravity flow and true gravity flow. Nevertheless, since the depressions usually occur at altitudes below the névé line, the only supply of ice to them is by pressure-controlled flow from the ice sheet. Consequently the volume of ice entering the basins in unit time from the ice sheet must approximately equal the volume flowing out of the basins through the outlet valleys, and, since the velocity of a given volume of ice passing different sections in unit time is inversely proportional to the areas of the sections, there must be a definite relationship between rates of flow within the outlets and within the ice sheet.

Near the medial line on the surface of the Great Karajak outlet glacier (latitude 70°25′ N. in West Greenland), at a point where the valley transects the coastal mountains, von Drygalski (1897, p. 269; Plate 2) measured velocities of 20 to 30 feet per day. Toward the bottom and sides of the glacier, however, velocities must diminish toward zero, so that the volume of ice passing a cross section must be considerably less than if all the ice were flowing at a velocity of 20 to 30 feet per day. A rough estimate based on these figures gives, for the entire cross section, an average velocity of about 5 feet per day. The width of the section is 4.5 miles, and the average thickness of ice, as determined from an approximate cross section of von Drygalski's (1897, p. 274), is of the order of 1000 feet. A section through the ice sheet at the head of the basin may measure about 75 miles in length and would be about 3000 feet in height. These last figures are more nearly guesses than estimates, but they are probably of the correct order. The area of the ice-sheet section is thus 50 times the area of the outlet section and it follows that the average velocity of flow must be but one-fiftieth of that through the outlet, that is, 0.1 foot per day. This is admittedly a very crude estimate, and it is subject to further error because the Karajak basin drains through a second smaller valley as well as through the Great Karajak outlet, and in addition, ablation must cause considerable losses of ice. Yet the order of magnitude is probably right, so that the figures are useful in showing that even within thick ice sheets, flow must be of very low velocities.

As was noted, the estimate applies only to the average velocity through the ice-sheet section, so it must be that the near-basal ice flows considerably faster, say 10 times as fast as the average. However, that is still a low velocity. On the other hand, the section is near the névé line. Consequently the average velocity is greater than elsewhere beneath the area of accumulation, and hence movements are there at a maximum for the pressure-controlled part of the ice sheet, diminishing toward zero under the locus of greatest accumulation.

These very slow rates of movement are to be expected since surface slopes on the interior parts of ice sheets are of very low magnitude. The following data, taken from a summary of published profiles of the Greenland ice sheet (Hess, 1933, p. 110), give the characteristic surface slopes between the 1000- and 1500-meter contours. At all the places given these contours lie within 40 miles of the edge of the ice sheet, and in many parts of Greenland the névé line lies between them.

Profile determined by	Nansen,	W.	Greenland	1° 20′
” ” ”	”	E.	”	0° 45′
” ” ”	Koch,	W.	”	0° 40′
” ” ”	”	E.	”	3° 30′
” ” ”	Quervain,	W.	”	0° 30′
” ” ”	”	E.	”	0° 50′

One would not expect the small differential pressures resulting from these slight surface slopes to produce more than a very slow motion even near the borders of an ice sheet. The exceptionally high slope value determined by Koch on the east side of Greenland may be disregarded, for the profile there was measured over the upper part of an outlet glacier that drains through the high mountains of Dronning Louise Land, and is therefore not characteristic of the interior parts of an ice sheet.

DRAINAGE-CONTROLLED GLACIERS.

The outlet glaciers of Greenland offer excellent examples of drainage control, for they occur where the pressure-controlled ice sheet flows out on to surfaces with sufficient slope to enable the ice to "drain" away by movements of gravity and obstructed gravity flow. In a similar way many, but not all, mountain glaciers drain through steeply sloping valleys and therefore are also drainage controlled.

Limiting Velocity and Limiting Thickness.

The mechanics of drainage movements and gravity flow have been demonstrated (Text Fig. 3 and accompanying text). Let us now consider a simplified and idealized outlet glacier (Text Fig. 12) that (1) is not affected by ablation or accumulation, (2) lies in a valley of uniform slope and cross section, and (3) terminates abruptly without thinning. Under these circumstances a constant rate of supply from the ice sheet maintains continuous flow with a constant velocity and ice thickness that are dependent upon the slope of the valley and the rate of supply. In other words, for a given slope and supply, an ideal drainage-controlled glacier, when not affected by ablation, has a definite velocity and thickness which will here be called the *limiting velocity* and *limiting thickness.*

This will be understood if we consider what would happen if our ideal glacier were initially of non-uniform thickness.

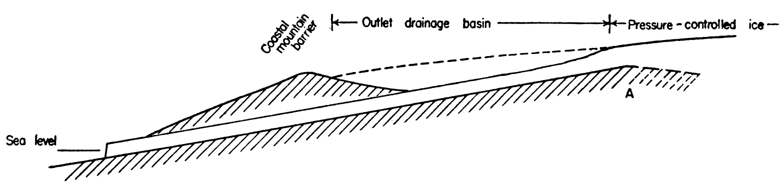


Fig. 12. Schematic long section through a Greenland outlet glacier and drainage basin. Pressure-controlled ice sheet is dammed back by a barrier of coastal mountains. Escape through outlet valley in the barrier allows drainage flow and results in thinner ice through the basin.

Suppose that the upper surface slope is less than the floor slope, so that ice thickness increases in the downstream direction. It would follow that both the average ice plasticity and the average effective shear stresses would also increase downstream, and as a result, a downstream segment of the glacier would flow more rapidly than an upstream segment. However, a downstream segment derives its entire supply from upstream, and since the upstream ice is necessarily moving more slowly, the supply is not sufficient to maintain the initial downstream thickness. Therefore the downstream segment thins and in so doing its velocity decreases. This must continue until the upstream and downstream segments are of equal thickness and therefore of equal velocity, or in other words, until the glacier is of uniform thickness and velocity throughout its length. It is in this way that reduced supply from an ice sheet or cirque must affect a drainage-controlled glacier, for under such circumstances the head of the glacier would immediately become thinner and slower. Then the segment just downstream from the head would necessarily suffer the same reduction, and so on until the whole length of the glacier were affected. In attempting to visualize the manner in which this thinning occurs, one finds an obvious deficiency in the analogy of sliding ice layers with the differential sliding of a pack of cards, for sliding cards cannot thin out as do the "layers" of streamline flow.

Transition from obstructed extrusion flow to obstructed gravity flow and gravity flow. We will next consider an ideal drainage-controlled glacier on which the slope of the upper surface is greater than the slope of the floor. This is a condition frequently found in nature since most drainage-controlled glaciers, both mountain glaciers and outlets, exist below the névé line where ablation causes increased thinning toward their termini. Where such thinning glaciers lie on horizontal floors they are pressure controlled, and movement is by obstructed extrusion flow in the manner discussed in connection with Text Fig. 7. It was pointed out that obstructed extrusion flow is a type of flow in which the stresses responsible for movement result from differential pressures and are proportional to the surface slope. In the examples discussed it was also pointed out that movements occur along planes that are directed forward and upward against gravity. However, within the ablation area of a pressure-controlled glacier, flow at the terminus necessarily occurs on nearly horizontal planes, and a relatively steep surface slope is required to maintain such near-horizontal flow. Stating the converse, we may say that within limits, the steeper the surface slope, the more nearly horizontal the planes of obstructed flow must be. Beyond those limits (which are quantitatively undetermined) further steepening of the slope must cause the planes to pass the horizontal so that they dip in the downstream direction and movement comes to have a downward component. In such circumstances flow must be more rapid than where there is an upward component, for not only is the surface slope steeper and the pressure differential greater, but, because of the downward component, motion is directly aided by gravity.

Gravity aid results from shear stresses of the sort illustrated in Text Figs. 2 and 3, but in the case now considered the effective shear components are not in the direction of slope of the glacier floor. Gravitative shear in the direction of the floor cannot occur because of the downstream obstructing effect produced by thinning. This sort of flow is characteristic of many drainage-controlled glaciers. It is the transitional flow type that we have called obstructed gravity flow.

Obviously the difference between pressure-controlled obstructed extrusion flow and drainage-controlled obstructed gravity flow is a gradational one, one in which both the surface slope with its accompanying differential pressures and the gravity components on the downward sloping planes of shear are effective in determining the stresses responsible for flow. Yet regardless of partial surface control the more fundamental control is the slope of the glacier floor. This is because surface slopes steep enough to allow a downward component of extrusion usually require steeply sloping floors. It may be correctly argued that if the surface slope of a pressure-controlled glacier should become sufficiently steep, a downward component of movement would occur even where the floor is horizontal; however, there is no reason why any glacier, other than those lying on steeply sloping floors, should ever acquire so steep a surface slope. Exception to this is found on pressure-controlled glaciers at parts of the termini where excessive ablation, influenced by englacial and surface debris, causes the formation of steep, cliff-like fronts, and it is probable that at such places downwardly directed shearing movements take place. But such exception does not alter the more general fact that the effect of pressure-controlled movements is to produce decreased rather than increased surface slopes. This is obviously in keeping with the fact that such movements are fundamentally parts of a leveling mechanism by which the glacier strives toward the attainment of a zero slope.

Thus glaciers can acquire surface slopes of sufficient steepness to allow a downward component of shearing in only those places where they lie on steep topographic slopes. The Greenland outlets are examples in which pressure-controlled movement extends up to a buried divide or break in slope (A of Text Fig. 12) beyond which the land surface is steeper than the critical surface slope at which obstructed extrusion flow (or extrusion flow) gives way to gravity flow. Hence any ice lying on that land surface must have a surface slope steep enough to promote downward shear movements. This is further illustrated by Text Fig. 13 which shows an ice sheet in the advancing hemicycle of glaciation. As the sheet is extended over the divide there is no way in which ice on the steep distal side can be given a sufficiently gentle surface slope for the maintenance of pressure-controlled movements. In short, the glacier floor exercises the fundamental influence and the ice becomes drainage controlled.

Gravity flow being more rapid than flow within the ice sheet, the drainage-controlled ice must be of less thickness than that on the crest of the divide where pressure-controlled flow gives way to gravity flow. Theoretically the actual thickness of the drainage-controlled ice will be the limiting thickness as determined by the degree of slope and the amount of supply. However, this thickness cannot be attained throughout the length of the glacier since the advancing terminus must thin out. Such thinning creates an obstruction and, as a result, movement is by obstructed flow, the effect of which is to build up the surface. In this way the glacier constantly attempts to attain its limiting thickness in all parts. But building up of the surface near the end of the glacier produces a steepened terminal slope and consequently increased movement, so that the glacier advances without attaining its limiting thickness.

Limiting thickness can be attained only where the obstructing effect of a thin terminus is eliminated. This might occur (if ablation were absent) in valleys like those of Greenland where the outlets flow into deep fiords and are abruptly terminated by wave action. A similar effect is produced where drainage-controlled glaciers pass over a steep break in slope of the glacier floor, so that icefalls or cascades are produced. Ice at the upper brink of the fall may be considered as an abrupt, unthinned terminus which can cause no obstruction to flow from upstream. Where the obstructing effect is thus eliminated the ideal glacier approaches its limiting thickness and in so doing the planes of obstructed flow gradually flatten out toward parallelism with the glacier floor. If the limiting thickness is actually attained, true parallelism must exist. Thus ideal gravity flow is but the end stage in a transition from pressure-controlled extrusion flow and obstructed extrusion flow through obstructed gravity flow. There is no sharply marked separation between these types of glacier motion, yet each type is distinctly characteristic of different glacial conditions.

ADJUSTMENTS TO CHANGES OF ABLATION AND ACCUMULATION.

Equilibrium requirements for drainage-controlled glaciers are fundamentally the same as for pressure-controlled glaciers. Supply must equal ablation and movement must maintain an unchanging profile. The equilibrium relationship between movement and ablation is the same as within the ablation area of a pressure-controlled glacier, for obstructed flow movements to any part of the surface are trimmed back as rapidly as advances are made. Increased rates of ablation must cause a glacier to retreat up valley to higher and cooler altitudes where ablation will be reduced to equality with supply, and decreased ablation must cause an expansion into lower and warmer altitudes. However, as already stated, retreat and expansion are seldom a result of changes in ablation alone. Usually there are accompanying changes in the altitude of the névé line and consequent variations in supply. Where increased supply accompanies glacial expansion the limiting velocity and limiting thickness (dependent on slope and supply) must also increase, so the glacier must accelerate and thicken at the same time that it advances. In a similar way the limiting velocity and thickness of a shrinking glacier are usually reduced so that deceleration and thinning accompany retreat.

Adjustments to Changes of Valley Width.

Changes of limiting velocity and thickness are also caused by changes of valley width. Changes of velocity due to this cause have frequently been mentioned in glaciological literature, but changes of thickness are seldom noted, perhaps because they

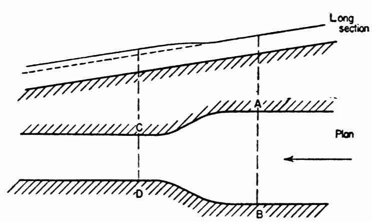


Fig. 13. Pressure-controlled ice sheet advancing over buried divide, the distal side of which is steep enough to demand drainage flow.

Fig. 14. Constriction in valley of a drainage-controlled glacier—causes increased thickness as well as increased velocity.

are not so readily measured or so obviously to be expected. We can understand these changes by reference to Text Fig. 14 in which the cross section of the glacier at AB is greater than at CD. In the maintenance of continuous flow the amount of ice passing AB in unit time must also pass CD, or in other words, the volume of ice flowing through AB in unit time is equal to the supply at CD. If for the moment we consider the glacier to be of uniform thickness at both sections, it must be that the area of the section at CD is less than that at AB. Therefore as flow proceeds, the given volume of ice constitutes a smaller supply per unit area at AB than at CD; in short, the effective supply of ice at CD is greater than at AB, and the result (at CD) is an increase of both the limiting velocity and limiting thickness.

The fact that both velocity and thickness must increase at the same time is apparent when one considers that the immediate effect of increased thickness is to produce a greater average plasticity of ice at the given section. As a result the ice is more mobile, and the active stresses are capable of producing a greater average rate of flow. When finally the rate of flow and increased thickness are such that the volume of ice passing CD in unit time is equal to the volume passing AB, conditions of continuous flow are re-established, and further changes at CD do not occur.

Adjustments to Changes of Slope.

We hardly need repeat that changes of slope as well as of supply are effective in controlling the limiting velocity and limiting thickness of a glacier. There is a difference in slope control, however, for in this case increased velocity is accompanied by decreased instead of increased thickness, and vice-versa. This is because the effective gravitative shear stresses in a glacier are proportional to the sine of their slope, so that on a gentle slope the stresses are smaller and flow is slower than on a steep slope. But, if a given volume of ice is supplied per unit time, slower flow must be accompanied by thickening of the glacier. Such thickening, in turn, is accompanied by an increase of average plasticity. As a result the given stresses can produce a somewhat greater rate of flow, and the loss of velocity is less than as if no thickening occurred. Balance is established when finally the slower but thicker ice moves at a rate just sufficient to handle the given amount of ice supplied in unit time. In a similar way ice on a steeper slope must become thinner though its rate of flow increases.

Thinning of Glaciers During Recession.

Quantitative data on the amounts of relative change in thickness and velocity are not known, but if, as is probable, ice plasticity increases as an exponential function of pressure (i. e., of glacier thickness), it must be that velocity changes are greater than thickness changes. In other words, a small increase of thickness causes a disproportionately large increase in plasticity and therefore in velocity. Conversely, a small decrease of thickness would cause a large decrease in velocity. These facts are particularly significant in showing that changes of velocity, rather than changes of thickness, are the principal means by which drainage-controlled glaciers adjust themselves to changes of supply.

At first thought these facts may seem to contradict the idea that thinning plays an important part in glacier recession. However, where ablation is effective and rates of obstructed movement to the surface determine whether or not ablation can be counteracted, it must be that any decreased velocity is accompanied by ablation thinning. We have noted that velocity decreases are usually the result of reduced supply and thus far we have considered a rise of the névé line as the principal cause of supply reduction. Actually, however, surface ablation, where it produces thinning, causes an equally important supply reduction, for it has a direct and immediate effect on supply at all places within the ablation area. Consequently ablation not only thins the ice but in reducing supply it causes an additional loss of velocity. This in turn results in ablation taking an even greater toll, for the slower surfaceward movements are less effective in counteracting ablation. As a result the rate of thinning becomes accelerated. On glaciers where ablation affects a large area of ice this should expectably lead to complete loss of velocity—that is, to stagnation—before final disappearance of the ice. This is especially true since thinning produces disproportionately large losses of velocity.

Obviously within any given glacier the distribution and intensity of ablation and the amount of supply losses due to reduced snow accumulation will determine the extent to which actual stagnation may occur. In extreme cases where ablation is excessive only at the terminus and where supply is maintained without great loss in upstream parts of a glacier, retreat may occur without notable thinning or loss of velocity. However, such extreme conditions are not characteristic of most glaciers, so we may expect that retreat is more frequently accompanied by thinning.

As already noted, thinning is actually dominant in North-

East Land, where the ice sheet has become nearly stagnant though its terminus has retreated only a hundred yards or so from its last moraine. Similar conditions are found on small glaciers of the Rocky Mountains and the Sierra Nevada. Many of these glaciers, most of which are but small remnants left against the headwalls of large cirques, built small moraines during their last period of activity. Since that time they have wasted to a condition of almost complete inactivity. In warm summers névé is entirely ablated from their surfaces, even at their heads, so we may be sure that they are thinning as a result of both ablation and lack of supply. Yet in many places their termini are still in contact with the recent moraines or they have retreated only a few yards from them. This must certainly indicate a condition of dominant thinning in which terminal retreat is quite subordinate.

Renewed activity of these glaciers can occur only if new supplies of snow accumulate on their surfaces. Then the first change must be one of increased headward thickness. Following that, downstream parts will be successively thickened by obstructed flow until finally the end of the glacier becomes thickened sufficiently so that terminal advance may occur. This will continue until equilibrium is established, and if that is maintained long enough, new terminal deposits will accumulate to form moraines.

It is suggested that glacier retreat in many places has been of this sort in which periods of dominant thinning have alternated with periods of temporarily renewed activity. This must be particularly true in late stages of retreat, since the shorter a glacier is, the more likely it becomes that changes of ablation or supply at one part of a glacier will be accompanied by sympathetic changes at another part. Thus increased ablation at the terminus is likely to be accompanied by a rising névé line, reduced supply, and thinning; conversely, an increased supply is likely to be accompanied by a lowered névé line, decreased net ablation, and renewed activity.

In thus emphasizing the occurrence of thinning in the late stages of glacial retreat, it is not intended to deny that terminal recession occurs. The fact that glaciers become progressively shorter is obvious proof of such recession. However, the interesting point is that recession is not independent of thinning—in fact it is the result of thinning, for the initially thinner terminal zone of a glacier must usually be the first to suffer stagnation and final disappearance. It is only where thick accumulations of surface debris retard terminal ablation or where subglacial topography is responsible for relatively thin upstream ice, that the ice may thin to disappearance at some place other than the terminus.

Examples of upstream thinning and disappearance of ice as a result of subglacial topography are found where valleys rise in a step-like series of cirques. In such valleys ice over the tops of the step risers is necessarily thinner than at places either upstream or downstream. Consequently late-stage thinning may cause the glacier to become separated into a series of tandem glacierets.

When glaciers are greatly extended, as at the time of maximum glaciation, there is less likelihood of sympathetic changes of ablation and supply, for with great distances between the head and terminus of a glacier there may be considerable differences in the meteorologic and topographic controls affecting ablation and supply at the two places. Consequently changes of terminal ablation are not so apt to be accompanied by sympathetic changes of supply, and terminal retreat is more likely, though not certain, to occur without appreciable thinning.

SUMMARY.

Analysis of glacier regimens shows that four types of ice movement may occur within glaciers, each type being characteristic of different conditions of underlying topography and surface configuration. These types are extrusion flow, obstructed extrusion flow, obstructed gravity flow, and gravity flow.

The two types of extrusion flow are called pressure-controlled types because they result from and are directly controlled by differential pressures. True extrusion flow is characterized by radial or downstream extrusion of ice at depth from below the overlying ice. It is found only within glaciers of very slight surface gradients, e. g., the interior parts of ice sheets and ice caps, expanded foot and piedmont glaciers, and some mountain névés and glaciers. Rates of flow increase in the downstream direction, being generally proportional to the distance from the locus of maximum accumulation; however, in all parts of a glacier, even where movement is most rapid, the absolute rates of flow are necessarily very low. In most cases even the maximum rates are probably to be measured in inches or fractions of an inch per day.

Obstructed extrusion flow is found where pressure-controlled glaciers encounter any sort of obstruction. Probably the most common, though not the most obvious, obstruction results from excessive thinning of ice in the downstream direction. This is characteristic of the ablation areas of pressure-controlled glaciers and results in forward and upward movements in such a manner that internal ice is extruded to the upper surface of the glacier.

The two types of gravity flow are called drainage-controlled types because they occur only on relatively steep topographic slopes where ice "drains" away from higher-level sources of supply. Many mountain glaciers and outlet glaciers from ice sheets are of this sort. The mechanism of true gravity flow involves differential shearing of infinitesimal "layers," the greatest differential between layers occurring at the base of the glacier, but the greatest absolute motion occurring at the surface which is moved downstream by the sum of all underlying differential movements. The stresses responsible for movement are shear components of direct gravitative forces.

Obstructed gravity flow is a transitional type connecting obstructed extrusion flow and true gravity flow. It is similar to obstructed extrusion flow in that obstruction by thinning causes a forward and surfaceward extrusion of ice. However, there is no upward component of motion in this surfaceward movement. Rather there is a downward component along which gravitative shear stresses are operative. Movement toward the surface occurs only because the downward slope of the surface is steeper than the slope of the planes of movement.

Glacier equilibrium is discussed and defined as a condition in which total accumulations of supply in unit time are equal to total losses by ablation, and internal movements are such as to maintain an unchanging surface profile.

It is demonstrated that there is a theoretical limiting velocity and limiting thickness for drainage-controlled glaciers. These limits are determined by the amount of ice supplied to the glacier per unit time and by the slope of the glacier floor. On this basis it is shown how changes of supply, valley width, and slope may affect the regimen of a drainage-controlled glacier. In general it appears that these factors have a greater effect on limiting velocity than on limiting thickness. In other words, in adjusting itself to variations of these factors, a glacier suffers a greater change of velocity than of thickness. Nevertheless it happens that within the ablation area of a glacier (this applies to pressure-controlled as well as to drainage-controlled glaciers) excessive thinning is produced as a secondary result whenever velocities are decreased. Thus retreating glaciers, which are generally characterized by reduced supply and therefore by reduced velocity, are also thinning glaciers.

In conclusion it must be emphasized that this study is based on deductions from ideal glaciers. The behavior of actual glaciers may be complicated by departures from ideal conditions; yet it is only through a study of ideals that complications can be understood. It is hoped that a basically sound approach has been made to such an understanding and that the general conclusions reached may be applied in analyzing the characteristics of all glaciers.

It must also be noted that though the author may be the first to organize and relate the various concepts discussed, he is not the first or only one to appreciate some aspects of these concepts. The fundamentals of gravity flow have been recognized since the early days of glaciology. In 1910 R. S. Tarr (1910, p. 30) called attention to the probability of something like extrusion flow. R. Streiff (1940), in an article that came to the author's attention since writing this paper, also shows an understanding of the difference between extrusion flow and gravity flow. And H. W. Ahlmann, in papers too numerous to mention, shows an appreciation of many of the implications of glacier equilibrium and extrusion flow.

(Part II will appear in the next issue of this journal)

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