Art. V.—*On Crystal Drawing;* by S. L. Penfield.

Introduction.—The methods commonly employed for representing crystals consist in drawing their edges as they appear when projected upon a plane. A peculiarity of the methods used is that the eye, or point of vision, is regarded as being at an infinite distance from the object, so that all edges which are parallel on a crystal appear as parallel lines in the drawing. Thus true perspective, whereby parallel edges would appear in a drawing as lines approaching one another in the distance, is lost sight of. Furthermore, two kinds of projection are employed: *orthographic*, where the lines of projection fall at right angles, and *clinographic*, where they fall at an oblique angle on the plane upon which the drawing is made. Most of the figures found in works on mineralogy and crystallography are drawn in clinographic projection.

The data generally employed in constructing a crystal figure are the inclinations and lengths of the axes and the symbols of the forms, while interfacial angles are not made use of directly, other than as they may have been employed for determining the axial relations and the symbols of the several faces.

To be really successful in drawing, it is essential that one should have a thorough understanding of the form or combination to be represented, and that every step in the process of constructing a figure should be fully comprehended. The reason for offering the present communication is the hope entertained by the writer, that by developing the subject of crystal drawing in a manner somewhat different from that generally adopted, the processes involved may be comprehended more readily and the work accomplished with greater facility and accuracy.

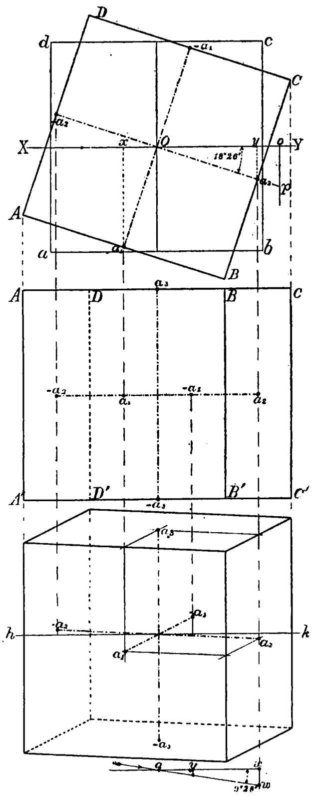
Projection of the Axes of the Isometric System.—It is believed that figures 1 to 4 will make clear the principles upon which the projection of the isometric axes are based. Figure 1 is an orthographic projection (a *plan*, as seen from above) of a cube in two positions, one, *a b c d*, in what may be called normal position, the other, *A B C D*, after a revolution of 18° 26′ about its vertical axis. The broken-dashed lines throughout represent the axes. Figure 2 is likewise an orthographic projection of a cube in the position *A B C D* of figure 1, when viewed from in front, the eye or point of vision being on a level with the crystal. In the position chosen, the apparent width of the side face *B C B′ C′* is one-third that of the front face *A B A′ B′*, this being dependent upon the angle of revolution 18° 26′, the tangent of which is equal to $\frac{1}{3}$. To construct the angle 18° 26′, draw a perpendicular at any point on the horizontal line, as at *o* figure 1, make *op* equal one-third *Oo*, and join *O* and *p*. The next step in the construction is a change from orthographic to clinographic projection. In order to give figures the appearance of solidity it is supposed that the eye or point of vision is raised, so that one looks down at an angle upon a crystal which is figured; thus, in the case under consideration, figure 3, the top face of the cube comes into view. The position of the crystal, however, is not changed, and the plane upon which the projection is made remains vertical. From figure 1 it may be seen that the positive ends of the axes a_1 and a_2 are forward of the line *XY*, the distances $a_1 x$ and $a_2 y$ being as 3:1. In figure 2 it must be imagined, and by the aid of a model it may easily be seen, that the extremities of these same axes are to the front of an imaginary vertical plane (the projection of *XY* above) passing through the center of the crystal, the distance being the same as $a_1 x$ and $a_2 y$ of the plan. In figure 4 the distance *ax* is drawn of the same length as $a_1 x$ of the plan, and the amount to which it is supposed that the eye is raised, indicated by the arrow, is such that a_1 instead of being projected horizontally to *x*, is projected at an inclination of 9° 28′ from the horizontal to *w*, the distance *xw* being one-sixth of *ax*; hence the angle 9° 28′ is such that its tangent is $\frac{1}{6}$.

1

2

3

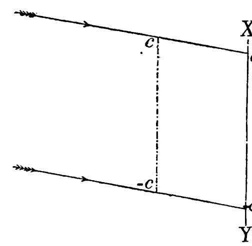
4



FIGS. 1–4.—Development of the axes of the isometric system in orthographic and clinographic projection.

Looking down upon a solid at an angle, and still making the projection on a vertical plane, may be designated as *clinographic projection;* accordingly, to plot the axes of a cube in clinographic projection in conformity with figures 1, 2 and 4, draw the horizontal construction line *hk*, figure 3, and cross it by four perpendiculars in vertical alignment with the points $a_1, -a_1$ and $a_2, -a_2$ of figures 1 and 2. Then determine the extremities of the first, a_1, $-a_1$ axis by laying off distances equal to *xw* of figure 4, or one-sixth $a_1\,x$ of figure 1, locating them below and above the horizontal line *hk*. The line a_1, $-a_1$ is thus the projection of the first, or front-to-back axis. In like manner determine the extremities of the second axis, a_2, $-a_2$, by laying off distances equal to one-third *xw* of figure 4, or one-sixth $a_2\,y$ of figure 1, plotted below and above the line *hk*. The line a_2, $-a_2$ is thus the projection of the second, or right-to-left axis. It is important to keep in mind that in clinographic projection there is no foreshortening of vertical distances. This is evident from figure 5, where c, $-c$ is supposed to represent a vertical axis and *XY* the trace of a vertical plane on which the projection is made. The parallel lines of sight, indicated by the arrows, project the axis c, $-c$ to c', $-c'$ without change of length. In figure 3 the axis a_2, $-a_2$ is somewhat, and a_1, $-a_1$ much foreshortened, yet both represent axes of the same length as the vertical, a_3, $-a_3$, and of the plan above, when plotted in clinographic projection. The completion of the cube about the clinographic axes, as indicated by the construction lines, figure 3, is too simple to need special comment.

5



It is wholly a matter of choice that the angle of revolution shown in figure 1 is 18° 26′, and that the eye is raised so as to look down upon a crystal at an angle of 9° 28′ from the horizontal, as indicated by figure 4. Also it is evident that these angles may be varied to suit any special requirement. As a matter of fact, however, the angles 18° 26′ and 9° 28′ have been well chosen and are established by long usage, and practically all of the figures in clinographic projection, found in modern treatises on crystallography and mineralogy, have been drawn in accordance with them. The development of the axes as indicated by figures 1 to 4 yields the same result as that obtained from following the scheme found in almost all text-books of crystallography, accredited to Naumann.*

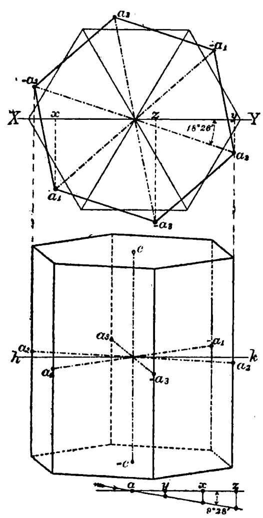
It will be observed that figures 1 and 3 are in vertical alignment, and one of the chief features of this communication will be to emphasize the value and importance of two projections, orthographic and clinographic. The object of the upper figure or *plan* is twofold: (1) it may be employed as a help in the construction of the more complex clinographic projection below, and (2) it serves to make clear certain relations which

* Lehrbuch der Krystallographie, 1830, Band II, p. 400.

at times are only with difficulty, if at all, comprehended from a clinographic projection alone. Figures 2 and 4 have been introduced merely as helps in the development of the clinographic projection. It is also worthy of note that in the majority of cases a plan and its accompanying clinographic projection may be drawn more readily than a single figure in clinographic projection alone.

No originality is claimed for the idea of making use of a plan in connection with a clinographic projection. The principles are those commonly made use of in mechanical drawing, though generally in dealing with that subject orthographic projection alone is employed. In Kokscharow's Atlas accompanying his "Mineralogie Russlands" it will be found that a plan accompanies almost every figure drawn in clinographic projection, while Miller in his "Treatise on Mineralogy" employs orthographic projection almost exclusively. Lastly, students of crystallography may use an orthographic and its accompanying clinographic projection much as a carpenter or builder uses a plan and its accompanying elevation. The one supplements the other.

6
7
8

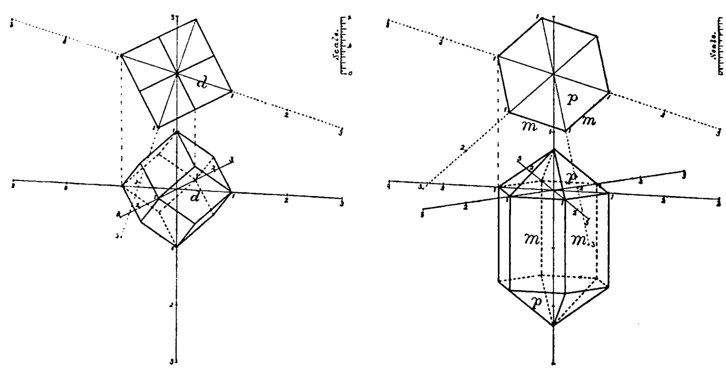


FIGS. 6, 7 and 8.—Development of the axes of the hexagonal system in orthographic and clinographic projection.

Projection of the Axes of the Hexagonal System.—For projecting the hexagonal axes exactly the same principles may be made use of as were employed in the construction of the isometric axes. Figure 6 is an orthographic projection, *a plan*, of a hexagonal prism in two positions, one of them, a_1, a_2, etc., after a revolution of 18° 26′ from what may be called normal position. In figure 7 the extremities of the horizontal axes of figure 6 have been projected down upon the horizontal construction line hk, and a_1, a_2 and $-a_3$ which are forward of the line XY in figure 6 are located below the line hk in the clinographic projection, the distances from hk being one-sixth of a_1x, a_2y and $-a_3z$ of figure 6. Figure 8 is a scheme for getting the distances which the extremities of the axes are dropped. The vertical axis in figure 7 has been given the same length as the axes of the plan.

Engraved Axes.—For the purpose of facilitating crystal drawing the writer has had the isometric and hexagonal axes engraved, and impressions of them made on good quality of drawing paper have been found very useful. To insure accuracy they were plotted on a large scale (the vertical axis 28^{cm} in length) and they are shown very much reduced in figures 9 and 10. Each axis from the center is divided into thirds, and generally the lengths marked 1, when taken as *unity*, will give a figure of convenient size for drawing. In figure 9 an orthographic and a clinographic projection of a dodecahedron are shown, and in figure 10 corresponding projections of a combination of prism *m* and pyramid *p* of apatite, $c = 0{\cdot}735$. As is evident from the figures, the upper axes are for orthographic,

9 10



FIGS. 9 and 10.—Scheme of the engraved axes of the isometric and hexagonal systems, one-sixth natural size.

the lower for clinographic projections. The sections of the axes marked *2* and *3* are lengths most frequently needed in the construction of complex figures. Printed on each sheet is a scale which will be referred to as the *scale of decimal parts.* Its length is equal to that of *unity* on both the vertical axis and the axes for orthographic projection. As printed on the original sheets the scale is divided into one hundred parts.

Axes of the Tetragonal and Orthorhombic Systems.—For drawing tetragonal and orthorhombic crystals the engraved isometric axes may be used, after changing certain lengths. The vertical axis for both systems is changed by taking the desired length from the scale of decimal parts, referred to in the previous paragraph. For an orthorhombic crystal the length of the brachy, or $\breve{a}$, axis is first laid off on the front-to-back axis of the orthographic projection above by means of the scale of decimal parts, and is then projected down upon the front-to-back axis below by means of a vertical line. Thus with facility and accuracy the engraved isometric axes may be modified to suit the requirements of any tetragonal or orthorhombic crystal.

11

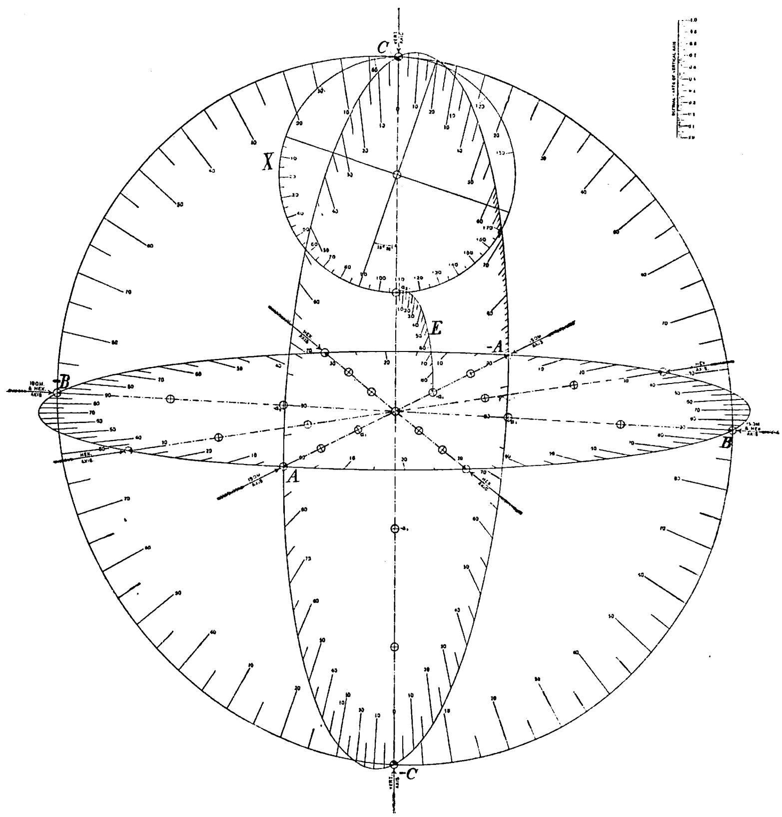


FIG. 11.—Protractor for plotting crystallographic axes; one-third natural size.

Projection of the Axes of the Monoclinic and Triclinic Systems.—These axes are obtained from those of the isometric system by giving the lines suitable inclinations, and varying their lengths. Instead, however, of using the methods generally employed for inclining the axes, it occurred to the writer that both time and accuracy might be gained by constructing a suitable protractor, which is shown one-third its natural size in figure 11. At the top is a graduated circle, *X*, two of the diameters of which inclined at 18° 26′ to the vertical and horizontal, represent unit lengths of the *a* and *b* axes in orthographic projection. The uses of the circle and its graduation will be explained later. The three large ellipses are the clinographic projections of three circles uniting the ends of the isometric *A*, −*A*; *B*, −*B* and *C*, −*C* axes; they represent, therefore, the paths which the extremities of the axes would follow if the latter were revolved in the three axial planes. The ellipses may also be regarded as the clinographic projection of three great circles of a sphere; an equator, crossed by two meridians at 90° to one another. The ellipses and their graduation were plotted with much care, and the engraving was skillfully executed by Messrs. Bormay & Co. of New York. Each axis is divided into thirds, and a scale giving decimal parts of the vertical axis accompanies the protractor. The quadrant of a small ellipse *E* has a radius equal to one-third that of the large ellipse. It is intended for getting one-third the length of an inclined *a* axis, but it has not proved to be of much value. Printed on cardboard, the protractor may be used for a long time, it being intended that the axes shall be transferred to a sheet of drawing paper by superimposing the protractor and puncturing the unit lengths of the axes with a needle point.

The axial protractor has been in use in the writer's laboratory for four years, and has been found very convenient, not only for plotting axes of the monoclinic and triclinic systems, but, also, for constructing the axes of twin crystals. It may be said of the protractor and also of the engraved axes that they have proved to be not only time-savers, but they have also helped to make the work of crystal drawing more accurate and better understood. Students frequently encounter difficulties in crystal drawing because the axes with which they are working have not been plotted with accuracy, but by the use of the engraved axes this difficulty, at least, is eliminated.

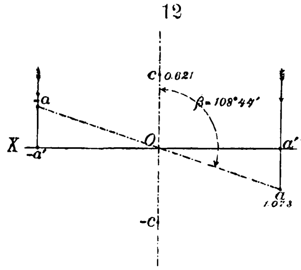
A few examples will serve to illustrate the methods of using the axial protractor in plotting inclined axes.

In both the monoclinic and triclinic systems the same method is used for plotting the *a* axis at the inclination β, hence one example in the triclinic system will serve for the two classes of crystals. The example chosen is rhodonite, and the data needed are as follows:

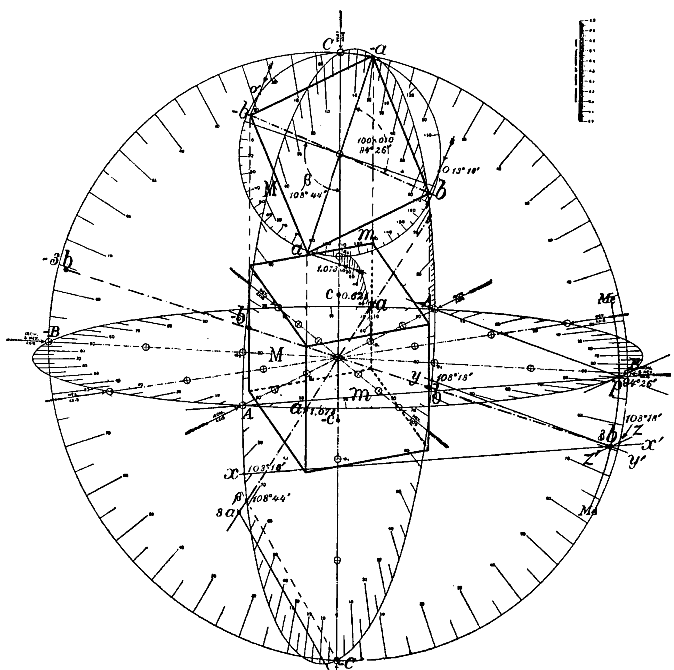
$$a : b : c = 1{\cdot}073 : 1 : 0{\cdot}621$$
$$\alpha = 103°\ 18' ;\ \beta = 108°\ 44'.$$
$$a \wedge b,\ 100 \wedge 010 = 94°\ 26'.$$

The projection of the *a* axis, which is the same for both the monoclinic and triclinic systems, will first be explained: When *a* is not at right angles to *c*, it must appear somewhat foreshortened in orthographic projection, as shown in figure 12, which represents the relations of the *a* and *c* axes of rhodonite: *XY* being the trace of the horizontal plane on which the orthographic projection is made, the *a* axis, length 1·073, will appear foreshortened to the length *Oa′*. Applying the foregoing principle to the upper circle of the protractor, figure 13, draw a radius at the inclination *β*, 108° 44′, making use of the graduation of the circle, lay off on this

12



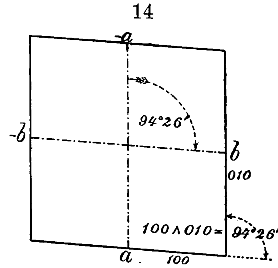
13



radius the length of the *a* axis (1·073 in figure 13) using the scale of decimal parts, and then project at right angles to the direction *a*, −*a*, as indicated by the arrow, thus determining the length of the foreshortened a axis. For the clinographic projection locate β, 108° 44′, on the graduation of the ellipse passing through A and C, draw a diameter through the center and fix the length of a by projecting down vertically from a of the orthographic axis above. If one does not wish to make use of the orthographic axes, draw the diameter of the ellipse at the inclination β, and find the length $3a$ by laying off a distance equal to $3a$ on the vertical axis (3·219 in figure 13), using the scale of decimal parts, and then transpose the length thus found to the inclined a axis by drawing a line parallel to β, $-C$, as shown in the figure: One-third of the length thus determined is the desired length of the a axis.

Two processes are involved in plotting the b axis of a triclinic crystal. (1) The vertical plane in which the b and c axes are located is revolved about the c axis so as to conform to the measurement $a \wedge b$, 100 $\wedge$ 010. Care must be taken to note the direction in which the plane of the b and c axes is turned: (1) As shown in figure 14, since 100$\wedge$010 (angle between normals) is 94° 26′ in rhodonite, the right-hand end of the b axis is first swung forward 4° 26′ in the plane of the equator. Carrying out the foregoing process in figure 13, a point p is located on the equator, 94° 26′, measured from $-A$, and likewise b of the orthographic projection above is brought forward to a position 94° 26′ from $-a$. (2) The horizontal b axis, in its new position, must next be inclined to the vertical axis at the angle α, which in rhodonite is 103° 18′. For the orthographic projection above, this inclination of the b axis causes some foreshortening, which is determined by laying off two points o and o', figure 13, 13° 18′ (103° 18′ $-$90°) on either side of where the b axis intersects the divided circle, and projecting through the points thus formed at right angles to the direction b, $-b$, as indicated by the arrows. To give the b axis of the clinographic projection its proper inclination, the value of α, 103° 18′, is laid off on two, or preferably three, of the vertical ellipses, as at x, y and z, figure 13, measured from C. Next draw three chords, Ap, $-Ap$ and Bp, on the plane of the equator, and parallel to them, respectively, the chords xx', yy' and zz'. The common intersection of the three chords determine a point $3b$, on the surface of an imaginary sphere and on a meridian Me passing through p. The point $3b$ is 13° 18′ below the equator and 103° 18′, that is α, from C. A line from $3b$ through the center is the projection of the b axis, and a perpendicular from

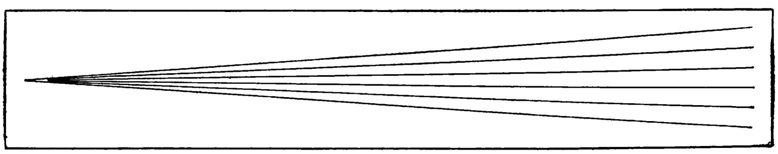
14



b of the orthographic projection above will intersect the axis at one-third of its length.

The principle involved in the projection of the clinographic *b* axis, as given above, is very simple. Imagine a sphere with two points fixed on its equator corresponding to *A* and *p* of figure 13, and then a chord *Ap* through the two points; it then follows that a series of chords parallel to *Ap* drawn through the 5°, 10°, 15°, etc., graduation points of the meridian through *A* would all emerge from the imaginary sphere on a meridian *Me*, figure 13, passing through *p*, at points 5°, 10°, 15°, etc., from the equator. By drawing two chords, *xx′* and *yy′*, as in figure 13, or a third *zz′* so as to make more certain of the intersection, any desired point on the meridian through *p* is quickly found. In figure 13 a combination of the prisms *m* (110) and *M* $(1\bar{1}0)$ and the base *c* (001) has been drawn.

15

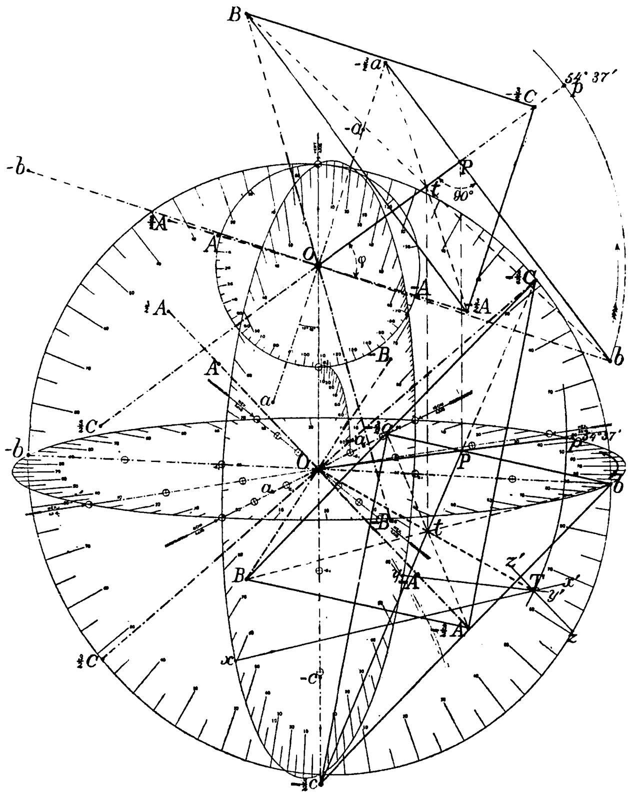


It may be said concerning the protractor that it has been plotted on a large scale to insure accuracy, and that lengths corresponding to one-third those of the axes will generally be found convenient for drawing simple crystal figures. In connection with the protractor it is recommended to use a scale, corresponding to figure 15, printed or drawn on tracing cloth or paper. When the outer lines of such a scale are adjusted between the five degree graduation marks of the ellipses, the intermediate lines will serve to subdivide the space into fifths, or degrees.

Projection of the Axes of twinned Crystals.—The axial protractor furnishes a ready means for plotting the axes of twin crystals, a problem which at times presents considerable difficulty, especially to beginners, hence two examples may be cited explaining the uses of the protractor. In staurolite, twins according to a pyramid are common, and in the example chosen it will be assumed that a face having the symbol $\bar{2}3\bar{2}$ $(-\frac{3}{2}a : b : -\frac{3}{2}c)$ is the twinning plane. The data employed in plotting the axes are the axial lengths, $a : b : c = 0{\cdot}473 : 1 : 0{\cdot}683$, and the ϕ and ρ angles of the twinning plane; $\phi = 010 \wedge \bar{2}30 = 54° \ 37'$ and $\rho = 001 \wedge \bar{2}3\bar{2} = 60° \ 31'$. To insure accuracy in plotting, the full lengths of the axes of the protractor have been regarded as unity. In figure 16 the axial lengths $-a$ and

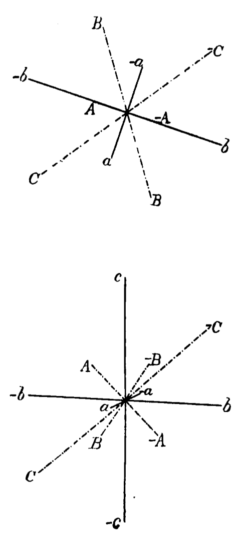
$-\frac{3}{2}a$; b ; and $-c$, and $-\frac{3}{2}c$ are laid off both on the orthographic and clinographic projections of the axes, and the twinning plane $-\frac{3}{2}a : b : -\frac{3}{2}c$ drawn. The value of ϕ, 54° 37′, is laid off at p on the equator, measuring in the direction of the arrow from b, and the radius from the center O to p makes an angle of 90° at P with the line $-\frac{3}{2}a : b$. The twinning axis, a line

16

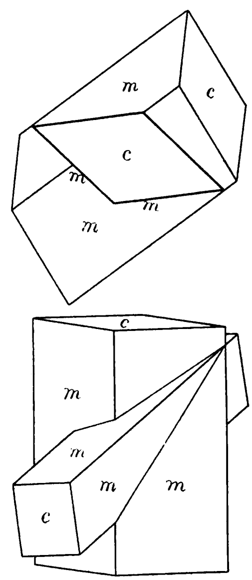


from the centre at right angles to the twinning plane, is now plotted on the clinographic axes by finding a point T, on the meridian through p, 60° 31′ (the value of ρ) from the south pole of an imaginary sphere. This is done by locating x, y and z on the graduated ellipses at 60° 31′ from the south pole, and drawing the chords xx', yy' and zz' parallel, respectively, to chords on the plane of the equator through p and the intersections of the a and b axes with the equator. The intersection of the three chords determine the desired point T at the surface of an imaginary sphere on the meridian through p, and OT is the twinning axis. The point t, where the twinning axis pierces the twinning plane, is determined by the intersection of the twinning axis OT with a line drawn from $-\frac{3}{2}c$ to P. The points p, P and t of the orthographic projection are in vertical alignment with corresponding points on the lower axes, and need no further explanation. Having found t on both the clinographic and orthographic axes, the ends of the axes, $-\frac{3}{2}a$, b and $-\frac{3}{2}c$, are shifted respectively to $-\frac{3}{2}A$, B and $-\frac{3}{2}C$, equidistant from t, as would result from a revolution of

17



18



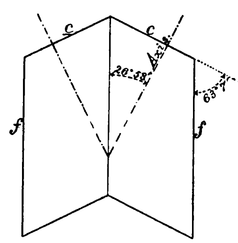
180° about the twinning axis. Lines from the centers of the two projections through $-\frac{3}{2}A$, B and $-\frac{3}{2}C$ are the axes in twin position. In figure 17 the axes are shown without construction lines, a and b being one-third as long as in figure 16, and in figure 18 two projections of interpenetrating prisms, m, terminated by basal planes, c, are shown.

A problem encountered by W. E. Ford and the writer in the study of twin crystals of calcite from Union Springs, N. Y.,* may be cited as a second example for illustrating the uses of the axial protractor in plotting the axes of twin crystals. It was desired to represent a scalenohedron, twinned about the rhombohedron f $(02\bar{2}1)$, so drawn that the

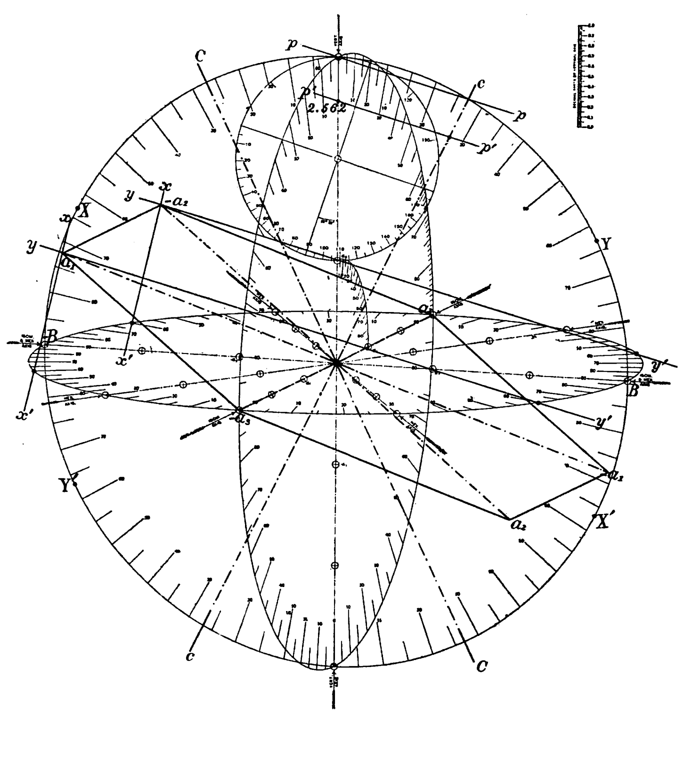
* This Journal (4), x, p. 237, 1900.

twinning plane should be vertical and have the position corresponding to that of the side pinacoid b (010) of an orthorhombic crystal. The solution depends upon the angle of base on twinning plane, $c \wedge f = 63° \ 7'$, from which the inclination of the vertical axes, 53° 46′ from one another, or 26° 53′ from the twinning plane placed in vertical position, as shown in figure 19, is derived. As indicated by figure 20, the inclinations of the vertical axes, c and C, 26° 53′, from the perpendicular, are determined by the graduation of the vertical ellipse through B. Also the intersections of the planes of the horizontal axes with the same ellipse are located at X and X', and Y and Y', 26° 53′ from B and $-B$. In order to have the twinning plane correspond with the side pinacoid 010 of the orthorhombic system, it is necessary to make one of the horizontal axes $-a_3$, a_3 of the hexagonal system correspond with the front and back or a axis of the orthorhombic system. The

19

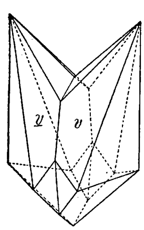


20

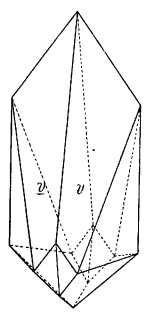


other hexagonal axes, therefore, must intersect great circles passing through $-a_3$ and X, and $-a_3$ and Y, at 60° from $-a_3$ and a_3. To find the desired intersections on the great circle at right angles to one of the twinned axes, c; through the 60° graduation points on the horizontal ellipse to the left, figure 20, draw the chords xx' parallel to a chord through $-B$ and X; likewise through the 60° points on the horizontal ellipse to the right draw the chords yy' parallel to a chord through B and X. The intersections of the chords xx' and yy' determine the extremities of the horizontal axes a_1, $-a_1$, and a_2, $-a_2$. To make the drawing somewhat more real, a hexagon at right angles to the twin axis cc has been constructed, by uniting the ends of the horizontal axes. Following a similar process (drawing chords parallel to BY and $-BY$ through the 60° graduation points of the horizontal ellipse) the extremities of the horizontal axes at right angles to the twinned axis C would be found, but it has not seemed best to complicate the figure by carrying out this construction. The length of the vertical axis of calcite is 0·854, and this is plotted on the vertical axis by laying off three times 0·854 (2·562) on the perpendicular, using the scale of decimal parts, and proportioning the length on the twinned c axis by constructing the parallel lines pp and $p'p'$, as indicated in figure 20. Figure 21 represents the scalenohedron v $\{21\bar{3}1\}$ of calcite drawn on the twinned axes, and figure 22 is a development like that observed on the crystals from Union Springs, N. Y., where the re-entrant angle is obliterated by the extension of four of the faces, resulting in a peculiar spear-head shaped development.

21

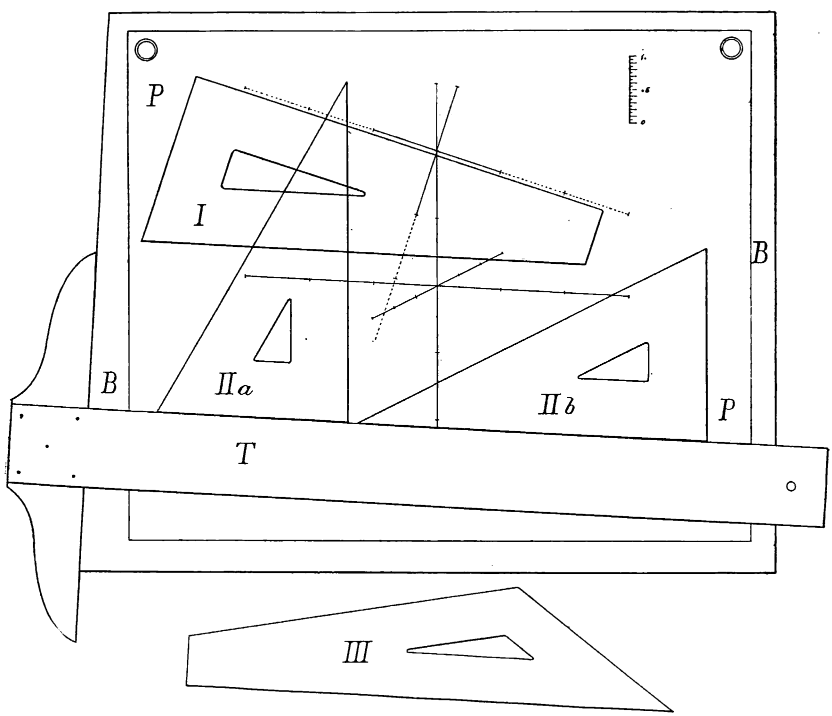


22



Use of T-square and special Triangles.—A T-square may be used to advantage in connection with the engraved axes, figures 9 and 10, the paper PP, figure 23, being adjusted on a drawing board BB so that the blade of the T-square is parallel with the right-to-left or b axis of the clinographic projection. If an ordinary rectangular drawing board is used, the paper may be fastened somewhat askew upon it, and it is not at all necessary to have a board with its right-hand edge cut at a special angle, as shown in figure 23. Special triangles have also proved to be very convenient. One of these is a truncated triangle I, figure 23, so made that when its lower edge is against the blade of the T-square its upper edge is parallel to the right-to-left of *b* axis, and its left-hand edge parallel to the front-to-back or *a* axis of the orthographic projection. A second triangle *II* is shown in two positions in figure 23; II*a*, when its shorter edge is against the blade of the T-square its right-hand edge is parallel to the vertical axis, and, II*b*, when one of its longer edges is against the blade of the T-square its upper edge is parallel to the front-to-back or *a* axis

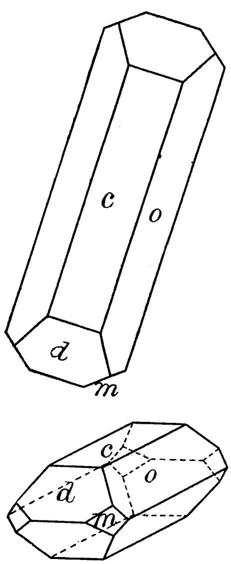
23



of the clinographic projection. A third triangle *III*, figure 23, is for the hexagonal system, and is so made that when its longer edge is against the blade of the T-square its upper left-hand edge is parallel to the a_1, $-a_1$ axis, and its upper right-hand edge parallel to the a_3, $-a_3$ axis of the clinographic projection; compare figure 10. Thus with T-square and triangles, the axial directions, the essential ones in the construction of a crystal figure, may be had almost instantly, excepting, of course, some of the directions of the monoclinic and triclinic systems.

Uses of the Linear or Quendstedt Projection.—In drawing crystals various methods may be employed for finding the direction of an edge made by the meeting of any two faces, but the principle depends generally upon locating two points, common to both faces, where they intersect certain axial planes. A line through the points thus found gives the direction of the edge. In general it will be found best to adopt some system for determining the direction of crystal edges, and to adhere to it rather strictly, and the writer has found the method based upon the linear or Quenstedt projection most useful. The projection is too well known to crystallographers to need discussion; as far as it relates to crystal drawing, however, it will be treated briefly in order to add to the completeness of the present article.

24



The principle upon which the projection is based is very simple: *Every face of a crystal (shifted if necessary, but without change of direction) is made to intersect the vertical axis at UNITY, and then its intersection with the horizontal plane, or the plane of the a and b axis is indicated by a line.* When it is desired to find the direction of an edge made by the meeting of any two faces, the lines representing the linear projection of the faces are first drawn, and the point where they intersect is noted. Thus a point common to both faces is determined, which is located in the plane of the a and b axes. A second point common to the two faces is *unity* on the vertical axis, and a line from this point to where the lines of the linear projection intersect gives the desired direction.

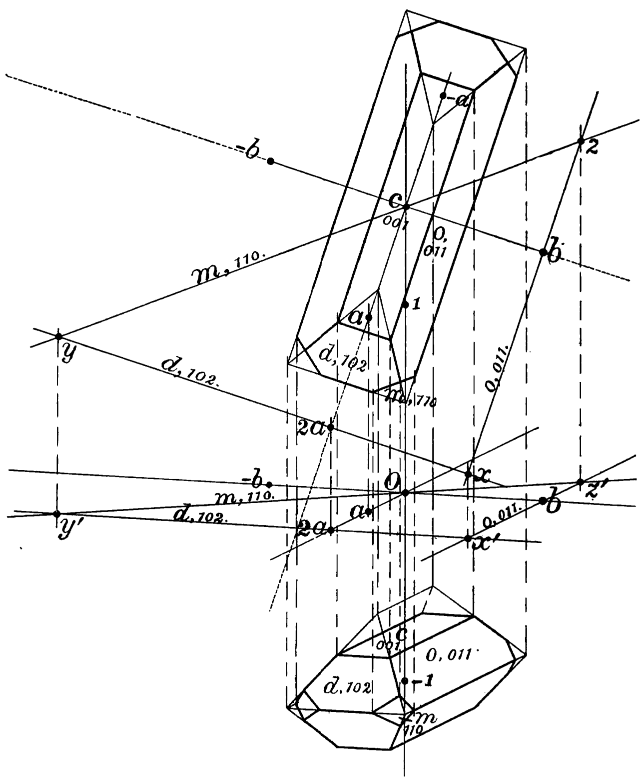
A simple illustration, chosen from the orthorhombic system, will serve to show how the linear projection may be employed in drawing. The example is a combination of barite, such as is shown in figure 24. The axial ratio of barite is as follows:

$$a : b : c = 0{\cdot}8152 : 1 : 1{\cdot}3136$$

The forms shown in the figure and the symbols are, base c (001), prism m (110), brachydome o (011) and macrodome d (102).

Figure 25 represents the details of construction of the orthographic and clinographic projections shown in figure 24. On the orthographic axes the axial lengths a and b are located, the vertical axis c being foreshortened to a point at the center. On the clinographic axes, centered at O, the ends of the axes a and b are located by dropping perpendiculars from corresponding points above, and the length of the vertical axis 1·316 is laid off above and below O by means of the scale of decimal parts, at points marked *1* and -1 in the figure. The lines of the linear projection needed for the two sets of axes are as follows: For the brachydome o, 011, the lines xz and $x'z'$, through b parallel to the a axis: For the macrodome d, 102 $(2a : \infty b : c)$, the lines xy and $x'y'$, through $2a$ parallel to the b axis: The prism m (110) is parallel to the vertical axis,

25



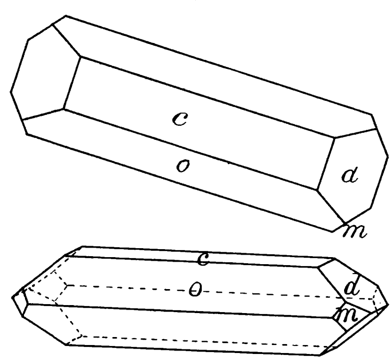
hence in order that such a plane shall satisfy the conditions of the linear projection and pass through *unity on the vertical axis*, it must be considered as shifted (without change of direction) until it passes through the center: Its linear projection therefore is represented by the lines yz and $y'z'$, parallel to the directions a to b on the two sets of axes. Since a linear projection is made on the plane of the a and b axes, the intersection of any face with the base (001) has the same direction as the line representing its linear projection. It is well to note that the intersections x, y and z and x', y' and z' are in vertical alignment with one another.

Concerning the drawing of figure 25, it is a simple matter to proportion the general outline of the barite crystal in orthographic projection. The direction of the edge between d, 102, and o, 011, is determined by finding the point x, where the lines of the linear projection of d and o intersect, and drawing the edge parallel to the direction from x to the center c. The intersection of the prism m, 110, with d and o is a straight line, parallel to the direction a to b or y to z. To construct the clinographic figure, at some convenient point beneath the axes the horizontal middle edges of the crystal may be drawn parallel to the a and b axes, their lengths and intersections being determined by carrying down perpendiculars from the orthographic projection above. The intersection between d, 102, and o, 011, is determined by finding the point x' of the linear projection and drawing the edge parallel to the direction from x' to 1 (*unity*) on the vertical axis, while the corresponding direction below is parallel to the direction x' to -1. The size of the prism m, 110, and its intersections with d and o may all be determined by carrying down perpendiculars from the orthographic projection above, but it is well to control the directions by means of the linear projection: The edges between m, 110, and d, 102; and m, 110, and o, 011, are parallel respectively to the directions y' to 1 and z' to 1. Having completed a figure, a copy free from construction lines may be had by placing the drawing over a clean sheet of paper and puncturing the intersections of all edges with a needle-point: An accurate tracing may then be made on the lower paper.

Should it happen that the linear projection made on the plane of the a and b axes gives intersections far removed from the center of the figure, a linear projection may be made on the clinographic axes either on the plane of the a and c or b and c axes, supposing that the faces pass, respectively, through *unity* on the b or the a axes.

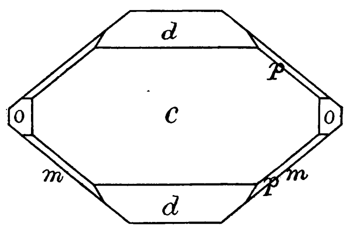
Importance of an Orthographic in connection with a Clinographic Projection.—There is no question in the writer's mind that many students, on commencing the study of crystallography, fail to derive the benefit they should from the figures given in text-books. Generally clinographic projections are given almost exclusively, with perhaps occasional basal or orthographic projections, and beginners find it hard to reconcile many of the figures with the appearance of the models and crystals which they are intended to represent. For example, given only the clinographic projection of barite, figure 24, it takes considerable training and knowledge of the projection employed to gain from the figure a correct idea of the proportions of the crystal which it actually represents. This may be shown by comparing figures 24 and 26, which represent the same crystal, drawn one with the *a*, the other with the *b* axis to the front. It is seen from figure 26 that the crystal is far longer in the direction of the *a* axis than one would imagine from inspection of only the clinographic projection of figure 24. The front or *a* axis is much foreshortened in clinographic projection, consequently by the use of only this one kind of projection there is a two-fold tendency to err; on the one hand, in drawing, one is inclined to represent those edges running parallel to the *a* axis by lines which are considerably too long, while, on the other hand, in studying figures there is a tendency to regard them as representing crystals which are too much compressed in the direction of the *a* axis. By using orthographic in connection with clinographic projections these tendencies are overcome. Having in mind the proportions of a certain crystal, or having at hand a model, it is easy to construct an orthographic projection in which the *a* and *b* axes are represented with their true proportions; then the construction of a clinographic projection of correct proportions follows as a comparatively simple matter. Without an orthographic projection it would have been a difficult task to have constructed the clinographic projection of figure 26 with the proportions of the *a* and *b* axes the same as in figure 24, while with the orthographic projection orientated as in figure 26 it was an easy matter. Then again, given a model for study, say of barite corresponding to figure 24, a student holding the model properly orientated, over or near to the orthographic projection, and looking down on it from above, sees at once the relations between the model and the figure: Prismatic angles have their true value in the drawing, and the directions and relative lengths of all of the edges appear to be the same as on the model. From an orthographic projection alone, however, one can gain no conception of the length of a crystal in the direction of the vertical axis, nor of the steepness of its terminal faces: A combination of two projections is needed, and from two figures a proper conception of the development

26

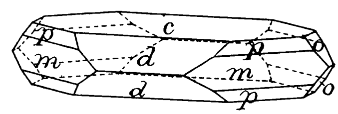


of a crystal may be had. Without question, in many and perhaps the majority of cases, figures in orthographic projection would be far more helpful to beginners, especially if studied in connection with models, than the ones so commonly used which are in clinographic projection alone. An architect in working out the details for any structure would never think of

27



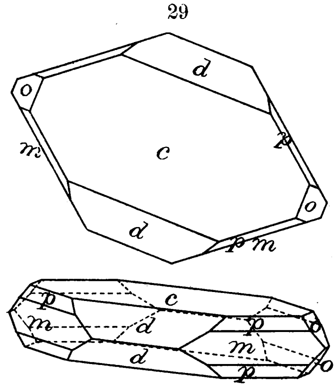
28



submitting to a builder a plan alone, or only an elevation: Two kinds of figures are considered as necessary, plans and elevations, and in like manner students of crystallography need figures drawn in two projections in order to derive the full benefit from them.

Position of Figures.—If orthographic and clinographic projections are to be used together there is some choice as to the

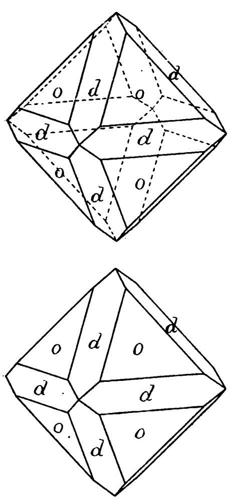
29



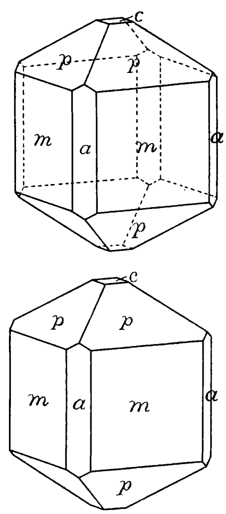
position in which the figures should be placed. Taking barite as an example: If an orthographic projection alone were employed there is no question but that the drawing should be orientated as in figure 27, with the direction of the *a* and *b* axes parallel respectively to the vertical and horizontal edges of the page. Provided two projections are used, however, if the clinographic, figure 28, is placed to one side of the orthographic, or directly below it, the apparent connection between the two figures is not at all evident: To place them thus is in violation of the principles of mechanical drawing and projection, and it is hard to realize that figures 27 and 28 are representations of the same crystal. Placed as in figure 29, however, it takes but little study to understand how the two projections are related. It is true that it may at first seem strange to see the orthographic projections skewed around at an angle of 18° 26′, but this is a condition to which one would soon become accustomed. If orthographic and clinographic projections are to be used together for purposes of illustration, it is believed that the orthographic projections should be left in the position in which they were drawn, and printed as in figure 29, although this is a matter which need not be insisted upon.

Stereoscopic Effect.—It has frequently been observed that the figures in text-books do not convey to many students the impression of solidity, and this is a defect which probably has

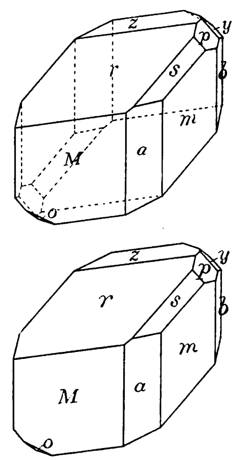
30



31

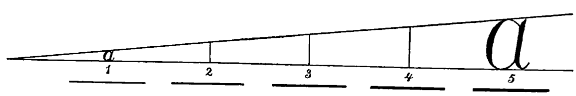


32



been generally recognized. Some have sought to overcome the difficulty by making use of two projections drawn at slightly different angles, as a crystal would appear if seen from the positions of the right and left eyes, and then viewing the two pictures with a stereoscope. The effects produced are most satisfactory, but for purposes of text-book illustration and for class-room work the method is scarcely practical. If a clinographic projection is well drawn, with the front edges represented by full lines and the back edges by somewhat lighter, dashed lines, a very satisfactory and at times quite remarkable stereoscopic effect may be had by viewing the figure through a tube. The practice is one commonly employed by artists in studying effects. The tube may be a roll of light or dark paper, either cylindrical or conical, quite variable in size (6^{cm} long by 1^{cm} diameter gives good results), while the most convenient thing to use is one's hand, doubled up so as to form a sort of tube. Stereoscopic effects are more pronounced with some figures than with others, but they would seem to depend to a large extent upon the proper proportioning of the heavier front and lighter (dashed) back lines. It is believed that the reason for the stereoscopic effect is not far to seek;—it seems to be wholly an optical illusion.—By looking through a tube the attention is concentrated on a single figure, and the heavy lines produce the effect of being near, the fainter, dashed lines of being farther away; hence the conception of solidity. In order that the stereoscopic effect may be observed by the reader, illustrations of three crystals are given for comparison, drawn with and without dashed back lines; Figure 30 is a combination of dodecahedron, *d*, and octahedron, *o*, magnetite; figure 31 is a combination of prisms of the first and second order, *m* and *a*, terminated by pyramid and base, vesuvianite; and figure 32 is a combination of triclinic forms observed on axinite. Except for stereoscopic

33



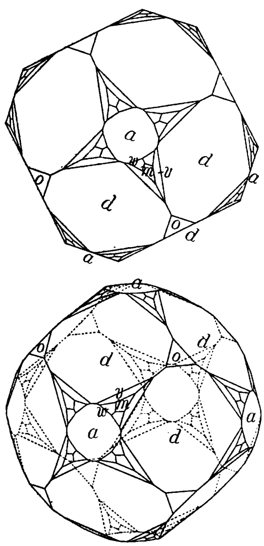
effect it may be questioned whether dashed back lines are not at times as much of a hindrance as a help in the understanding of crystal figures, because of the complexities which they introduce. As a rule they certainly add to the effectiveness of a figure, but not always; for example, in figure 18, page 50, it seemed far better to do without them.

Size of Original Drawings; Lettering.—Generally speaking, the size of an original drawing should depend to a large extent upon the complexity of the figure. It may be recommended to draw simple figures three or four times as large as needed for illustration, while with a complex subject like figure 34 it is almost impossible to make a drawing with accuracy except on a scale seven or eight times the size of the illustration. Figure 34 represents a crystal with 240 edges; hence it is evident that it is necessary to make the original drawing on a large scale in order to preserve with accuracy the directions of the many short lines.

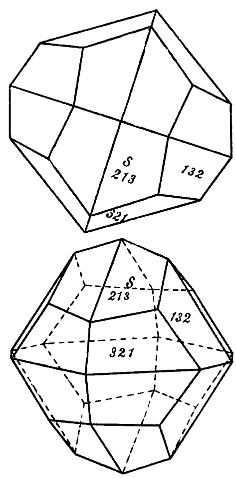
If figures are to be reduced by the photo-engraving process, they must be drawn in ink and lettered to suit the reduction. Figure 33 gives the approximate width of line and size of letter to be used with various degrees of reduction indicated by the numbers. Almost any one can succeed fairly well in forming letters who will take pains and make use of good models.

Uniformity of Lettering.—A gain has been made in recent years in adopting some uniformity in lettering, as must be appreciated by all who are accustomed to use Dana's System of Mineralogy. The scheme there adopted is in general to indicate the three pinacoids, 100, 010 and 001, by *a*, *b* and *c*, respectively, and the prism 110 by *m*. In the hexagonal and rhombohedral systems the prisms of the first and second orders are designated by *m* and *a*, respectively, in conformity with the usages of the tetragonal system, and the unit rhombohedron is designated by *r*. In the isometric system the cube, octahedron and dodecahedron are lettered *a*, *o* and *d*, respec-

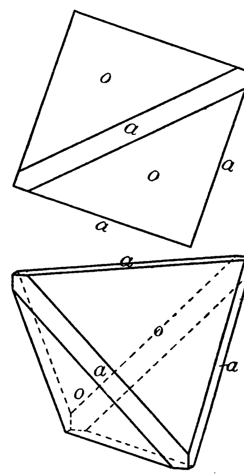
34



35



36

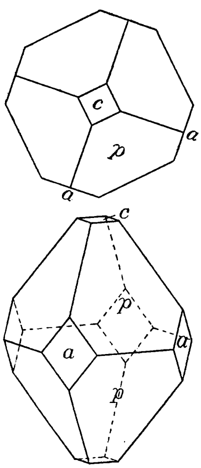


tively. The writer recommends going still one step further and designating the form 111 (isometric system excepted) always by *p*, but to carry the scheme beyond this point would be cumbersome and scarcely practicable. In using Dana's Mineralogy, or reading any article in which the scheme as outlined above is followed, a glance at the figures will generally serve to indicate the character of the forms, for however complicated a crystal may be, it is almost certain that some of the above mentioned forms will be present. It is hoped that the scheme will be more generally adopted than it is at present.

Examples.—In conclusion some figures will be given illustrating numerous advantages derived from drawing crystals in both orthographic and clinographic projection.

For the normal group of the isometric system, the forms observed on a specimen of magnetite in the Brush Collection, from Achmatowsk, Ural Mts., figure 34, has been chosen. The figure was drawn by Mr. R. G. Van Name when a student in the writer's laboratory. The combination is unusually complex, trapezohedron *m* (311) and two hexoctahedrons, *v* (531) and *w* (21 . 7 . 5), besides the simple forms *a*, *o* and *d*. A similar combination, but with somewhat different development of the forms, is described by Kokscharow.* In the construction of the complex clinographic figure, the orthographic projection proved to be a great help.

37



Both in drawing and in the study of forms of lower symmetry, orthographic projections are very helpful. Figure 35 represents the diploid *s*, (321), and figure 36 a combination of cube *a* and tetrahedron *o*. It is the writer's experience that the average student has great difficulty in gaining an idea of tetrahedral forms from figures in clinographic projection, yet a combination of cube and tetrahedron if orientated and looked at from above, in the direction of the vertical axis, will appear exactly like the orthographic projection of figure 36, hence the value of the figure.

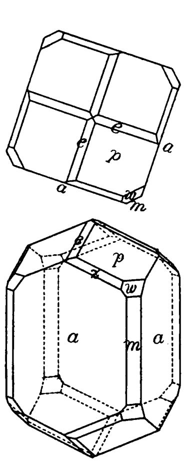
Figure 37 represents a simple combination of the tetragonal system observed on apophyllite; prism of the second order *a*, base *c*, and pyramid of the first order *p* (111). The clinographic projection alone gives a very satisfactory idea of the general proportion of the crystal, but the imagination must be drawn on to grasp the idea that the pyramid is tetragonal, a property which is brought out by a glance at the accompanying orthographic projection.

Figure 38 is a combination belonging to the tri-pyramidal group of the tetragonal system, observed on scapolite from Templeton, Canada. The forms are two prisms *a* and *m*, terminated by pyramids of the first order *p* (111) and *w* (331), of the second order *e* (101) and of the third order *z* (311). From the standpoint of a student desiring to understand the relations of the three kinds of pyramids of this group, it is believed that the orthographic is the most helpful of the two projections, although the clinographic is needed to give an idea of the general proportions of the crystal.

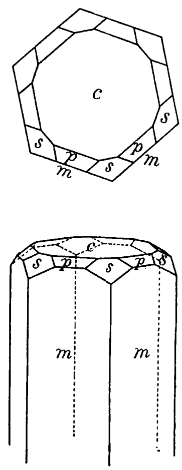
* Mineralogie Russlands, vol. iii, p. 47.

In the combination observed on beryl, *c* (0001), *m* $(10\bar{1}0)$, *p* $(10\bar{1}1)$ and *s* $(11\bar{2}1)$, figure 39, it takes considerable imagination to grasp the idea of the hexagonal shape and distribution of the pyramidal forms from the clinographic projection alone, relations which are at once brought out with distinctness by means of the accompanying orthographic projection. In the rhombohedral group of the hexagonal system clinographic projections alone are at times quite inadequate for representing the shapes of crystals. For example, given the clinographic

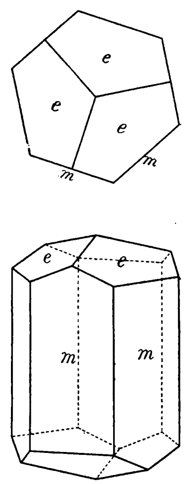
38



39



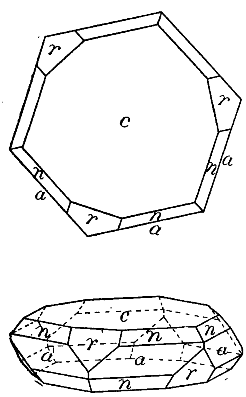
40



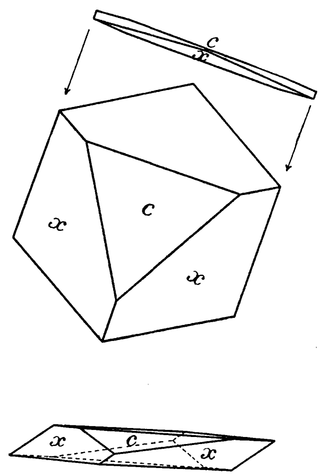
projection alone, figure 40, it may well be imagined that beginners have difficulty in understanding the simple type of calcite crystal represented, prism *m*, terminated by the flat negative rhombohedron *e* $(01\bar{1}2)$, but with the accompanying orthographic projection, the hexagonal nature of the prism and the distribution of the terminal faces about the vertical axis with trigonal symmetry is evident. The two projections, figure 41, supplement one another in giving an idea of the proportions and arrangement of the faces observed on a crystal of corundum from Cowee Creek, Macon Co., N. C. Figure 42 represents a crystal of hematite from Fowler, N. Y., showing the combination of the base *c* and a very flat rhombohedron *x* $(0{\cdot}1{\cdot}\bar{1}{\cdot}12)$. In this case the clinographic projection alone is quite inadequate, for although the figure is a correct representation in so far as the projection is concerned, it is next to impossible to gain from it a correct conception of the shape and proportions of the crystal which it is intended to represent. The orthographic projection above, accompanied by the statement that the rhombohedron is very flat, $c \wedge x = 7° 29'$, enables one to gain an idea of the shape of the crystal, while a second orthographic projection which represents the crystal when viewed edgewise, that is so that the base is foreshortened to a line, has been introduced to indicate how very thin the crystal really is.

For the orthorhombic system, illustrations have already been given of the use of orthographic projections both in drawing

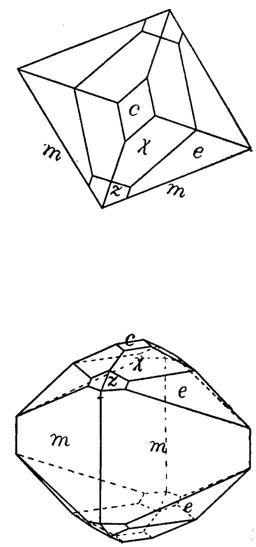
41



42



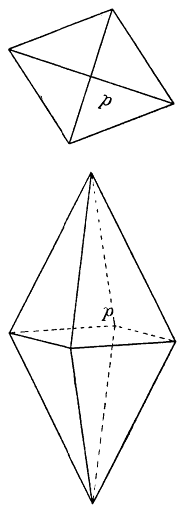
43



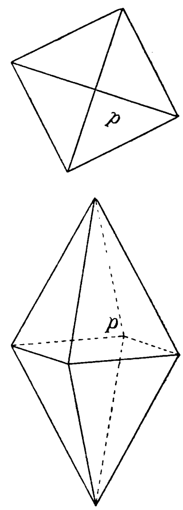
and in the understanding of the forms of barite crystals. Figure 43 is offered as an additional illustration: It represents a combination observed on brookite from Magnet Cove, Arkansas. From the clinographic projection alone it is very difficult to gain an appreciation of the proportions of the crystal; while the orthographic projection is excellent for showing the distribution of the terminal faces and zonal relations. Figures 44 and 45 represent pyramids of sulphur and octahedrite, respectively. Considering the clinographic projections alone, it takes careful inspection to discover any difference between the two figures, while the accompanying orthographic projections indicate at a glance that the pyramid is orthorhombic in the one case and tetragonal in the other.

In the monoclinic system, the clinographic projections alone, figures 46, 47 and 48, need to be supplemented by the accompanying orthographic projections in order that the real shapes of the crystals may be fully appreciated. Taking another example; it has always seemed to the writer that the clinographic projection of epidote, figure 49, was poorly adapted for showing the form of so simple a crystal. It represents a combination lengthened in the direction of the *b* axis and terminated by two faces *n* ($\bar{1}11$), one of which, however, in the position adopted, happens to be foreshortened to a line. The accompanying orthographic projection, especially if studied in connection with a model, helps to give an understanding of the shape. A clinographic projection better adapted for giving an idea of the development of the crystal is shown in figure 50.

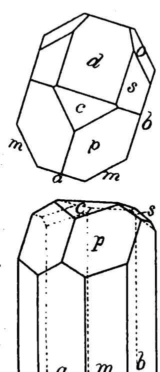
44



45

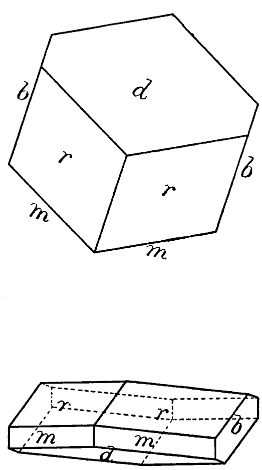


46



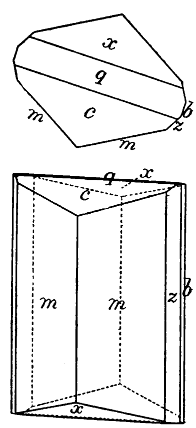
Pyroxene.

47



Tremolite.

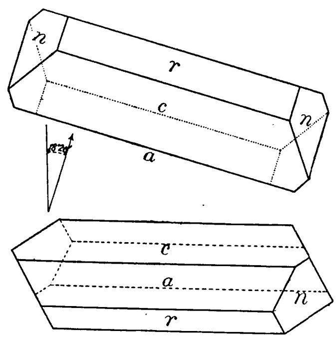
48



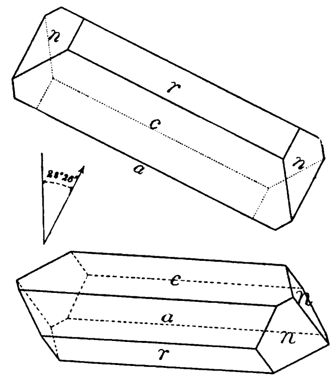
Adular.

In this case the revolution about the vertical axis is 28° 26′ instead of 18° 26′, as in the previous illustrations, and both terminal faces are thus shown in the lower figure. By means

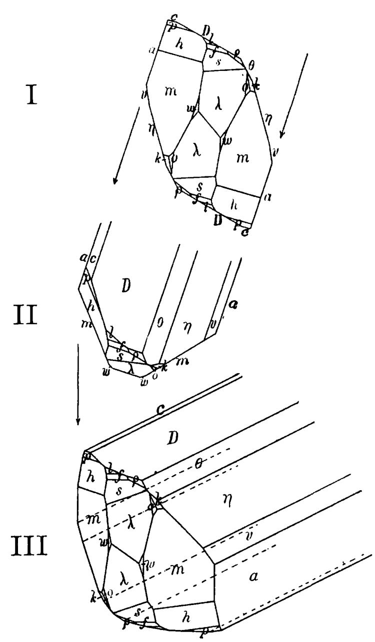
49



50



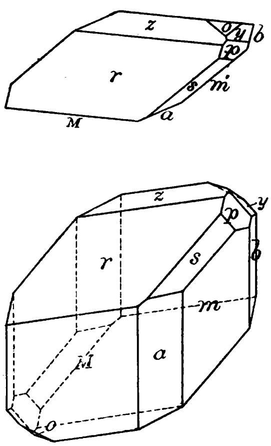
51



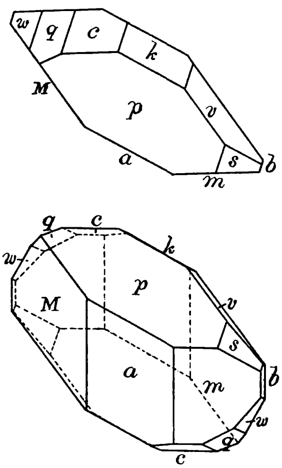
of the axial protractor, page 44, it is an easy matter to plot the axes in the position chosen.

Owing to foreshortening, without the use of an orthographic projection it often becomes a very difficult matter to construct a figure in clinographic projection in which the relative proportions of the several faces of a crystal are preserved with accuracy. A case illustrating this, encountered in the study of some very beautiful and complex crystals of azurite from Broken Hill mines, New South Wales, figure 51, may be cited. The drawings were made by Mr. R. G. Van Name when a student in the writer's laboratory. The crystals, lengthened like epidote in the direction of the *b* axis, showed only one termination, and the clinographic projection III represents the crystal turned so that the *b* axis runs from front to back. The list of forms is not given here because, if needed, it may be found in an earlier publication.* Endeavoring to preserve the true proportions of the faces, it proved to be a difficult matter to construct the orthographic projection II, as seen in the direction of the vertical axis. An end view of the crystal, an orthographic projection as seen in the direction of the *b* axis I, was therefore first drawn, a comparatively easy task, and tilting it at an angle of 18° 26′, as shown in the figure, the

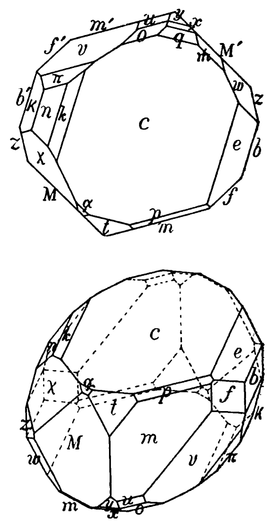
52



53



54



orthographic projection II was readily made by projecting as indicated by the arrows, and, finally, the clinographic projection III, in exactly the desired proportions was made. In studying the forms of a complex crystal, such as represented by figure 51, the orthographic projection I (end view) is doubtless more satisfactory than either of the other projections.

For the triclinic system three illustrations are offered. The clinographic projection of axinite, figure 52, is very satisfactory, but its proportions are made more real by the accompanying orthographic projection. In the examples chalcanthite and anorthite, figures 53 and 54, the clinographic projections taken alone are inadequate because of the foreshortening of several of the prominent faces, but supplemented by the accompanying orthographic projections the combinations are readily

* This Journal (4), xiv, p. 278, 1902.

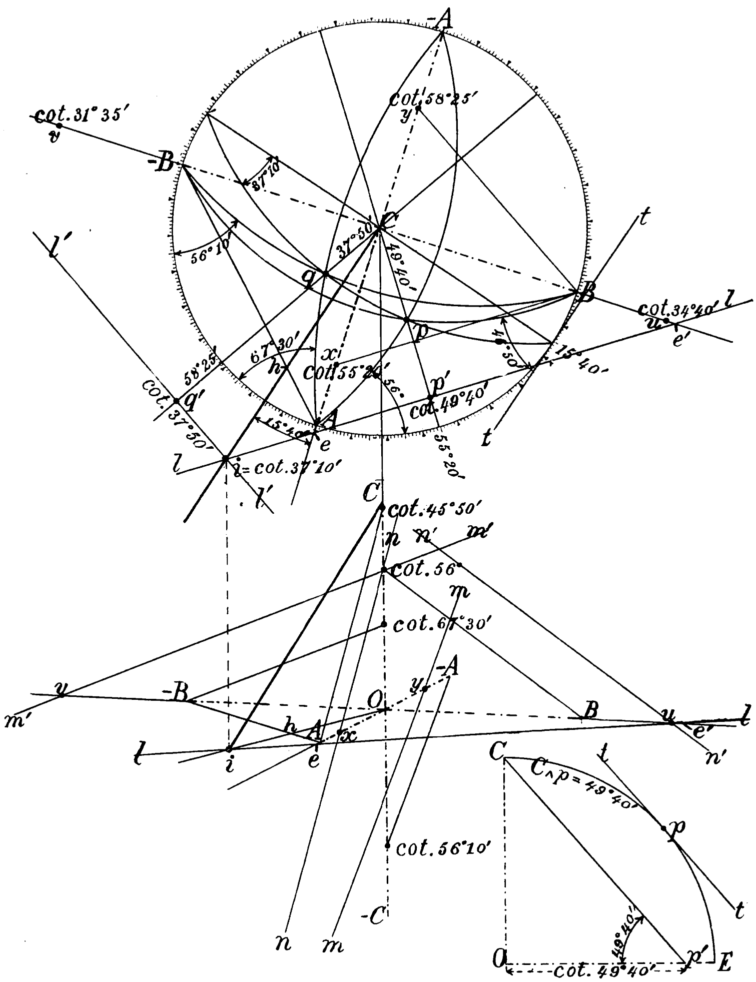
understood. The complicated figure of anorthite was drawn by Mr. J. C. Blake of the writer's laboratory.

Conclusion.—It is not the object of the present communication to make the subject of crystal drawing easy. The drawing of a complex combination requires patience, skill, and above all a knowledge of the principles of crystallography and mechanical drawing. For some persons the subject is a very easy one, while others acquire it only with difficulty, differences depending upon personal peculiarities. That correct ideas of the shapes of crystals should be obtained from figures is evident, and those who are familiar with crystallography, especially if they are not called upon to teach it, have difficulty perhaps in appreciating how hard it is for some persons to see the relations between a figure and the crystal which it represents. The clinographic projection is undoubtedly as good a one as can be found for representing the shapes of crystals, but, as has been pointed out, in many cases a figure thus drawn should be supplemented by one in orthographic projection. Orthographic projections are so simple that they may be made easily, even sketched free hand with some approximation to accuracy, and it is especially desired to emphasize their value as a help both in drawing and in the understanding of crystal figures. In the majority of cases two figures, one in orthographic and the other in clinographic projection, may be made in less time than a single figure in clinographic projection. The engraved axes, axial protractor and special triangles, having been in use for more than four years in the writer's laboratory, have proved their efficiency: by means of them increased accuracy in drawing is attained, time is saved, and, what is of no little importance, strain on the eyes is materially lessened.

Drawing from the Stereographic Projection.—A stereographic projection of the faces of a crystal, or, for that matter, of any geometrical figure with plane surfaces, furnishes all the data needed for constructing figures in both orthographic and clinographic projections. In the methods to be described use will be made of three lines or axes; one a vertical, corresponding to the north and south axis of a sphere, the others at right angles to one another in the plane of the equator. In the upper part of figure 55 the two diameters of the graduated circle, A, $-A$ and B, $-B$, represent the front-to-back and right-to-left axes in the plane of the equator, the vertical axis, C, $-C$, being foreshortened to a dot at the center. The axes have been turned as it were through an angle of 18° 26′ in order to make A, $-A$ and B, $-B$ correspond with the directions of the axes for orthographic projection of figure 1. It is supposed that in figure 55 p is the pole of some crystal face: From the graduated circle it is seen that p is on the meridian

55° 20′, measured from *B*, and the distance from *C* is easily determined as 49° 40′ by means of a stereographic scale.* If

55



we imagine a pole corresponding to *p* located on a spherical

* For a description of the stereographic scales and protractors mentioned in this and the succeeding paragraphs, the reader is referred to the earlier publications of the writer: "The Stereographic Projection and its Possibilities from a Graphical Standpoint," this Journal (4), xi, pp. 1–24 and 115–144, 1900; and "On the Solution of Problems in Crystallography by Means of Graphical Methods, based upon Spherical and Plane Trigonometry," Ibid., xiv, pp. 249–284, 1902.

surface, a plane surface tangent to the sphere at p would be parallel to the crystal face under consideration, and, if extended, it would intersect the plane of the equator on a line at right angles to a radius drawn through the intersection of the meridian of p and the equator. In the lower right-hand corner of figure 55 the arc CE is supposed to represent a portion of the meridian through p; C is the north pole of the sphere, OE the trace of the plane of the equator and tt the trace of the tangent at p: If now the tangent plane is shifted, without change of direction, until it intersects C (*unity* on the vertical axis) it will intersect the radius OE in the plane of the equator at p'. The linear projection of p is therefore found by determining the point p', where a plane parallel to the tangent at p and intersecting the vertical axis at C cuts the radius drawn through p, and then drawing the line of the linear projection, ll, at right angles to the radius. Knowing the distance C to p in degrees, the point p' where the line ll crosses the radius through p may be readily found in three ways: (1) Graphically, as shown in the lower right-hand corner of figure 55; (2) From the same figure it is evident that Op' is the cotangent of the angle $Cp'O$ or of the arc $C \wedge p$; the value of the cotangent may be found from a table of natural tangents and cotangents and laid off on the radius through p by means of a scale of decimal parts; (3) A cotangent scale may be prepared, based on the radius of the circle as *unity*, and the distance Cp' laid off directly from the graduation. The latter method is probably the best, and a scale for laying off cotangents may be easily had by a simple modification of the stereographic scale, No. 3, of the engraved sheets described by the writer.* The basis of the stereographic scale is that the distance from the center to any pole, for example, C to p, figure 55, is equal to the tangent of half the arc $C \wedge p$; hence in order to prepare a scale for laying off tangents and cotangents it is only necessary to take a stereographic scale and renumber it, making 20° of the one equal to 10° of the other. On applying such a scale to a radius of the graduated circle for laying off cotangents, 90° is located at the center (the cotangent of 90°=0), and 0° falls at infinity. The reason for using a cotangent instead of a tangent scale (when the numbering would run in the opposite direction) is that cotangents are better adapted to the ϕ and ρ angles of the two-circle goniometer. Having a second pole q, 37° 50′ from C, figure 55, its linear projection is the line $l'l'$. The two lines of the linear projection ll and $l'l'$ intersect at i, and the direction of the edge made by the intersection of p and q will be parallel to the line joining C and i.

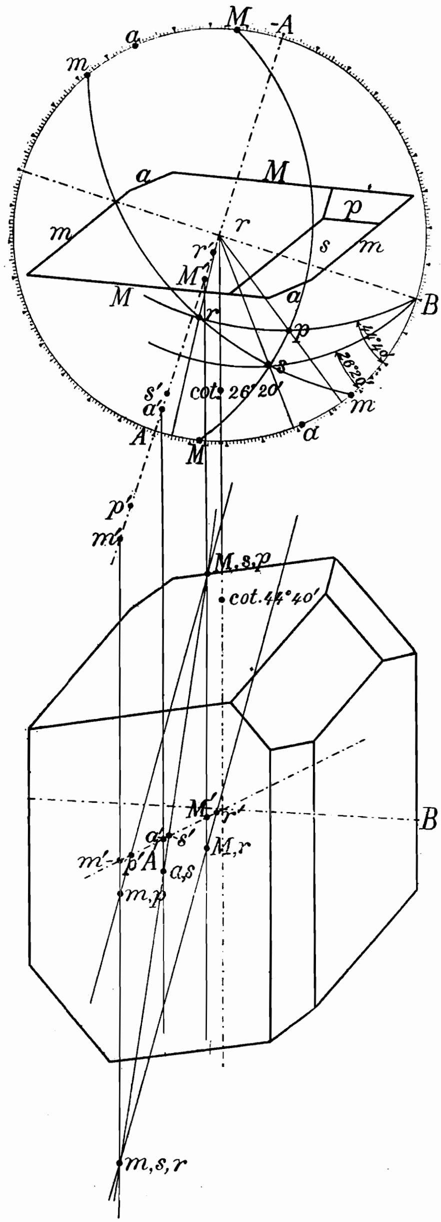
*Loc. cit.

Still another way in which the direction *Ci* may be found is as follows: Among the stereographic protractors described by the writer there was one consisting only of great circles printed on celluloid (Protractor No. *IV*). Having *p* and *q* located, the protractor is centered over the projection and turned until *p* and *q* fall on the same great circle, and then the points where the great circle intersects the divided circle (15° 40′ from *B* in figure 55), are noted, although it is not necessary to draw the great circle as in the figure. It follows from this that *p* and *q* are in a zone with a vertical plane, the pole of which is located at 15° 40′ from *B*: The intersection of such a vertical plane with the plane of the equator would be parallel to the line *tt*, tangent at 15° 40′, or, simpler, it would be parallel to a line from the center *C* to a point on the graduated circle 15° 40′ from *A*, which is identical with the direction *Ci* found by means of the linear projections of *p* and *q*. The method of the great circle protractor has one decided advantage; it is not necessary to make any construction lines; the position of the protractor alone determines the desired direction. The line *Ci* in orthographic projection may be regarded as representing two things: (1) a radius drawn on the plane of the equator, and (2) the projection of the edge between *p* and *q*, passing through unity on the *C* axis and intersecting the plane of the equator at *i*: the point *i* is an important one to determine, and may be found by noticing the angle which the great circle through *p* and *q* makes with the diameter, 37° 10′ in figure 55, and locating *i* by means of the cotangent scale.

In order to find the intersection between two planes in clinographic projection, *p* and *q*, figure 55, proceed as follows: Through *C* and a point 18° 26′ to the right of *A* on the graduated circle, draw a line, and continue it for some distance below the circle, to represent the vertical axis. As shown in figure 23, page 53, the vertical axis is next made parallel with the edge of the special triangle II*a* resting on a T-square, then, at some convenient distance *O*, the lines *B*, −*B* and *A*, −*A* are drawn with the aid of a T-square and the special triangle II*b* to represent the right-to-left and front-to-back axes. Unit lengths on the axes are determined by projecting down from *A*, −*A* and *B*, −*B* of the orthographic axes above, and a distance equal to the radius of the graduated circle is laid off above and below *O*, at *C*, and −*C*, to represent unity on the vertical axis. If the special triangle referred to is not at hand, the clinographic *A*, −*A* and *B*, −*B* axes may be constructed readily from the details given on pages 40 and 41, in connection with figures 1 to 4. If on the orthographic axes above the linear projection of *p*, that is the line *ll*, has been drawn, its intersections with the A and B axis, e and e', are noted and points corresponding to this are projected down on the clinographic axes beneath. The line ll, through e and e' on the lower axes, is the linear projection of p. The point i, the intersection of ll and $l'l'$ of the orthographic projection above, may now be transferred to the line ll of the lower axes by projecting down parallel to the vertical axis: the intersection between p and q is parallel to the line from C to i. If the point i on the upper axes has been determined by means of the cotangent scale, without the use of the linear projection, the corresponding point i on the lower axes may be found as follows: On both the upper and lower axes draw lines from A to $-B$, and on the upper axes note the point h where the lines A to $-B$ and Ci cross; on the lower axes find the corresponding point h on the line A to $-B$ by projecting down from h above, draw a line from O through h and find i by projecting down from i above.

In following out the methods just described, two conditions may be encountered which give rise to difficulties; (1) the pole of a certain crystal face may be located within a few degrees of the center of the stereographic projection, in which case the line representing its linear projection would be so far removed from the center that it is difficult to construct it, and (2), two lines of a linear projection may happen to be so nearly parallel that their intersection falls too far from the center of the figure for convenience of drawing. Such difficulties may be overcome easily by making the linear projection either on the plane of the A and C axes, supposing the faces to pass through *unity* on B; or on the plane of the B and C axes, supposing that the faces intersect *unity* on A. To illustrate how a linear projection may be made on the plane of the A and C axes:—The pole p, figure 55, is on the meridian 55° 20′ from B, and a crystal face corresponding to p would intersect the plane of the equator at right angles to a radius drawn to a point on the equator 55° 20′ from B; such a plane if shifted so as to intersect B at *unity* would intersect the A axis at the point marked x, cot. 55° 20′ (best laid off with the cotangent scale), which is projected down upon the A axis beneath. The great circle stereographic protractor is next centered over the projection, and it is found that the great circle passing through B and p makes an angle of 45° 50′ with the equator at B; hence it follows that all the possible faces in the zone Bp, if made to intersect A at unity, would intersect the vertical axis at a distance equal to the cotangent of 45° 50′ measured from the center. By means of the cotangent scale the point cot. 45° 50′ is laid off from O on the vertical axis and the linear projection of p is the line nn, drawn through x, previously determined, and parallel to the direction from A to the point cot. 45° 50′ on the vertical axis. In the case of q, figure 55, being on the meridian 58° 25′ from $-B$, a little explanation is necessary: A crystal face corresponding to q, when shifted so as to intersect $+B$, will intersect the negative ends of the A and C axes; the former at the point marked y, cot. 58° 25′, which is projected down on the A axis beneath. The great circle through $-B$ and q makes an angle of 56° 10′ with the equator, and the point cot. 56° 10′ is laid off in this case on the negative end of the C axis. The linear projection of q is then the line mm, drawn through y and parallel to the direction from $-A$ to the point cot. 56° 10′ on the vertical axis. The lines nn and mm intersect at some distance from the center of the axes, but if continued it is found that a line from $+B$ to their point of intersection is parallel to the direction Ci. If it is desired to make the linear projection

56

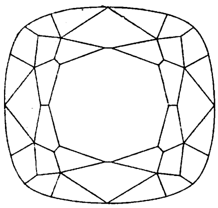


on the plane of the *B* and *C* axes, the data are as indicated in figure 55: The meridians of *p* and *q*, 34° 40′ and 31° 35′, measured from *A* (their cotangents plotted at *u* and *v*), and the angles which the great circles through *A* and *p* and *A* and *q* make with the equator, 56° and 67° 30′, respectively. The linear projection of *p* is the line *n′n′*, drawn through *u* parallel to the line from *B* to the point cot. 56° on the vertical axis; and the linear projection of *q* is the line *m′m′*, drawn through *v*, parallel to the line from $-B$ to the point cot. 67° 30′ on the vertical axis. A line drawn from *A* to the point of intersection of *n′n′* and *m′m′* is the desired direction, and is parallel to the line *Ci*.

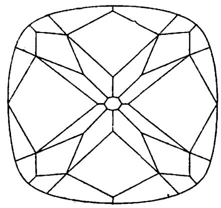
As an illustration of the application of the methods just described, the details of a drawing of a crystal of axinite may be cited. The forms present are shown in the stereographic projection, figure 56: *m* (110), *a* (100), *M* (1$\bar{1}$0), *p* ($\bar{1}$11), *r* (1$\bar{1}$1) and *s* (201). It was found that on making the linear projection on the plane of the *A* and *B* axes, several of the lines were so nearly parallel that it was difficult to determine some of the intersections. It was decided, therefore, to make the linear projection on the plane of the *A* and *C* axes as shown in the figure, the data needed being derived from the stereographic projection, as follows: Meridians of the poles, measured from *B*: *m* 32° 47′; *p* 35° 50′: *a* 48° 21′; *s* 51°; *M* 77° 16′ and *r* 85°; also the angles made by the great circles *Bpr* and *Bs* with the equator, 44° 40′ and 26° 20′, respectively. The cotangents of the meridians of the several poles are laid off on the orthographic *A* axis at *m′*, *p′*, *a′*, *s′*, *M′* and *r′*, and projected down on the clinographic *A* axis. The linear projections of the faces of the prismatic zone are vertical lines through *m′*, *a′* and *M′*; those of *p* and *r* are lines through *p′* and *r′*, parallel to the direction from *A* to the point on the vertical axis marked cot. 44° 40′; and that of *s* the line through *s′*, parallel to the direction from *A* to the point on the vertical axis marked cot. 26° 20′. All of the intersections of the figure in clinographic projection are parallel to lines drawn from *B* to points of intersection on the linear projection, indicated by the lettering. Thus the orthographic and clinographic figures of axinite were made wholly without reference to the lengths and inclinations of the triclinic axes and the symbols of the faces.

Figure 57 represents a cut stone (brilliant) as seen from above, and figure 58 as seen from below in orthographic projection, while figure 59 is a clinographic projection of the same. These drawings were made from two-circle goniometer measurements plotted in the stereographic projection. The object measured was a glass model of the Regent or Pitt diamond.

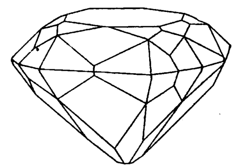
57



58



59



It is scarcely necessary to state that drawings may be made from gnomonic as well as from stereographic projections, with but slight modifications of the methods just described.

It is the writer's belief that the average student will find it easier to draw crystals from axes and the symbols of crystal faces, as set forth in the earlier part of this paper, than from the stereographic projection. Cases may arise, however, in which the latter methods may be found useful, as, for example, in finding the intersections between faces of twin crystals, or in representing some odd shapes which can not be referred to the axes of the crystal systems.

Mineralogical Laboratory of the
Sheffield Scientific School of Yale University,
New Haven, Conn., November, 1904.

NOTE.—If any desire to make use of the Engraved Axes, page 43, the Protractor for plotting Crystallographic Axes, page 44, or the Special Triangles, page 53, the writer will be glad to answer any communications and see that the necessary articles are supplied from his laboratory.