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ART. XXVII.—*On Galvanometers of High Sensibility*; by  
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AFTER considerable experience with sensitive galvanometers we have been led to the design and construction of an instrument which, while it is not more sensitive than the best hitherto made, still possesses, we think, advantages as regards convenience and ease of manipulation which make this short account worth while. We also give the results of some experiments with minute magnets which have a bearing upon the general subject of sensitive galvanometry.

The galvanometer is of the ordinary four-coil Thomson type, the materials used being brass and glass, with insulating fiber supports for the coils. The dimensions are in general smaller than is customary, as can be seen from fig. 1, which shows a simplified elevation (details omitted), and which is drawn to scale, so that from the dimensions given a general idea of the proportions of the instrument can be obtained. It should be said that the tube for the quartz fiber is supported from the base independently of the outer cylindrical glass case, but the supporting pillars are omitted from the drawing for the sake of simplicity. Figure 2 shows a plan of the working arrangement of galvanometer, control magnet support, etc., while fig. 3 is a photograph of the instrument mounted for use, showing again the adjustments for control magnets, and the method of viewing the needle-system as it hangs in position.

The two pairs of coils are supported on two sliding carriages (suggested by the instrument of Mr. Abbot's design at the Smithsonian Astrophysical Observatory), but these can be given to-and-fro rectilinear motion by screws from outside the case. The coils have adjustments about vertical and horizontal axes, and vertical and horizontal sliding adjustments, so that they can be centered and placed parallel and vertical.

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The glass tube carrying the quartz fiber passes through the top of the instrument nearly down to the top of the coils, and

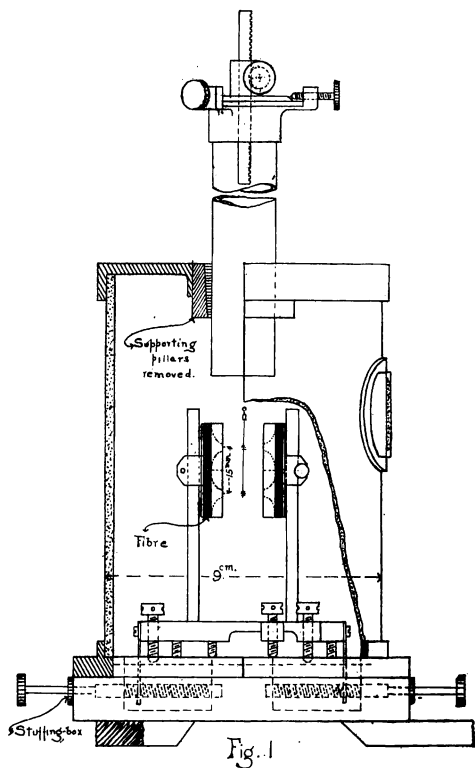


Fig. 1

carries at its upper end a rack and pinion motion for raising and lowering the fiber and system. By having the coil sup-

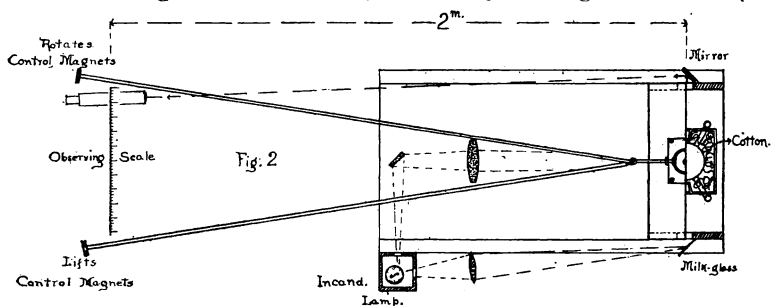


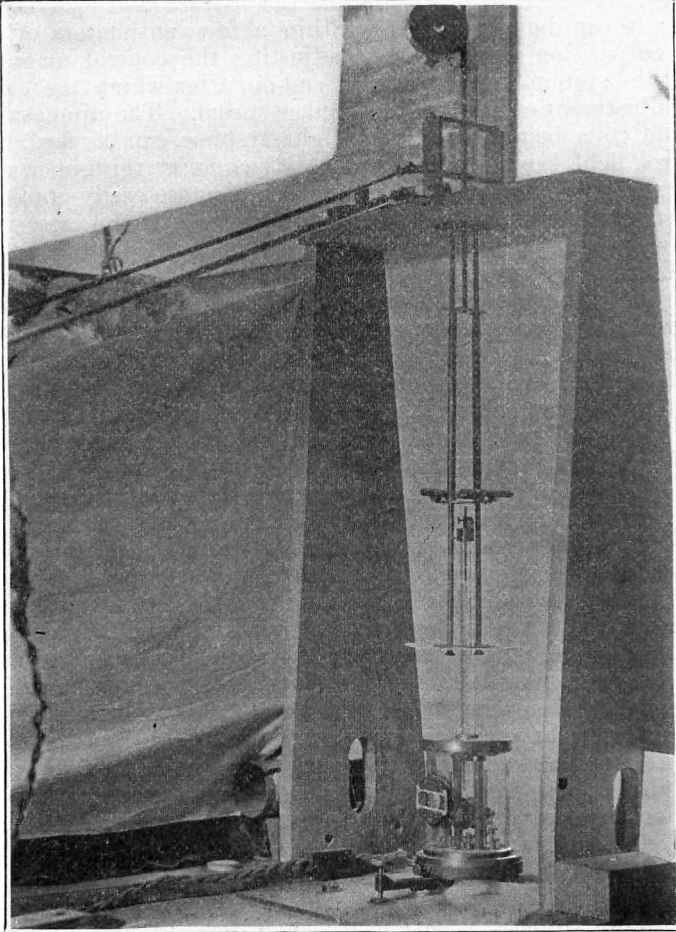
Fig. 2

ports slender and out of the way, and the outer case of glass, the space between the coils is well illuminated and easily visible.

The movable coils furnish a quick means of varying the sensibility; and besides, they render the insertion of light and

delicate systems a simple matter. The length of the quartz fiber is so chosen that the entire system can be drawn up into the glass tube; while in this position the tube can be placed in the galvanometer without danger to the fiber or system.

Fig. 3



The coils being now pulled apart, the system can be lowered into place, after which the coils can be approached as carefully as is desired. The easy visibility of the system is another important consideration; in particular we have found the arrangement of telescope, mirror and milk-glass screen, as shown in fig. 2 and fig. 3, extremely convenient, as enabling the operator at the scale to know the position of the system independent of the spot of light. The telescope is adjusted so

that the field of view includes the space between the coils, and shows the needle-system clearly defined against the illuminated milk-glass; when the system hangs in the desired position parallel to the coil-faces, the magnets appear end-on, and they are so thin that a very slight departure from this end-on position can be detected. In fact, "finding the spot of light" becomes easy; it can be adjusted to within a few centimeters of its desired position on the scale by adjusting the control magnets until the system-magnets appear end-on, after which the careful adjustment of zero can proceed as usual. The dimensions of the coils being small, very light systems can be used, the control field can be made very nearly equal at the upper and lower coils, and magnetic shielding can be more easily applied. Construction is simplified by the circular form of the case, permitting the extensive use of lathe work.

We would suggest the change of the top of the fiber tube from its present form to that given in fig. 1; the centering screws there shown would allow considerable horizontal adjustment of the fiber, and would clamp it in any desired position. A larger but shorter fiber tube—say 2<sup>cm</sup> internal diameter, a heavier base, and a method of clamping the outer case above the base, so as to permit free access to the interior, would also be better.

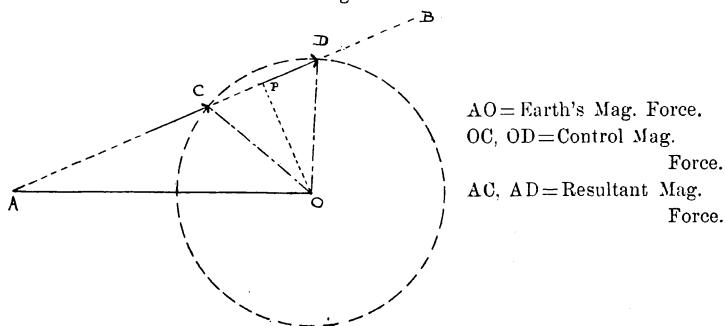
In connection with this instrument we have used a support for control magnets shown in plan in fig. 2, and in perspective in fig. 3; this permits an independent rotation and vertical motion of the symmetrical pair of control magnets, by the observer at the scale, using the long shafts shown in fig. 2. Independent changes in direction and magnitude of the control vector are thus possible.\* In our apparatus, one turn of the observer's handle rotates the control magnets through 20°, and this is sufficiently delicate for most circumstances. It is convenient to have the magnetic force due to the control magnets nearly the same at the upper and lower coils, because the behavior of the needle can then be more easily predicted; with our control magnets 35<sup>cm</sup> above the coils, there is a difference of about 15 per cent, which is ordinarily not enough to confuse matters. For example, if (fig. 4) AO represents in magnitude and direction the earth's magnetic force, AB the direction in which it is desired to have the resultant field, then for a given height of the control magnets there are two positions of the magnets, OC and OD, which give a resultant in the desired direction; one, OC, giving a long period of vibration, the other a short period. These two periods will be obtained provided the force OC due to the control magnets is greater than

\* See Waidner, chap. on galvanometers, Ames & Bliss, Laboratory Manual; also, Rogers, in Nichols, "The Galvanometer."

OP and less than OA, and one of the periods increases and the other diminishes as OC approaches AO in magnitude.

Very light systems are of course extremely sensitive to air currents, such as will be produced by very slight inequalities in temperature; it is therefore important that the entire galvanometer be protected somewhat as indicated in fig. 2. The coil faces are covered with gold-leaf—and static charges of electricity upon them have given little or no trouble; but magnetic impurity in the material of the coils is very troublesome, and necessitates a free space of at least a millimeter between the coil faces (see Nichols, "The Galvanometer"). White silk insulation is somewhat better than green, though still very bad; and a part

Fig. 4



of the difficulty is with the copper itself. The trouble seems to be in part due to a permanent magnetization of the coils, producing sometimes a rather complicated distribution of poles over the coil faces, which can be largely removed by a careful demagnetization of the coils; and in part to the poles induced by the approach of the magnets of the needle system itself—which can only be helped by using purer material.

In winding the coils the ends of all the sections were brought out, and small soldered junctions made on the back of the coil; hence six sizes of wire, viz.—40, 38, 36, 34, 32, 30, could be used without danger of wasting space in junctions. But the gain is probably hardly worth the trouble.

In making very light systems it is convenient to begin by fastening the magnets in the proper relative position, to a piece of plate glass with a solution of sugar or flour paste; after the glass staff and mirror have been fastened on, the system can be loosened with a drop of water. One of our best systems developed after use a second position of equilibrium, which it would assume if given a large deflection; it seemed to be due most probably to a change in strength of the magnets caused by the inductive action of the field, rather than to torsion of the staff of the system—as before suggested.\*

\* *Astrophys. Journal*, Jan., 1901, p. 47.

*Changes of zero, and changes in sensibility.*—In general both the zero and the sensibility of a galvanometer will be affected by the changes constantly going on in the direction and magnitude of the earth's horizontal force, and it becomes of interest to know whether there is any position of the galvanometer with respect to the magnetic meridian which will give minimum values for these disturbances. An examination of the magnetic records of the Naval Observatory shows that rather quick changes of  $H$  to the amount of nearly 1 per cent may be expected rarely, and to the amount of  $\frac{1}{8}$  per cent frequently, while the declination sometimes changes rapidly by 19' and frequently by 3'. A consideration of the vector diagram previously given, involving the earth's horizontal force  $H$ , the force due to the control magnet  $M$ , and the resultant force  $R$ , supposing the above changes to take place in the direction and magnitude of  $H$ , and assuming various directions for  $R$ , shows that there is no position of the galvanometer for which the resulting changes of zero or sensibility are markedly small. For null methods, however, in which changes of sensibility are of less moment than changes of zero, it would be somewhat advantageous to have the plane of the coils parallel to the magnetic meridian. It is interesting to note the magnitude of the changes produced by the above mentioned changes in  $H$  and the declination in the case of two identical systems, both arranged to have a complete period of  $10^s$  in the galvanometer, but one having a period of  $1^s$  in the earth's field, while the other is astaticised to a  $4^s$  period. In the first case the resultant field in the galvanometer would equal  $\frac{H}{100}$ , in the second case  $\frac{H}{6.25}$ ; and we would have, if the plane of coils is perpendicular to magnetic meridian:—

(a) non-astatic,				
change of zero due to 19' change in declination,				= 0°
“ “ “ “ “ “ “ “ H of 1%				= 90°
change of sensibility due to 19' change in declination				= 55%
“ “ “ “ “ “ “ “ H of 1%				= 50%
(b) astatic				
change of zero due to 19' change in declination				= 0°
“ “ “ “ “ “ “ “ H of 1%				= 3°40'
change of sensibility due to 19' change in declination				= 3.5%
“ “ “ “ “ “ “ “ H of 1%				= $\frac{1}{5}$ %

With the astatic system, but for the smaller changes in  $H$  and declination of  $\frac{1}{8}$  per cent and 3', the change in zero would be less than 30' (or  $18^m$  at  $1^m$  distance), and the change of sensibility about 0.6 per cent. This emphasizes the great importance of careful astaticising unless magnetic shielding is available.

*Galvanometer Magnets.*

The problem usually at hand in the construction of highly sensitive galvanometer systems is to construct a system which shall have the greatest attainable sensibility for a given period of vibration, as it is a great advantage to have a short working period, especially where magnetic disturbances cause constant variations in the zero. If we let

$m_u$  = magnetic moment of the upper group of magnets,

$m_l$  = magnetic moment of the lower group of magnets,

$M = m_u + m_l$

$K$  = moment of inertia of the system (including also the non-magnetic material),

$H$  = strength of the resultant controlling field,

$T$  = time of a complete vibration (undamped),

then we have the following relations

$$T^2 = \frac{4\pi^2 K}{(m_u - m_l)H}$$

But the deflection for a given current and set of coils.

$$d \propto \frac{(m_u + m_l)}{H(m_u - m_l)}$$

or, substituting,

$$d \propto \frac{(m_u + m_l)T^2}{4\pi^2 K}$$

And for a given period this may be written

$$d \propto \frac{M}{K}$$

Consequently the greatest sensibility is attained by building up the system of such magnets that the relation  $\frac{M}{K}$  shall be a maximum. It was with this object in view of determining the best dimensions of magnets, quality of steel, the mutual action of magnets on one another when placed near together as they are in galvanometer systems, the effect of boiling, and of mechanical jars, the magnetic moments and inductions attained in these short magnets, etc., that the experiments, recorded in the following tables, were made.

The magnetic moments of these small magnets were studied by mounting a number of them, as nearly alike as possible and whose dimensions had been carefully measured, on small brass discs whose moments of inertia would be accurately computed and were large in comparison with the moment of inertia of the magnets, which entered as a slight correction term. The

magnets were then magnetized in the uniform field of a small electromagnet and vibrated in an exhausted receiver placed at a point where the horizontal intensity of the earth's field had previously been determined. The slight controlling moment of the silk suspension was determined from the time of oscillation of the brass disc without any magnets on it, and allowed for in all the experiments. The results of these experiments are given in the following tables. These magnets were mounted on the face of the disc, side by side, so that they occupied the same relative positions that they would in a galvanometer system.

Designation of Magnet.	Quality of Steel.	Length mm.	Width mm.	Thickness mm.	Volume per Magnet ccn.	Weight per Magnet grms.	Magnetizing Field, C. G. S. Lines.	Magnetic Moment per Magnet.	Average Intensity of Magnetization.
H <sub>1</sub>	Fine Watch Spring	5.30	0.170	0.061	0.000055	0.00043	3400	0.02580	469
H <sub>2</sub>	"	4.15	0.172	0.060	0.000043	0.000337	"	0.0184	422
H <sub>3</sub>	"	3.16	0.174	0.059	0.000032	0.000251	"	0.0122	382
H <sub>4</sub>	"	1.95	0.175	0.058	0.0000197	0.000155	"	0.00485	246
H <sub>5</sub>	"	1.24	0.175	0.059	0.0000128	0.000100	"	0.00215	168
H <sub>6</sub>	"	1.02	0.173	0.059	0.0000103	0.000081	"	0.00143	139
H <sub>7</sub>	"	0.650	0.170	0.061	0.0000067	0.000053	"	0.000542	81
G <sub>1</sub>	"	3.00	0.197	0.050	0.0000295	0.000232	"	0.0110	372
G <sub>2</sub>	"	1.81	0.197	0.050	0.0000178	0.000140	"	0.00405	228
G <sub>3</sub>	"	0.98	0.210	0.050	0.0000103	0.000081	"	0.00130	127
I <sub>1</sub>	J. and C. Spec. Mag. Steel.	1.12	0.252	0.050	0.0000141	0.000111	"	0.00252	179
I <sub>2</sub>	"	1.15	0.161	0.052	0.0000096	0.000076	"	0.00193	201
B <sub>1</sub>	"	4.12	0.89	0.120	0.00044	0.00345	2200	0.1251	284
B <sub>2</sub>	"	1.53	0.79	0.126	0.000152	0.00119	5200	0.1248	283
"	"						800	0.0102	67
"	"						1300	0.0107	70.5
B <sub>3</sub>	"	1.09	0.74	0.125	0.000101	0.00079	3400	0.0107	70.5
"	"						"	0.0052	51
A <sub>1</sub>	Watch Spring	3.35	0.52	0.115	0.000200	0.00157	3400	0.0407	202
A <sub>2</sub>	"	2.29	0.51	0.115	0.000135	0.00105	"	0.0198	144
A <sub>3</sub>	"	1.67	0.52	0.115	0.000100	0.00078	"	0.0100	99
A <sub>4</sub>	"	0.80	0.50	0.115	0.000046	0.000361	"	0.0031	67
A <sub>5</sub>	"	1.53	0.254	0.114	0.000044	0.000345	"	0.0049	111
C <sub>1</sub>	Boker's Music Wire	3.13	D=	0.22	0.000119	0.00094	"	0.0211	177
C <sub>2</sub>	"	1.14	D=	0.22	0.000043	0.00034	"	0.0024	55
J <sub>1</sub>	Extra fine Steel	2.20	D=	0.078	0.0000105	0.000082	"	0.00284	270
J <sub>2</sub>	"	1.21	D=	0.069	0.0000045	0.000035	"	0.00076	168

*Saturation of Magnets.*—The experiments with magnet  $B_1$  (see preceding table), magnetized in fields of 2200 and 5200 lines, and with magnet  $B_2$ , magnetized in fields varying from 800 to 3400 lines per sq. cm., show that a limit to the intensity of magnetization attainable is soon reached, practically in a field of 1000 lines of induction per cm<sup>2</sup>. We undoubtedly have the isthmus effect coming in when these small magnets are magnetized by placing them between the faces of an electromagnet; thus in  $B_1$ , which was magnetized in a field (undisturbed) of 2200, an average intensity of magnetization of 284 is reached, which must correspond to an average remanent induction of at least 3500.

*Effect of Length and Cross Section of Magnets.*—The experiments made on the group of fine hair-spring magnets  $H_1$  to  $H_7$ , all taken from one piece tempered glass hard in water, shows the variation of the magnetic moment with the length. These results are plotted in a curve, fig. 6 (p. 260), which shows that, for short magnets, the magnetic moment decreases much more rapidly than in proportion to the length.

The effect of cross section on the intensity of magnetization is shown by comparing, e. g.,  $H_6$  and  $B_3$ , the average intensity of magnetization in  $H_6$  (although shorter) being more than 2.7 times that in  $B_3$ . Similarly  $A_5$  (width reduced by grinding), although shorter, has a considerably greater intensity of magnetization than  $A_3$ . This strongly emphasizes the fact that probably the greatest gain in sensibility at present at our disposal is in the use of a number of magnets of small cross section instead of one (or so) of large section, i. e., if you have a given mass of magnetic material (to be made up of magnets of a given length), the system will be considerably more sensitive if you use a number of magnets of small cross section. This brings up at once the question whether, when you divide up the magnetic material into a number of thin magnets, you do not in the end lose what you gained in magnetic moment by subdivision, by the fact that in galvanometer systems the magnets must be placed very near together and so tend by their mutual action to weaken one another.

*Mutual Action of Magnets.*—This question was investigated for many sizes of magnets and under various conditions; one or two experiments will, however, be sufficient to show that the effect is much smaller than is usually supposed. To study this the magnets  $B_1$  were fastened to one of the brass inertia discs, magnetized, and period determined, in an exhausted receiver, when the distance between the centers of the magnets was 3.3<sup>mm</sup>; the magnets were then pushed together until the distance between the centers of the magnets was 1.2<sup>mm</sup>, when the

period was again determined; magnets then pushed apart, etc. The results are shown in the following table:

Distance between Centers of Magnets.	T Time of Vibration of Disc.	M Magnetic Moment C. G. S. Units.
3·2 <sup>mm</sup>	2·618 <sup>secs</sup>	0·1251
1·2	2·718	·1160
3·2	2·659	·1212
1·2	2·725	·1155
3·2	2·663	·1209
1·2	2·727	·1153
3·2	2·666	·1206

These experiments show that the first near approach of the magnets produced a permanent decrease in the magnetic moment amounting to about 4 per cent (this was to be expected inasmuch as these magnets had not been aged). After that it will be seen that the magnets practically regain their former magnetic moment on separation. The temporary decrease in magnetic moment due to mutual action is less than 5 per cent.

A similar set of experiments with the shorter magnets, B<sub>3</sub>, gave a decrease of 5·5 per cent due to mutual action when the center lines of the magnets were 1<sup>mm</sup> apart. When the magnets were pushed together so that they touched their neighbors throughout their length, the magnetic moment continued to decrease for some minutes until the decrease amounted to 19 per cent, and on separating them they did not recover but were permanently weakened about 10 per cent.

*Quality of Steel.*—Under similar conditions magnets of Jonas & Colver's special magnet steel\* (probably tungsten steel) have moments about 25 per cent greater than those made from watch hair-spring, which is the next best material.

*Effect of Jars, etc.*—Magnets B<sub>1</sub>, 4<sup>mm</sup> long, dropped twenty times through 30<sup>cm</sup> on to a glass plate diminished only  $\frac{1}{5}$  per cent in magnetic moment; with magnets B<sub>3</sub>, 1<sup>mm</sup> long, the change was less than 1 per cent. Boiling immediately after magnetization reduces the moment by 10 per cent or 15 per cent, and most of the loss occurs during the first few minutes of boiling.

Paschen† has called attention to the fact that to increase the sensibility, for a given period, we must increase the total magnetic moment  $M$  and decrease the moment of inertia,  $K$ . Assuming that the magnetic moment is proportional to the volume, i. e., for a given cross section proportional to the length of the magnet, and that the inertia of the non-magnetic

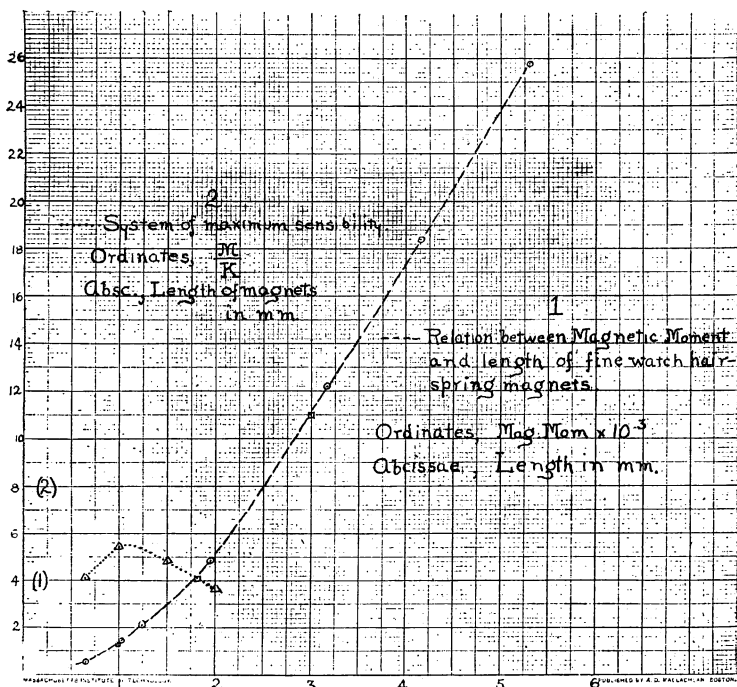
\* From Boker & Co., N. Y.

† Wied. Ann., xlviii, 282.



sensibility would continue to increase as the magnets decreased in length. This not being the case in general, it is possible to use too short magnets. To show this we have calculated the sensibilities to be expected with four systems made of the same material and cross section as magnets  $I_2$  (see table), which showed a greater intensity of magnetization than any others of the same length. Furthermore we assumed that only four such magnets were to be used in each group, to be  $.65^{\text{mm}}$  long in the first system, and  $1^{\text{mm}}$ ,  $1.5^{\text{mm}}$ ,  $2^{\text{mm}}$  in the others; that the mirror weighed  $.23$  mg., and was  $1^{\text{mm}} \times 1.1^{\text{mm}}$  in area, the staff

Fig. 5



and shellac weighed  $.09$  mg., and the hook  $.06$  mg.—these values being found by trial. Except for the omission of the hook, which is readily dispensed with, it would be difficult to reduce the non-magnetic mass much below this. The results are shown in curve 2—where ordinates are proportional to sensibility, and abscissæ to length of magnets used. A very decided maximum is shown, for a length of  $1.1^{\text{mm}}$ . It must be said, however, that the magnets could be put closer together without undue demagnetization, and that two sets could be

mounted, one on each side of the staff, thus increasing the number from eight to twenty; this would require shorter magnets for maximum sensibility.

We have not yet exactly realized this system in practice, but give the result obtained with a system having only six magnets, of the same material and cross section as  $I_3$ , and an average length of about  $1.16^{\text{mm}}$ ; the non-magnetic moment of inertia was probably a little less than that assumed in the above calculation. The sensibility actually obtained, and also the sensibility reduced to the Ayrton-Mather scale, is given in the following table, together with the sensibilities of a few other recent galvanometers, taken from the table of Ayrton and Mather (Phil. Mag., Oct., 1898).

It should be noted that with these very light systems the "period" when it is at all long—say  $5^{\text{s}}$  to  $10^{\text{s}}$  complete—is very difficult to determine, on account of the great damping. Moreover, it is unfair to compare systems which were tested at considerably different periods, and reduced to the same period by assuming the sensibility  $\propto T^2$ —for this is on the basis of a *free period*.\* In computing moments of inertia for the above systems, we have assumed that the axis of rotation passed through the center of gravity of the system and was parallel to the staff; if the system is badly constructed so that the axis of rotation departs  $\cdot 2^{\text{mm}}$  from the staff at its upper extremity, the moment of inertia will of course be increased, and the sensibility (for a given period) decreased by about 25 per cent. The moments of inertia of the four systems computed for use in plotting curve 2 varied between  $44 \times 10^{-8}$  and  $529 \times 10^{-8}$  C. G. S. units.

TABLE.

Galvanometer and Constants when tested.	Suspension System.	Sensibility and conditions of use when tested.	Deflections per micro-ampere when scale dist. = 1000 scale divs., complete period = 10 secs. Resistance = 1 ohm.
Snow (Wied. Ann. xlvii. p. 218; Phys. Rev., i, p. 37). 4 coils, 30mm. ext. and 6mm. int. diam. Each coil wound in 2 sections, with 1800 turns. $R = 140 \text{ ohms}$ (series)	6 small watch-spring magnets in each group, 3 on each side of staff. Magnets 3-4 mm. long. Mirror 5 mm. dia., 0.14mm. thick. Quartz fibre 40cm. long. Wt. of system = 80mg.	$C = 1.5 \times 10^{-11} \text{ amp. per mm.}$ Scale dist. = 300cm. $T$ (complete) = 10 secs. $R = 140 \text{ ohms.}$	470

\* See Ayrton and Mather, Phil. Mag., pp. 366, 1898.

Galvanometer and Constants when tested.	Suspension System.	Sensibility and conditions of use when tested.	Deflections per micro-ampere when scale dist. = 1000 scale divs. complete period = 10 secs. Resistance = 1 ohm.
<p>Wadsworth (Phil. Mag., xxxviii, p. 553). 4 coils, 50mm. ext. diam., 2mm. int. and 40mm. deep. Each coil wound in 5 sections, with 2396 turns. <math>R=86\text{ ohms}</math> (series)</p>	<p>10 small magnets in each group (5 on each side of staff, varying from 2-3mm. in length) made from smallest size sewing needles (untreated). Glass 150 mm. long, weighs 5mg. Mirror (concave) 2.5mm. diam. and weighs 12mg.</p>	<p><math>C=4 \times 10^{-11}\text{ amp. per mm.}</math> Scale dist. = 100cm. T (complete) = 20 secs. <math>R=86\text{ ohms.}</math></p>	675
<p>Nichols E. L. and E. F. (Phys. Rev., i, p. 2). <math>R=9.3\text{ ohms}</math> (parallel)</p>	<p>Wt. of system = 48mg.</p>	<p><math>C=1.3 \times 10^{-10}\text{ amp. per mm.}</math> Scale dist = 150cm. T (complete) = 10 secs. <math>R=9.3\text{ ohms.}</math></p>	2200
<p>Paschen Wied. Ann., xlviii, p. 284). 4 coils, 40mm. external and 5mm. internal diam. Wound with graded wire, about 1200 turns in each coil. <math>R=60\text{ ohms.}</math></p>	<p>Each group has 13 magnets 1 to 1.5 mm. long, on both sides of glass staff 0.3mm. apart; Mirror 2mm. dia. 0.03mm. thick. 5cm. quartz fibre. Wt. of system = 5mg.</p>	<p><math>C=3.3 \times 10^{-12}\text{ amp. per mm.}</math> Scale dist. = 300cm. T = 15 secs. (aperiodic, i. e., has only one turning point). <math>R=60\text{ ohms.}</math></p>	5800
<p>Mendenhall and Waidner. 4 coils, ext. diam. = 15mm., internal diam. = 2 mm. 6 sizes of wire, 500 turns on each coil. <math>R=3\text{ ohms}</math> (in parallel)</p>	<p>3 magnets in each group. Total wt. 1mg. Wt. of magnetic part of needle = .68mg., length of magnets = 1.15mm. Plane mirror, .9mm. x 1mm.</p>	<p><math>C=5.6 \times 10^{-11}\text{ amp. per mm. deflection.}</math> Scale dist. = 200cm. T = 9 sec. complete aperiodic (i. e. has only one turning point). Zero stable. <math>R=3\text{ ohms.}</math></p>	6090

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