

ART. L.—*Some of the results of the International Cloud Work for the United States*;* by Professor FRANK H. BIGELOW.

THE general scheme of the survey of the clouds proposed by the International Cloud Commission is so widely understood, that it will not be necessary to describe it again, beyond saying that the observations undertaken by the U. S. Weather Bureau began on May 1, 1896, and ended on June 30, 1897, employing one primary base station at Washington, D. C., and 14 nephoscope stations distributed quite uniformly throughout the territory east of the Rocky Mountains. The computation of the resulting data and the arrangement for the publication follow closely the prescribed forms submitted in the circulars of the commission, and while the labor of preparation up to this point was considerable, there would be nothing of special interest to say regarding that portion of the report, the whole of which will form Part VI of the Report of the Chief of Weather Bureau for 1898.

The possession of much new data, contained in the 6,600 single theodolite observations and in the 25,000 nephoscope observations, afforded, however, a favorable opportunity for considering several of the fundamental problems of Meteorology, especially in view of the fact that they develop in the most perfect manner on the North American continent, and therefore, the discussion of the observations has been pushed far beyond the limits implied in the scheme of the commission. It will be admitted, no doubt, by all those who are conversant with the true state of meteorology, that in spite of much good work on the part of able investigators, there are still serious gaps in the series of facts needed to construct a sound theory of the history of cyclones and anti-cyclones; and, furthermore, that the existing theories are neither in agreement among themselves nor with all the known facts. It was important, therefore, to develop the facts regarding the circulation of the atmosphere without bias *ab initio*; and it was essential to so far correlate the existing mathematical analyses that their true meaning as to one another and as to the results of the observations should appear. Meteorology must always remain, not a crude branch of science as some writers erroneously maintain, but a difficult one, on account of the complications attending the physical processes and the fluid motions in the complex form presented by the atmosphere. We have attempted to show how some of the apparent obstacles can be overcome by employing the methods used in these observations and reductions, and the results are such as to stimulate students to continued efforts to finally resolve these interesting problems.

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A Standard System of Constants and Formulæ.

Part of the difficulty in making students generally realize that meteorological mathematics already stands upon a definite fundamental basis is due to the fact that while many papers of great merit exist, they are detached from one another, and there is no well defined system of formulæ which is common to all such related investigations. Professor Ferrel's treatises, it is true, in spite of his inattention to a consistent and clear notation, cover the ground, as he conceived the solution of the problem in a consecutive order from beginning to end. Yet many of his primary developments are exceedingly complicated; other valuable mathematical analyses have been discovered since his day; his main theory of the local cyclone has been found to be loaded with objections, so that students have expected that before long improvements would be introduced. The German school of authors, including Guldberg and Mohn, Oberbeck, Sprung, Hann and others, have followed substantially one line of thought which is characteristic of them, and though they reach many results in agreement with Ferrel's, especially in regard to the general cyclone covering a hemisphere of the earth, they have in reality radically different conceptions regarding the structure of the local cyclone. Thus, in Ferrel's case, it was assumed that the general and the local cyclone are examples of the same type of circulation, wherein the inner and the outer region of the cyclone are separated by a region where the gyratory velocity about the central axis is reduced to zero, having a positive direction inside and a negative direction outside in the lower strata, with a complete reversal as to gradient and direction in the upper strata, the entire system embracing the same fluid material in a continuous motion. The German school, on the other hand, began with the principle of the logarithmic potential, of which a common example is found in the motion of the ether as an electric current through a wire which is surrounded by a magnetic whirl. In this case there is no reversal of the direction of the gyratory motion, but instead of being a minimum at the boundary of the inner and the outer regions, it is there a maximum. The inner region is distinguished from the outer, however, by the fact that it alone has a vertical motion. This is evidently an entirely different type of local cyclone from Ferrel's. In the case of the general cyclone the American and the German schools are in much closer accord. Furthermore, some important difficulties arose from the attempt to account for the energy expended in the local cyclone, on the theory of a vertical convection due to the buoyancy of air expanded by the liberation of latent heat in large quantities

upon the condensation of aqueous vapor into water. Also, some observations discussed by Dr. Hann seemed to show that the distribution of the temperature in the upper strata of cyclones and anticyclones is not consistent with the principles of the vertical convectional theory. Since there exists this lack of harmony as to the main theory of the motions of the atmosphere, it is no wonder that progress has been very slow in reducing meteorology to a strictly scientific basis on its theoretical side. Accompanying this confusion in the theory, the authors have seldom been fortunate enough to adopt the same notation for their mathematical discussions, so that the study of this subject has been unusually wearisome to all those who have had no strong motive for undertaking such work.

It seemed to me, therefore, desirable to construct a standard system of equations covering the entire subject, and to transpose the most important papers into that system, at least to such an extent that a student would have but little trouble in following the writings of one author and comparing the others, by means of this exposition. Several original solutions covering important ground have been introduced, with the object of bringing the formulæ into practical working forms. These include the development of the equations of motion in rectangular, cylindrical, and polar coördinates, the treatment of the humidity term in the barometric formulæ, the transformation of the thermodynamic equations in the stages represented by the α , β , γ , δ , processes in the formation of clouds, and in the treatment of the equation of continuity by which the vertical component of motion is connected with the horizontal in the case of the local cyclone. A complete new series of tables, adapted to practical work, was computed from these sets of formulæ, and applied throughout the discussion of the cloud observations.

The Weather Bureau Tables.

As a basis for the construction of the new tables, a system of the constants employed in meteorology was selected, and many of the immediate minor relations defined by suitable brief formulæ, the entire set showing numerous useful cross connections between the several parts. The primary constants are substantially those adopted by the International Committee, and they are so arranged in parallel columns for the Metric and the English systems as to be convenient for reference; the logarithms of the numbers are also given. Many minor problems in meteorology, which are often obscure in a wordy exposition, are readily explained by means of these defining formulæ, since these are more definite than any general explanation.

In preparing to discuss the physical processes which occur in the several cloud strata at heights ranging from the surface to

an elevation of at least 15,000 meters, wherein the pressure B , the temperature t , and the vapor pressure e , pass through great changes, it was found that the existing tables were wholly inadequate for the purpose. The International and the Smithsonian Barometric Tables extend only to 2,000 meters, but the new tables are computed in metric measures from 0 to 15,000 meters, and for temperatures ranging from -50° C. to $+40^{\circ}$ C. for $h = 0$ to 5,000 meters, from -50° to $+30^{\circ}$ for $h = 5,000$ to 10,000 meters, and from -60° to $+20^{\circ}$ for $h = 10,000$ to 15,000 meters; similar tables have been made in English measures up to 10,000 feet, which is sufficient for our weather map reductions. There are certain practical difficulties with the existing tables in other particulars. The formula employed by them is of the form, $B_0 - B = B(10^m - 1)$, where $B_0 > B$, and m is a function of the temperature, humidity, gravity, altitude, and surface topography. It gives the correction which, added to the pressure B at a given altitude, will reduce it to B_0 , the pressure at sea level. It is perceived that this is a very special case of reduction, namely, downward to sea level, whereas in cloud work we must be prepared to reduce upward as well as downward, and also where neither pressure is that at sea level. If by the above formulæ we wish to reduce upward, it must be done through approximations, because the value of B at the upper station is involved in the formula, and not the value of B_0 with which we begin. There is trouble with the humidity term, especially in the Smithsonian tables, where a certain average value of the vapor pressure is included permanently within the m , so that the humidity does not stand out by itself, and is, therefore, not available for an independent discussion. But in cloud work this is the very element most required, and it is not proper to assume either an invariable law of variation of the vapor contents, nor, as in the International tables, is it possible to measure the humidity term at the top and bottom of a column which is not in contact with the ground. For example, in reducing from the bottom to the top of a cumulus cloud, I have taken the following form of equation,

$$\log. B_0 = \log. B + m - \beta m - \gamma m,$$

where m includes the temperature, the altitude, and the topographic terms, β the humidity and γ the gravity. What is wanted is the value of B_0 , and not the correction $B_0 - B$, which involves one superfluous operation in computing. The humidity term with its assumed law of vertical variation, and the gravity term here stand out distinctly by themselves, and the whole subject of humidity is easily open to treatment, and even to employing a different law without disturbing the main term, which is limited to the dry air pressures. By simple

transformation the formula is available for reduction upwards; this may take place between any two fixed points whatsoever; the set of special tables to determine the heights by the barometric pressure is dispensed with entirely, since the m table is arranged for double entry with the arguments, h = height, t = temperature; with h, t, m , any two being given, the other follows. These new tables give identical results with the others for special cases; they work rapidly in practice; one can compute with accuracy to the one-thousandth of a millimeter, so far as the data are concerned.

The second important group of tables contains the four thermodynamic processes, which take place in the formation of clouds, the unsaturated, the saturated, the freezing and the frozen stages, designated as the $\alpha, \beta, \gamma, \delta$, stages, respectively. This subject has been discussed by Ferrel, Hann, Lord Kelvin and others along one line, and by Hertz and von Bezold along another line, though both came to the same conclusion so far as the results are concerned. Hertz has constructed a diagram which graphically deals with the four stages, but it was necessary for him to neglect in part the vapor contents, so that although the divergence is no more than 7^{mm} of pressure between the rigorous and the approximate solutions, yet all the fine accuracy which should pertain to good cloud computations is sacrificed. The direct application of the rigorous formulæ, which are very complex, would require an excessive amount of labor to use them, and they are never utilized by meteorologists. But it seemed to me essential to overcome this obstacle, and accordingly the formulæ were transformed so as

to depend upon three arguments, namely, $B, t, \frac{e}{B}$. pressure, temperature, and the ratio of the vapor pressure to the barometric pressure. The tables are simple in structure, and involve only moderate interpolations. They work rapidly and have proven to be perfectly satisfactory by use in the actual reductions. The results of this discussion have led to much definite information regarding the physics of clouds in many connections, but only a few of them can be mentioned here.

(1) In the case of air rising from the lower strata to form cumulus clouds there exists a definite level at which saturation takes place, namely the base of the cloud. It is necessary to clearly distinguish between true adiabatic saturation, and the saturation as it takes place in the actual atmosphere. The formulæ of the tables as they stand deal only with adiabatic processes, but in order to apply them to the atmosphere the value of the ratio $\frac{e}{B}$ must be observed at the base of the

cumulus cloud. In the adiabatic process the ratio $\frac{e}{B}$ is constant in the unsaturated stage, that is from the ground to the cloud base, and by two or three easy approximations, after starting with *B. t. e.* at the ground, we compute B_s, t_s, e_s and the height h_s of saturation. Now the question is, does this computed height h_s agree with the measured height of the cumulus base h_i ? The result of our work is to show that the observed height h_i is greater than h_s . We must, therefore, determine the values B_i, t_i, e_i at the base of the cloud accurately, and thus find the relation between the adiabatic $\frac{e}{B}$ and the actual $\frac{e_i}{B_i}$. A considerable number of kite ascensions were made in the summer of 1898 by the Weather Bureau, and more than 100 cases occurred in which the B_k, t_k, e_k, h_k were measured by the kite instruments on entering the base of the cloud. These have enabled us to study this important question carefully. It may be stated that four distinct ways have been developed of finding the temperature quite approximately at the cloud base, and hence the vapor tension and the pressure, so that for usual conditions, that is to say excepting the strongly stratified condition which occurs when currents of very different temperatures flow over one another, we can compute the pressure at the height of a mile with an error usually of ± 0.02 and always of less than ± 0.04 inch, which insures good map drawing at that height. The determination of the divergence of the actual from the adiabatic atmosphere is valuable in its application to several meteorological problems.

(2) It has been assumed that the value of the ratio $\frac{e_i}{B_i}$ obtained for the base of the cumulus cloud holds true throughout the cloud itself, and that in this space the adiabatic laws prevail. The theodolite measurements give the height of the top of the cloud where the process of saturation ends. The saturated or β stage has two cases for consideration, the first being where the top of the cloud is lower than the beginning of the freezing stage, and the second where it passes into or through that stage. We computed the $B'. t'. e'. h'$ at the top of the cloud in the first case, but the corresponding B_0, t_0, e_0, h_0 at the bottom of the freezing or γ stage in the second case. Then the thickness of the γ stage with the value of $B^\circ, t^\circ, e^\circ, h^\circ$ at the top of it followed, these being the same as $B_{11}, t_{11}, e_{11}, h_{11}$ the bottom of the frozen or δ stage. Finally with the observed h'' , the top of the cloud, $B''. t''. e''$ were computed. This gives the heights at which the several stages begin and end, and hence the thickness of each stage; thence the gradients of

B.t.e. per 100 meters in each stage were computed and tabulated. The work was so arranged as to deal with the mean normal meteorological elements prevailing in each of the 12 months, so that the annual variations in all these quantities were found. Also selected cases, as of the towering cumulonimbus clouds, some of which reach to 14,000 meters, were computed throughout. The details are so instructive that several of the computations are reproduced in full. The tops of the lofty cumulonimbus give a temperature of -30° or -40° C. in several cases, and of -59° C. in one high cloud. This method of computing the temperature at the top of lofty clouds is a welcome addition to the method of the balloon ascensions for determining the meteorological elements in the highest strata, since the clouds may be considered as accurate sounding gauges. The mean heights of the stages show that the γ stage begins at about $\frac{e}{B} = 0.0090$ and develops as a wedge-shaped space up to a thickness of about 500 meters for $\frac{e'}{B'} = 0.0300$. In this the hail forms, and especially in summer when t, e, h , have large values. I am inclined to think that the stratified appearance of hail stones is due to the fall through a series of these γ spaces alternating with warmer β stages, which may form at different heights in the congested state of the atmosphere accompanying thunder storms, rather than to any vertical orbital circulation such as Ferrel suggested. At every point of these computations the checks are so perfect that we can work accurately to 1 millimeter of pressure and to 0.1° C. temperature, when the trial approximations are repeated two or three times.

(3) It is a most interesting problem to determine just how much heat must be added to an ideal adiabatic atmosphere to produce the actual atmosphere in the several levels. Two preliminary discussions were required to develop this subject. The first was to determine the normal distribution of temperature as observed each month at all altitudes up to 16,000 meters. For this purpose all the available results of balloon ascensions were collected and discussed by tabular and graphic methods, involving a balanced network of mutually dependent lines, by which the average temperature topography was made up to that elevation. Upon the reliability of these observations and this method of treatment the accuracy of the results required must depend. The second discussion was the determination of the mean heights of the several types of clouds from the stratus to the cirrus in each month of the year. This was found by means of the theodolite observations at Washington, D. C., and from them the region covered annually by each

kind of cloud was carefully mapped out. Beginning with the mean meteorological elements *B.t.e.* at the surface for each month, purely adiabatic values were computed at the required heights; and then the actual state of the atmosphere was computed by using the temperatures derived from the balloon ascensions. Subtracting these values at the same heights, the difference is the quantity of heat required. In integrating $\int \frac{dQ}{T_m}$ I was obliged in this preliminary work to regard T_m as constant, and to take as its value the mean of the adiabatic and the observed temperatures. The formula employed is,

$$\int dQ = Q = T_m \left[\begin{array}{l} \left(.2374 + .1512 \frac{e}{B} + .0232 \frac{e^2}{B^2} \right) \log T - \left(.06858 + .02592 \frac{e}{B} \right) \log B \\ - \left(.2374 + .1512 \frac{e}{B} + .0232 \frac{e^2}{B^2} \right) \log T_0 + \left(.06858 + .02592 \frac{e}{B} \right) \log B_0 \end{array} \right]$$

the upper terms being the observed and the latter adiabatic. In computing, the dry air and the vapor terms for temperature and for pressure, four in all, were carried through separately; finally the values for each 1000-meter level were interpolated, so that we have in a table the calories required to effect the change from an adiabatic to the actual atmosphere. This is at least a fair effort to elucidate quantitatively the problem of the absorption of heat by the earth's atmosphere. Its interest and importance would justify a special campaign of operations devoted to its more careful study.

The Motions of the Atmosphere.

Besides these mathematical discussions and physical researches, a considerable portion of our labor was expended upon the determination of the stream lines and vectors of motion, which occur throughout anti-cyclonic and cyclonic regions in the United States. The complexity of this subject is so great that it is necessary to refer the reader to the charts of the Report itself for a complete presentation of the result. We had two sources of information to depend upon, namely, the long series of cloud charts which are used in the daily forecasts, but are not published, and the nephoscope observations of the International cloud year. These charts contain blue arrows, showing the direction of motion of the lower or cumulus clouds, and red arrows giving the direction of the upper or cirrus clouds. The United States was divided by me into six areas, the northern Rocky Mountain region, the Lake region, the New England districts, the southern Rocky Mountain region, the west Gulf States and the south Atlantic States, for the purpose of discussion. Then for high and low areas

respectively in each district, for winter and also for summer, a set of composite charts was constructed by placing a transparent sheet of paper over a series of the maps, selected to show the same weather type for each district, and tracing in the arrows, from which finally a set of resultant vectors for equal squares was computed by counting the number of compass point directions thus recorded. From 40 to 70 maps were used in making each chart, and the resulting vectors were reduced to an average of 40 vectors in each square. If the frequency of direction is proportional to the prevailing movement of the air, then we obtain a chart of relative motions in all parts of the high and low areas. The result is most instructive, in many respects, of which a few are mentioned. The wind and the lower cloud circulation up to the strato-cumulus type are quite the same in form, though the cloud level is rather more rounded; this movement is very independent of the upper cloud region, which is due eastward, or only a little sinuous over the highs and lows. This is true of ordinary cyclones, but in the case of hurricanes for the South Atlantic States the penetration of the lower circulation into the higher is very pronounced, showing a much deeper disturbance of the air. Cyclones are very thin, only 2 or 3 miles deep, while hurricanes are certainly 5 or 6 miles deep. The anticyclonic and cyclonic areas are hardly to be considered as centers of motion except in the very lowest strata, since currents of air blow directly across them from west to east, even in the cumulus region of the Rocky Mountain districts. It is shown that remarkably long streams of air, as from the North Pacific to the Lake region, and from the Gulf of Mexico to the Lake region, counterflow against each to form the cyclonic circulations. We cannot consider these to be due to vertical convections drawing in these distant masses of air by indraft, since the vertical component ceases at 2 or 3 miles high. Rather the great horizontal convections of the lower strata, caused by the interchange of air between the polar and the tropic zones, produce counter currents at the cyclone centers, which develop vortices discharging upward into the permanent eastward drift. The study of these normal charts of circulation will tend to correct some prevailing erroneous conceptions regarding the structure of cyclones. It will surprise many to see that a strong and warm current in the cumulus region blows directly from the Pacific Ocean eastward across a cold-wave area, showing that cold waves are thin masses of air, hardly one mile thick, produced by surface radiation on the eastern or lea side of the mountains. It is no less remarkable to find that the center of the high areas formed by the isobars drawn from reductions made by the Hazen method, now employed by the Weather Bureau, is often 500 miles distant

from that indicated by the vectors of motion. The discrepancy between gradients and wind directions in the mountain districts is already well known, but the problem acquires a special interest from the study of these new charts.

The discussion of the nephoscope observations was very laborious in consequence of the necessity of handling the large mass of figures several times. For this purpose, the area surrounding a center of motion was subdivided into 20 parts, symmetrically disposed on three circles about the center, so that the transference from rectangular to cylindrical coordinates should be simple. The right hand (anti-clockwise) rotation, with positive direction "as the arrow flies," was also adopted. Each observation was located in the proper sub-area according to its own district; each cloud type, at a given mean height, was computed separately; the northern districts were compiled by themselves and the southern by themselves; mean resultants for the vectors were found for each sub-area in 8 levels, and charts of the circulation were constructed by accurately plotting in these vectors. The result shows that a slightly sinuous eastward movement prevails over the high and low areas in the cirrus stratum, gradually deepening as the surface is approached, till in the strato-cumulus the gyrotory movement is very marked, and in the cumulus, stratus and wind levels predominant. The actual velocities diminish from 40 meters per second in the cirrus to 5 or 6 m. p. s. at the surface. Next on the theory that the sinuous motion is due to components in composition, the mean rectangular N-S and W-E components were found algebraically, subtracted from the total vectors and the residuals were combined in a secondary system of vectors, which were also transferred to charts. These are the true local gyrotory vectors as distinguished from the general motions on the hemisphere. In the cyclone they show an inward radial component from the *bottom to the top*, and nothing outward in the upper strata, as Ferrel's circulation requires. They do not show a maximum velocity at a certain distance from the center with falling off nearer it, as Oberbeck's solution demands, but they increase from the outside up to the center. The components are strongest in the strato-cumulus region and diminish above and below; they show a continuous inflow everywhere together with a strong rotation about the center, such as to cause a true vortex with discharge upwards throughout, the forced upflow being injected into the eastward drift which carries it off, while at the same time the flow is somewhat deflected anti-clockwise. In the anticyclone on the two outer circles 750 and 1250 k. m. radius, there is outflow from top to bottom on all sides; near the center there is inflow at the top, reversal at the middle, and outflow at the bottom, thus causing reversal of gradients in the

interior of the anticyclone. The entire system of high and low areas seems to be constructed by the counterflow, chiefly in the cumulus and strato-cumulus levels, of long currents due to horizontal convection, the double action on the pressure, that is the formation of high and low pressures simultaneously in adjacent districts, being referred to the general circulation of the atmosphere, especially the deflecting and centrifugal forces, rather than to local temperature accumulations. The North American Continent is the region where cyclones *form* in large numbers, and Europe-Asia the region where they *dissipate*, so that the violent general circulation over the United States in the lower strata, as compared to that of Europe, is chiefly responsible for this excess in the production, near or in the United States, of the local storms of the northern hemisphere.

A careful study of these vectors in all strata up to 11,000 meters, 7 miles high, reveals the very important fact that there is little disposition to conform to the canal theory of the circulation over the hemisphere, as ordinarily taught, namely consisting of a southward movement in the lower strata from the polar zone towards the tropics, with reversal of the component from east to west at latitude 35° , together with an overflow northward in the higher strata from the tropics towards the poles. While the general circulation conforms to this type in many features, there has always been the greatest difficulty in accounting for the comparatively slow eastward drift in the upper strata of the higher latitudes. Ferrel attributed a large part of the required retardation to the effect of friction, but this is in reality a comparatively small term. Also he stated that the difference in the eastward velocity of the northward and southward moving strata at different elevations represented the expenditure of retardational energy. As a matter of fact the lower strata do not move southward *as a whole*, and our observations do not indicate that the higher strata are vigorously moving northward, because that component is very small. What takes place is this: in each stratum from the surface to the cirrus level about as much air moves north as south, for there are enormous counter currents *passing by each other at the same level, and not over one another at different elevations*. This puts a new aspect on the entire problem of the general circulation. It looks as if the solar radiant energy was absorbed chiefly in the lower strata, and that instead of going the rounds, overflowing above from the tropics, there is developed a continuous leakage in the lower strata, which is observed as our persistent winds from the south. These meet the north winds which flow in obedience to the general circulation, as figured by the form of the land and ocean areas. This escape from the tropical belt diminishes the pressure in low latitudes,

which would require to be balanced by an excessively rapid eastward drift. Furthermore, the formation of cyclonic vortices discharging into the eastward drift and distorting it, also retards the eastward velocity. It is along these lines that a more probable explanation of the existing moderate eastward motion may be found than in the Ferrel theory, which has been widely accepted by students.

There is a chapter treating of the barometric diurnal wave and its relation to the magnetic diurnal vectors, as developed in Bulletin No. 21, 1898, together with a comparison of the diurnal components of the motion of the atmosphere locally, which shows some interesting relations. I have been unable, in the time at my disposal, to utilize the new general tables of motion in connection with the vectors just described. Something has been done in the way of a theory of the local cyclone and the tornado, which is promising, though its completion must be postponed to a future day. I have been most efficiently assisted in this work by the faithful labor of Messrs. H. H. Kimball, H. L. Heiskell and R. H. Dean, who have taken great interest in the observations and the computations. The Chief of the Weather Bureau has always placed at our disposal all the resources of the office, and the other officials have uniformly rendered all the aid in their power.