AN IMPROVED TORSION GRAVITY METER.

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ABSTRACT.

In the improved instrument, recently built at the Geophysical Laboratory, the torsion element is the same as that used heretofore, but its other parts have been rearranged to provide simpler control for operation in the field. The present apparatus is lighter than its predecessor and gives promise of even better field performance. The last instrument, during one period of eight months' service in 1937 in occupying and reoccupying old and new gravity stations, yielded results that, for any given station, agreed within one milligal. The new instrument is mounted inside a covered, insulated motor truck. Under ordinary conditions twenty stations or more per day can be occupied. The torsion element in this balance is under load only during the time required to make a gravity measurement (either 4 or 6 minutes); otherwise it is at rest and the small residual strains are dissipated. This procedure maintains the characteristics of the spring remarkably constant.

Experiments and tests with torsion gravity meters have been made at the Geophysical Laboratory for a number of years. In 1930 a gravity balance was constructed that has proved quite successful in field use. For one period of eight months in 1937 during which many stations were occupied repeatedly, the values obtained for any given station agreed within one part in a million ($\pm$ 1 milligal). Laboratory study has yielded much information on the elastic behavior of various materials and on the factors that affect the accuracy of the readings of the instrument.

In the light of these experiences an improved instrument, based on the type of elastic system originally adopted, has recently been built in the Laboratory and been found to be better adapted to field use than its predecessors; it weighs less, is more effectively insulated, and is of sturdier construction.

Its torsion element consists of a helical spring of fine, well annealed tungsten wire; it is mounted at its two ends to a horizontal rotating frame and tapers from the ends to the center at which a cross-boom is attached with a small mass of platinum at its outer end. The measurement is made by rotating the frame first in a clockwise direction, thereby winding the spring, which in turn raises the cross-boom against gravity to a horizontal position. The greater the weight or gravitational attraction on the platinum mass at the end of the boom the larger the number of turns of the frame required to raise the boom to its horizontal position, at which the reading on the graduated circle attached to the frame is recorded. The frame is then rotated counterwise, the boom is lowered gradually, passes through its original hanging position of rest, and is then slowly raised to the horizontal position on the left side. The angular rotation of the supporting frame required for the boom to pass from the first horizontal position to the second is the measure of the relative value of gravity at the station as compared with that of another station previously occupied.

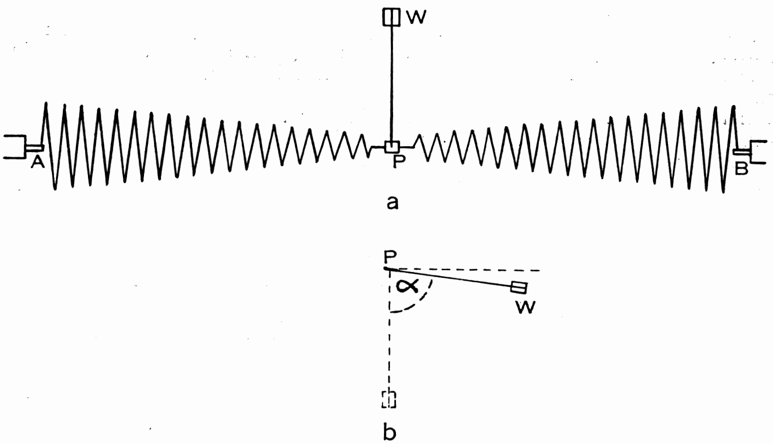

Fig. 1. (a) Schematic plan of helix spring and cross-boom, *PW*. (b) Diagram of vertical section through boom *PW*, which includes an angle α with the vertical. The horizontal direction is indicated by the dotted line.

The positions of horizontality are chosen because they are near the upsetting positions of the boom (3° or less) where a small angular rotation of the frame produces a large movement of the boom and where the vibration of the elastic system is non-harmonic. The elastic system as a whole is astatic and self-damping to such a degree that all elastic vibrations are quickly eliminated. The boom near its upsetting position and the tapering form of the helical springs are features that decrease resonance of the oscillating boom to disturbances of any frequency.

The forces acting on the boom near the horizontal position are easily computed. In Fig. 1a the spring with cross-boom is shown schematically in plan; its ends are attached to the supporting frame at *A* and *B*; the cross-boom is attached at the center P and carries a small weight W at its outer end. Figure 1b is a vertical section showing the boom near the horizontal position. W is the center of gravity of the arm PW of length r; the arm includes the angle, α, with the vertical.

Let K be the moment necessary to rotate (including bending) unit length (1 cm.) of the torsion wire through unit angle (one radian); and l, the length of the torsion wire on each side of the boom. The total torsional moment is then $\frac{2K}{l}(\theta - \alpha)$ where θ is the angle (in radians) of rotation of the frame supporting the spring.

The moment due to gravity pull on the boom is $-mgr \sin \alpha$. At the position of rest the moment due to gravity is exactly counterbalanced by the moment of torsion; therefore

$$mgr \sin \alpha = \frac{2K}{l}(\theta - \alpha) \qquad (1)$$

or

$$mg \sin \alpha = C(\theta - \alpha) \qquad (2)$$

wherein

$$\frac{2K}{r \cdot l} = C$$

From this equation the behavior of the system can be ascertained for different positions of the arm (α), for different values of the mass (m) and of gravity (g) and for different angles of rotation (θ) of the frame. Thus for $\alpha = \frac{\pi}{2}$ equation (2) reads $mg = C\left(\theta - \frac{\pi}{2}\right)$, a linear relation between g and θ.

If the weight, $m\,g$, be held constant, we obtain on writing $\frac{mg}{C} = C'$ the equations

$$\theta = C' \sin \alpha + \alpha \qquad (3)$$

$$\frac{d\theta}{d\alpha} = C' \cos \alpha + 1 \qquad (4)$$

$$\frac{d^2\theta}{d\alpha^2} = -C' \sin \alpha \qquad (5)$$

In equation (3) we have for $\alpha = 0$, $\theta = 0$; for $\alpha = \frac{\pi}{2}$, $\theta = \frac{\pi}{2} + C'$ and $\frac{d\theta}{d\alpha} = 1$ (equation 4). For $\alpha > \frac{\pi}{2}$, $\cos \alpha$ is negative and the value of $\frac{d\theta}{d\alpha}$ decreases and vanishes for $C' \cos \alpha + 1 = 0$ or $C' = - \frac{1}{\cos \alpha}$. At this point the system is unstable and an infinitesimal increase in θ produces a very large change in α. For this upsetting point we have from equation (3)

$$\theta = - \tan \alpha + \alpha \qquad (6)$$

an equation that can be satisfied for any value of θ greater than π; thus for $\alpha = \pi$, $\theta = \pi$; for $\alpha = \frac{\pi}{2}$, $\theta = \infty$. The important feature to be noted is that by use of an elastic system such that θ is large, the upsetting point is near the horizontal position of the boom and a high degree of sensitivity is obtained. In the gravity measuring instruments built in the Geophysical Laboratory the graduated circles read with vernier to 10″ of arc; in order that the instrument read directly to one part in a million the angle θ should be at least $10'' \times 10^6 = 2777.78°$; therefore an angular rotation of approximately 1400° of the frame is required to raise the boom from its vertical position of rest to the horizontal position. In this case the boom upsets at $\alpha = 92° \; 32'$, or roughly 2½° above its horizontal position.

To insure an accuracy of one part in a million or better, certain precautions should be taken. The elastic system should be under load only during either 4 or 6 minutes required to make the necessary measurements at a station. By introducing stresses into the system first in one direction and then in the opposite direction the successive strains thereby set up are in opposite directions and tend to counteract each other, so that at the end of the short exposure only small residual strains remain. Between stations the helical spring is held at rest, supported by suitable pans with the boom at rest in the hanging vertical position and clamped so that it does not move. During the periods of rest the small residual strains are dissipated and the spring returns to its original unstrained state. There is good evidence that elastic after-effects or hysteresis are largely of the nature of surface phenomena, and that both the outer surface and the surfaces between individual crystals in a crystalline aggregate are involved.

To obtain reproducible results it is necessary that at each station a definite time schedule of operations be strictly followed in order that the elastic and other properties of the spring be kept constant. Elastic hysteresis sets in immediately after a load is applied to the spring. Hooke's law is valid only to approximately one part in 10,000. The elastic hysteresis effect is roughly exponential in character and enters into any elastic system under load and increases with the load. Therefore in an elastic system intended to be used for the measurement of gravity to one part in a million or better, it is wise for the observer to introduce as small loads as possible, and to recognize that the elastic response is imperfect and that the deformation is dependent on the rate at which the spring is loaded and unloaded; also to maintain the elastic system free from load except during the short measuring periods.

The temperature of the spring should be held constant to 0.02° C. and all measurements should be made at the same temperature; otherwise the elastic response changes and introduces an uncertainty into the result that cannot be completely controlled by use of a correction factor especially if the elastic system is under continuous load. It has long been known that the deformation response of an elastic system to an applied load of fixed amount depends on the previous treatment of the system. For this reason a disturbing factor is avoided by maintenance of constant temperature. In the present instrument this is attained either by surrounding the instrument with ice or by use of an electrically controlled thermostat.

The air pressure inside the instrument should be held constant and preferably at 1 or 2 mm. of mercury. A change in pressure of 4 to 5 millimeters air pressure decreases the gravity reading by one milligal.

The air in the instrument should be kept dry by use of a suitable drying agent.

The instrument should be equipped with accurate levels, reading to 20″ of arc, to enable the observer to level the instrument accurately in both lengthwise and crosswise directions.

For convenience and speed of operation the instrument may well be transported in a motor truck with insulated walls and so carried that at a gravity station it can easily be transferred to a heavy tripod resting on the ground and reaching into the truck through holes in the floor. With this arrangement the apparatus is not exposed to rapid changes in outside temperature and to inclement weather. In actual transport the gravity meter rests on an inflated inner tube of an automobile tire to reduce the jolts and jars of transportation.

The shell of the new gravity balance is made of an aluminum alloy of high mechanical strength; it is shown in vertical section in Fig. 2. Its walls are sufficiently thick to permit without deformation a reduced inner air pressure of 1 mm. of mercury. It comprises a forward tubular section with an outwardly

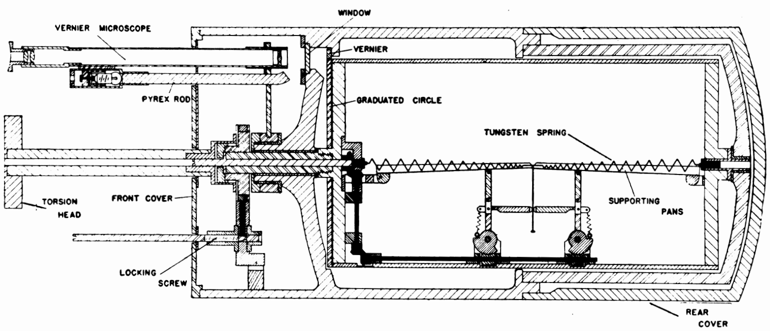

Fig. 2. Vertical section through gravity balance.

domed transverse wall, and a similarly domed rear cap that fits tightly into it. To the same end of the section a yoke is secured, which is axially bored to receive the rear axis of the rotating frame that supports the helix. The front axis of this frame rests in the axial bore of the transverse wall.

To the front end of the rotating frame a graduated circle is attached; the vernier is secured to the housing. The circle and vernier are mounted inside the shell in order to obtain a more satisfactory support for the circle than has heretofore been possible, and to insure better thermal insulation. The graduations on the circle and vernier are viewed through a low-power microscope mounted outside the vacuum chamber on a rotatable arm so that any part of the vernier can be brought into the field. The circle is illuminated by a small electric bulb whose rays are transmitted through a rod of Pyrex glass or fused silica and a small plate glass seal of arcuate shape.

The rotating frame is a sturdy casting to insure absence of flexure. Attached to the inner surface of one side of this frame is the gear mechanism for raising the pans that support the helix when at rest; also thin S-shaped springs of flat phosphor bronze for holding the cross-boom of the spring against movement. The gear system is used in place of the flexible wire heretofore employed; the gears are operated from the outside by a rod that passes through the forward axle of the frame, as indicated in Fig. 2. The device for clamping the fine adjustment tangent screw is also shown in Fig. 2; the tapered end of the locking screw bears against the beveled end surface of a plunger and forces it forward to press against the friction plate.

The forward end of the housing is capped by a snugly fitting cover to prevent moisture from entering the enclosed, but not evacuated space. In this space several parts are mounted as shown in Fig. 3, which is a photograph of the forward end of the apparatus with the front cover removed.

The two microscopes for observing the end of the boom are seen in Fig. 3 on opposite sides of the cylindrical housing. The optical system of each microscope comprises two parts; an inner part consisting of a right angle prism (Fig. 4) and an objective lens which forms an image of a fine glass filament, attached to the end of the boom, on a reticle etched on the plane surface of a lens mounted 2 mm. in front of the plate glass window which serves as seal for the vacuum; these elements are mounted rigidly on recesses in the housing. The outer part is mounted in a tube (Fig. 3), and consists of an objective lens which images both the reticle and the image of the glass filament in the focal plane of the eyepiece. This disposition of the optical system was adopted to insure absence of movement of its parts, especially of the inner portion.

The glass filament mentioned above is one mil. (0.025 mm.) thick and is attached to a short piece of wire at the end of the cross-boom (Fig. 4). The filament extends parallel with the axis of rotation of the helix and its supporting frame; it is illuminated by a small electric bulb placed in front of the eyepiece of the microscope on the side opposite to that of the boom; the rays from the bulb pass through the microscope system and across the cylinder space to the glass filament which has the appearance, in transmitted light, of a narrow bright band between two black bars. The diameter of the filament is such that the light band is but slightly wider than the cross hair of the reticle, thus enabling the observer to make accurate settings. When the boom is on the opposite side the situation is reversed. A second glass filament, also attached to the boom and parallel with the first, is then viewed under conditions analogous to those of the first. The two glass filaments are used to offset

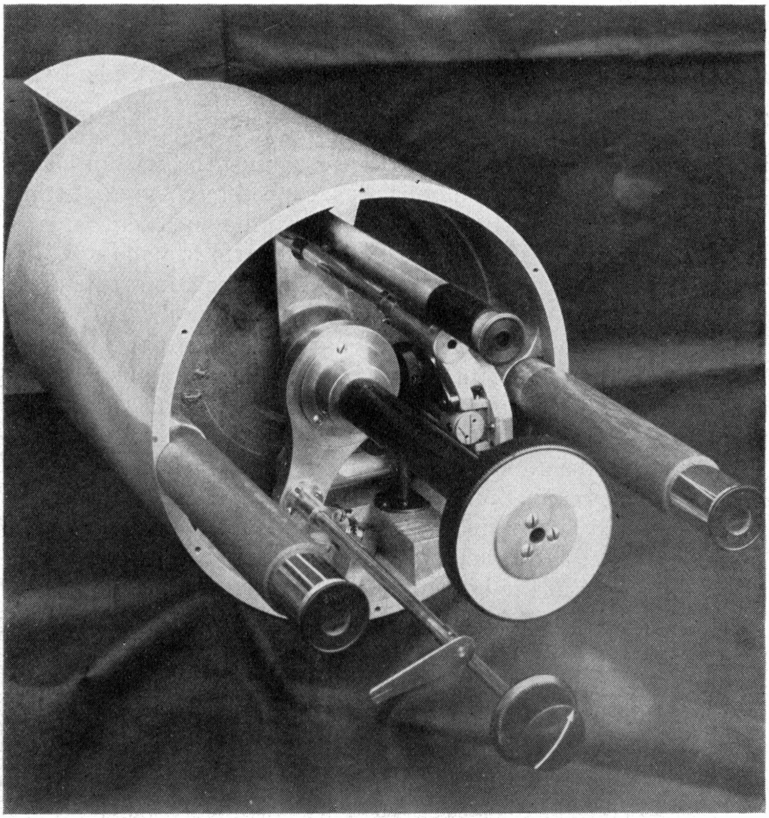

Fig. 3. Photograph of forward end of gravity balance. Front cover has been removed.

the sag in the helix, to provide satisfactory signals, and to facilitate the adjustment of the instrument. Experience with various devices has proved that this method is the most satisfactory thus far tested. It was first used on the gravity balance of 1930.

The temperature inside the vacuum chamber of the housing is measured at each station by two special mercurial thermometers reading to 0.01° C. and mounted on the inner surface of the rotating frame. Each thermometer is illuminated by the method used for the cross-boom signals; the thermometer scale is brought into focus by means of a negative lens set in the rotating frame on the side opposite to that of the thermometer. The effect of this lens when placed in front of the objective of the

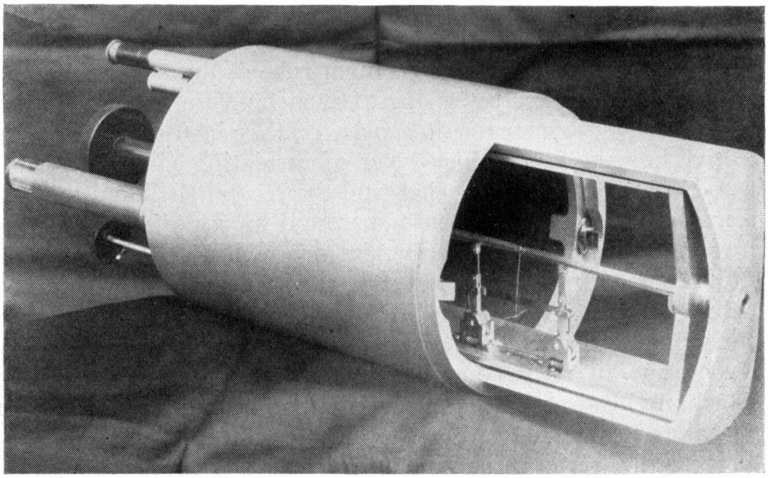

Fig. 4. Photograph of rear end of gravity balance with rear cover removed.

observing microscope is to increase its focal length so that the thermometer is imaged on the reticle rather than the glass filament of the helix boom which is held at rest during these readings by the supporting pans and clamps.

The reduced air pressure in the inner chamber of the housing is measured by means of a small mercury manometer mounted on the front panel of the apparatus. The air is kept dry by use of calcium chloride.

The instrument is leveled by means of screws in the metal table of the tripod on which the instrument is placed at each gravity station. The levels are "Zeiss" theodolite levels and read directly to 20″ of arc. They are mounted in the forward space of the housing (Fig. 3). They are viewed through a low-power lens so mounted that both levels, showing the longitudinal and transverse tilts respectively, are in the field of view.

They are illuminated by a small electric bulb whose rays are transmitted to them through a mounted Pyrex rod.

The protective housing of the apparatus is built of a series of concentric cylinders of stainless steel. The outer shell rests in, and is secured to, a cradle by clamp rods that are drawn down on shock-absorbing strips of sponge rubber resting on the upper half of the cylinder. The cradle rests on leveling screws that reach through the metal table of a heavy tripod whose legs extend through holes in the floor of the truck to the ground. During transportation the gravity balance is shifted to a platform that rests on the inflated inner tube of an automobile tire and is held in place by springs attached to the truck. Other springs hold the instrument in place on the platform. The protective housing has a thick layer of insulating material (balsa wood) between the outer shell and the second shell and the end pieces. These compartments surround an inner space that is water tight and contains either ice or an electrically controlled thermostat for maintaining constant temperature within the helix chamber.

During occupation of a gravity station the time for the schedule of operations is communicated to the observer through a telephone headset from a clock beating seconds and sending a signal at 50 seconds and on the minute.

Extensive series of routine measurements with the new gravity balance have not yet been made. With its predecessor, however, many series of tests have been carried out during the past eight years. Difficulties have arisen at times and much effort has been spent in ascertaining their causes and in eliminating them. Ordinarily the cause has been of a mechanical kind, such as the slight shift of an optical element, rather than imperfect response of the elastic element itself.

The faithful behavior of the elastic systems tested during this investigation has been a source of wonder to the observers. The behavior of an elastic element at any given time depends of course upon its previous treatment; at no time is it perfectly elastic; the hysteresis curve of deformation, which it follows under a given load, is determined quite exactly by its previous history; in other words, by the amount and distribution of stresses which it harbors at the given moment. If these stresses and resultant strains are large and non-uniform, its elastic behavior is erratic and unreliable. On the other hand, if the elastic element has been properly annealed and well seasoned and is maintained in that state, its response to a given small load is so constant, month after month, that measurements accurate to one part in a million or better are possible.

Special study has been made of the character of the curves of deformation followed by springs of tungsten and other materials, including single crystals, under load and their recovery therefrom. Many series of runs have been made in the Laboratory covering different time-periods from minutes to days and information has been gathered on the nature of elastic after-effects when measured with high accuracy.

Experience has proved that with proper treatment a spring maintains its characteristics remarkably well with no appreciable change in its behavior for long periods of time; and, that, as a result, no detectable shift arises in what would be called the zero point in other instruments. With careful handling and exact observance of a definite time schedule during the actual measuring periods and with freedom from load between these periods the spring preserves its state so satisfactorily that it can be relied upon to give reproducible results accurate to one part in a million. Its behavior under applied load is, however, not that of a perfectly elastic body but one in which certain departures from Hooke's law enter to modify the elastic response and to introduce a time element into rate of deformation that must be recognized, if the desired accuracy is to be attained. The realization of this condition led in 1923 to the adoption of the principle of the method followed in the series of gravity balances that have been constructed in the Geophysical Laboratory. The results obtained with these instruments have demonstrated the validity of the principle and have yielded valuable data on the deformation of certain materials when subjected to loads well within their elastic limit.