ART. XVIII.—*Hydraulics of the Report of Humphreys and Abbot on the Mississippi River;* by Prof. F. A. P. BARNARD. (Continued from p. 37.)

FOR the solution of most problems in practical hydraulics, it is necessary to establish the relations which exist between the cross-section of the stream, its mean velocity, and the slope of its surface. As a basis of this investigation, it is assumed by the authors of this report, as by other writers on the subject, that the condition of uniform motion is expressed by equating the accelerating force with the resistances. One side of the equation presents no difficulty; it is simply the expression for the force of gravity. The other requires consideration.

The authors of the report reject the idea that the cohesion of the particles of the liquid among themselves enters as an element into the resistance of the liquid to motion. They hold that this cohesion is concerned only in determining the distribution of the resistance through the mass; but that the resistance itself is simply the adhesion of the liquid to its bed. It is unnecessary to stop just here to discuss the question by what name it is most fitting that the resistance to flowing water shall be called. It is quite sufficient, if we agree that were the resistances, irregularities, and obstructions to motion, at the surfaces of contact between the water and the earth or the superincumbent air, to be totally annihilated, the whole body of water would descend with a uniformly accelerated velocity, as a solid descends an inclined plane; and there would be no subsurface curves. The resistances therefore come from the perimeter, and must be proportional to it, and to the length of the channel considered. In the perimeter, the results of this survey go to prove that we must include that part which is in contact with the air, as well as that which bounds the cross-section beneath the water. The question how there happens to be a resistance at the surface, and what is the cause or what are the causes producing it, is a question to be considered by itself. For the moment we accept the fact, as experiment has established it. The resistances must then be proportioned to the perimeter and length of channel; and, also, because, when there is no motion, there is no resistance, to some function of the mean velocity at the surfaces in contact. Now, if we put

a=cross-section of the river, W=width,

p=wetted perimeter, $r=\frac{a}{p}$=mean radius, or hydraulic depth,

l=length of channel considered, h=total head or difference of level,
$h_{,}$=part of head balanced against ordinary resistances of the channel,
$h_{,,}$=part of head neutralized by bends, and irregularities,

G=specific gravity of the water, g=measure of the force of gravity,
$s=\frac{h_{,}}{l}$=slope of surface expended against ordinary resistances,

$\frac{h_{,,}}{l}$=slope expended against bends, &c. V, U, v, b=same values as before,

we shall have, for the accelerating force of gravity, the expression, $Ggals$, and, for the resistances at the perimeter, the expression,

$$l(p+\text{W})\varphi\left(\frac{\text{U}_0\text{W}+\text{U}_r p}{\text{W}+p}\right),$$

which two expressions are to be put equal to each other. In a very large river, W and P are very nearly equal, though p of course always exceeds W. The expression may be simplified by putting them equal, and no appreciable error will result. Also, G and g are constants, and l is a factor common to both sides. Dividing by these, and putting p=W in the fraction, there will result the equation,

$$as=(p+\text{W})\varphi\left(\frac{\text{U}_0+\text{U}_r}{2}\right); \quad \text{or} \quad \frac{as}{p+\text{W}}=\varphi\left(\frac{\text{U}_0+\text{U}_r}{2}\right).$$

If we substitute the values of U_0 and U_r given above, the equation simplifies itself immediately to the following:[1]

$$\frac{as}{p+\text{W}}=\varphi(0{\cdot}93v-0{\cdot}167(bv)^{\frac{1}{2}}); \text{ which put } =\varphi(z).$$

Let $\frac{a}{p+\text{W}}$ be denoted by $r_{,}$=the radius of the entire perimeter: the expression then becomes

$$r_{,}s=\varphi({\cdot}93v-167(bv)^{\frac{1}{2}})=\varphi(z).$$

This preliminary equation differs from that which is usually assumed, in two particulars. First, the radius of the entire perimeter, $r_{,}$ is employed instead of r, the hydraulic mean depth (to which, however, it is in nearly a determinate ratio); and secondly $\varphi(z)$ is used instead of $\varphi(v)$, or a function of the *mean velocity at the perimeter*, instead of a function of the mean velocity of the river itself.

[1] In stating this expression, the authors have committed a sort of mathematical solecism, in arbitrarily changing the sign of the second term, to guard against the subsequent occurrence of what they conceived to be an inconsistency. We shall see that the apprehension was unfounded. The change of this sign, in fact, contradicts the original hypothesis. For the quantity under the symbol φ is the mean of the velocities at the upper and lower boundaries of the mean vertical plane; and this can never, by any possibility, be greater than the mean velocity in the same plane, which is ${\cdot}93v$; nor even equal to it, except in the case in which the parabola becomes a straight line, and v=0. Therefore, since $(bv)^{\frac{1}{2}}$ has been necessarily taken throughout as essentially positive, the written sign before it must be negative.

In order to determine the nature of the function $\varphi(z)$ and the constants which must enter into it, a collection was made of all the available data which had been furnished by the survey, or which could be gleaned from the publications of other observers; embracing thirty examples of area, width, wetted perimeter, maximum depth, mean velocity and slope of streams varying in magnitude from the dimensions of the Mississippi at high water, down to those of a small canal. In regard to slope, it was to be considered that a portion is expended in overcoming the irregularities of the channel and the changes of cross-section: and a portion, in compensating for the loss of living force at bends. It is only what remains after these effects have been subtracted, which constitutes the equivalent of the resistance of a straight and regular channel. The effect of bends must be provided for in an independent formula; and the amount of slope neutralized by them, deducted from the total slope observed. Irregularities and changes of cross-section in the channel are governed by no law, and therefore cannot directly enter into the formula; but they produce a mean effect, which is provided for in the modification which they introduce into the constants which are derived from observation, on the supposition that, after bends have been allowed for, the channel is straight and regular, and the movement in it uniform. The method pursued by most writers, of putting $\varphi(v)=Av+Bv^2$, and then seeking values for the indeterminate coefficients which shall most nearly represent the observations, was tried by the authors of the report, making

$$r_{\prime}s=Az+Bz^2, \quad \text{or} \quad \frac{r_{\prime}s}{z}=A+Bz,$$

in which $\frac{r_{\prime}s}{z}$ and z are co-ordinates in the equation of a straight line; but they found that a straight line would not represent the observations, and that the involution of z produced expressions of troublesome complexity. They then put

$$r_{\prime}s=Cz^2, \quad \text{or} \quad C=\frac{r_{\prime}s}{z^2}$$

and plotted the values of C as ordinates to $r_{\prime\prime}$, s, and v. The plots with $r_{\prime}$ and v produced irregular curves following no apparent law. That with s was quite regular. It was inferred therefore that C is some function of the slope. After a very long series of trials, with a view to discover this function, the expression

$$C=\frac{s^{\frac{1}{2}}}{195}$$

was adopted, as most satisfactorily fulfilling the required conditions. Substituting this, therefore, in the formula, it becomes

$$z=(195r_{,}s^{\frac{1}{2}})^{\frac{1}{2}}=\left(\frac{195as^{\frac{1}{2}}}{p+\mathrm{W}}\right)^{\frac{1}{2}}.$$

From this are deduced values for each of the variables in terms of the rest (regarding $p+\mathrm{W}$ as a single variable), viz:

$$s=\left(\frac{(p+\mathrm{W})z^2}{195a}\right)^2,$$

$$a=\frac{(p+\mathrm{W})z^2}{195s^{\frac{1}{2}}},$$

$$p+\mathrm{W}=\frac{195as^{\frac{1}{2}}}{z^2}.$$

Instead of $p+\mathrm{W}$, may be put, without appreciable error, for rivers, 2·015 W. Resuming the value of z, viz:

$$z=0{\cdot}93v-0{\cdot}167b^{\frac{1}{2}}v^{\frac{1}{2}}=(195r_{,}s^{\frac{1}{2}})^{\frac{1}{2}},$$

and solving with respect to $v^{\frac{1}{2}}$, we obtain

$$v^{\frac{1}{2}}=-\sqrt{0{\cdot}0081b+(225r_{,}s^{\frac{1}{2}})^{\frac{1}{2}}}+0{\cdot}09b^{\frac{1}{2}},$$

and

$$v=\left(-\sqrt{0{\cdot}0081b+(225r_{,}s^{\frac{1}{2}})^{\frac{1}{2}}}+0{\cdot}09b^{\frac{1}{2}}\right)^2$$

The negative value of the radical is that which it is necessary to take, in order to fulfil the condition that v shall become zero when s is zero.

For rivers, the value of b, as heretofore given, is 0·1856. The term containing it under the radical will have only the value ·0015, and may ordinarily be neglected. The expressions for the several variables will then become

$$v=(0{\cdot}0388-(225r_{,}s^{\frac{1}{2}})^{\frac{1}{4}})^2=\left(0{\cdot}0388-a^{\frac{1}{4}}\left(\frac{225s^{\frac{1}{2}}}{p+\mathrm{W}}\right)^{\frac{1}{4}}\right)^2$$

$$r_{,}=\frac{(v^{\frac{1}{2}}-0{\cdot}0388)^4}{225s^{\frac{1}{2}}} \qquad s=\left(\frac{(v^{\frac{1}{2}}-0{\cdot}0388)^4}{225r_{,}}\right)^2 \; *$$

If Q represent the amount of discharge per second, then

$$v=\frac{\mathrm{Q}}{a}, \text{ and } a=\frac{\mathrm{Q}}{v}.$$

If Q be given, along with any two of the foregoing variables, the rest may be computed by the help of this equation, unless the two given at the same time are v and a.

In estimating the effect of bends, the authors found the

* In the last two formulæ, the second term of the numerator has the negative sign, where the authors of the Report have made it positive. This difference results from our having chosen not to adopt the change of sign in the value of z, introduced by the authors, as explained in the note on p. 198. The two formulæ above are the only ones in which the original difference of proceeding involves any difference of final values; and as neither of these is employed at all in the subsequent test computations, the discrepancy has here no practical importance.

formula of Dubuat, with a modification of the constant, to represent very nearly the effect deduced from observation. This formula is (with the constant divisor reduced to English feet)

$$h_{\prime\prime} = \frac{v^2 \sin^2 \acute{a}}{266{\cdot}3},$$

in which $\sin^2 \acute{a}$ is the sum of the squares of the natural sines of the amount of bending, divided into angles not exceeding 36° or 40°.

Dubuat derived this formula from observation on the flow of water in pipes, in which the cross-section has no such variation as is always observed in the bends of rivers. The value of $h_{\prime\prime}$ is therefore too small, or the constant too large. Observations were made by the survey, to determine the amount of slope required to overcome a given bend. These observations were founded on a principle at once simple and ingenious. A level was run from one point of the river to another, several miles distant, embracing between them a bend and a long straight reach. Simultaneous readings of the stand of the river were made at both ends of the reach, and above the bend. If the bend had not existed, the slope in the reach multiplied into the distance by river between the extreme stations, should give the observed difference of level. The fall is always greater as observed than as computed, by the value of $h_{\prime\prime}$. From the data obtained by means of such observations, it was ascertained that Dubuat's formula, with the constant 134, would accord very closely with the observations. The authors therefore give, as the expression for the effect of bends,

$$h_{\prime\prime} = \frac{v^2 \sin^2 \acute{a}}{134}.$$

A table of the comparative values of $h_{\prime\prime}$ as computed by formula, and as obtained by actual measurement, over distances varying from five to nine miles, is given in the report, in which the differences are all very small, and are proportionally smaller as the distance is greater. In the application of the formula to cases in which an actual examination of every bend had not been made, resort was had to the best maps, and an estimate of the amount of bending made by measurement on the map. As a rough test of the correctness of these determinations, an independent formula was constructed, on the principle that the amount of bending between two points must be approximately proportional to the difference of distance between the points, as measured by an air line, and by the river. Denoting this difference in miles by M, it was found that $\sin^2 \acute{a}$ rarely differed essentially from 0·34 M. A series of comparisons somewhat extended, upon stretches of the river varying from three miles to more than eighty miles in length, gave, for the total of the observed values of $\sin_2 \acute{d}$, 140·81, and for that of the values computed by the last formula, 141·78. The total of the difference, taken, without regard to sign, was 22·83. This, observe the authors, "is given as an illustration that so far from being, as often declared in popular writings, a river without rule or beyond the restraint of law, the Mississippi is in reality controlled by laws which can be expressed in single algebraic formulæ."

We now come to that part of the report which has impressed us most forcibly with the value of the new formulæ, and the merit of the great labor by which they have been wrought out. This consists in an extended series of tests, in which the results of computation according to the formulæ are compared with actual measurements of the quantities computed. As no adequate idea of the severity and thoroughness of these tests can be formed without an inspection of all the data, along with the results deduced from them, we regard it as in justice due to the authors of the report to insert the following tables in full. It is to be observed that, in all cases, the slopes of rivers, as given in the first table, are the slopes as corrected from the measured slopes, by applying the formula for bends. The second table contains *differences* between the values of the computed mean velocity, and the mean velocity actually measured, in each of thirty cases. This table is rendered especially interesting by the comparison which it exhibits between the results given by the new formula for velocity, and those derived from the formulæ laid down by other writers. The following list embaces all these formulæ:

"*Chezy* ... $\begin{cases} \text{(Young's coefficient)} \ldots\ldots\ldots\ldots & v = 84{\cdot}3(rs)^{\frac{1}{2}}. \\ \text{(Eytelwein's coefficient)} \ldots\ldots\ldots & v = 93{\cdot}4(rs)^{\frac{1}{2}}. \\ \text{(Downing's and others' coefficient)} & v = 100{\cdot}0(rs)^{\frac{1}{2}}. \end{cases}$

$$\textit{Dubuat} \ldots v = \frac{88{\cdot}49(r^{\frac{1}{2}} - 0{\cdot}03)}{\left(\frac{1}{s}\right)^{\frac{1}{2}} - \mathrm{L}\left(\frac{1}{s} + 1{\cdot}6\right)^{\frac{1}{2}}} - 0{\cdot}086(r^{\frac{1}{2}} - 0{\cdot}03).$$

In which L= common logarithm multiplied by 2·302585.

$$\textit{Girard} \ldots v = (2{\cdot}69 + 26384\, r\, s)^{\frac{1}{2}} - 1{\cdot}64$$

De Prony $\begin{cases} \text{(For canals)} \ldots\ldots\ldots & v = (0{\cdot}0556 + 10593\, rs)^{\frac{1}{2}} - 0{\cdot}2357. \\ \text{(For canals and pipes)} & v = (0{\cdot}0237 + 9966\, rs)^{\frac{1}{2}} - 0{\cdot}1542. \\ \text{(Eytelwein's coefficient)} & v = (0{\cdot}0119 + 8963\, rs)^{\frac{1}{2}} - 0{\cdot}1089. \\ \text{(Weisbach's coefficient)} & v = (0{\cdot}00024 + 8675\, rs)^{\frac{1}{2}} - 0{\cdot}0154. \end{cases}$

$$\textit{Young} \ldots v = \left(\frac{rs}{3\mathrm{A}} + \left(\frac{\mathrm{B}}{12\mathrm{A}}\right)^2\right)^{\frac{1}{2}} - \frac{\mathrm{B}}{12\mathrm{A}}.$$

In which $\mathrm{A} = 0{\cdot}0000001\left(413 + \frac{1{\cdot}5625}{r} - \frac{90}{3r+8} - \frac{15}{4r + 0{\cdot}0296}\right)$,

$$B=0\cdot0000001\left(\frac{900r^2}{r^2+0\cdot5}+\frac{1}{(3r)^{\frac{1}{2}}}\left(271\cdot25+\frac{6\cdot88}{r}+\frac{0\cdot0001146}{r^2}\right)\right).$$

Dupuit.... $v=\frac{s\,r\,a}{0\cdot08W}+(0\cdot0067+9114\,rs)^{\frac{1}{2}}-0\cdot082.$

St. Venant.. $v=106\cdot068\,(rs)^{\frac{11}{21}}.$

Ellet...... $v=0\cdot64\,(\Delta H)^{\frac{1}{2}}+0\cdot04\,\Delta H.$

In which Δ denotes the maximum depth of the stream, and H the fall in water surface in 1 English mile."

Measurements of cross-section, slope, and resulting mean velocity of rivers

No. of observation.	Stream.	Locality.	Date.	Dimensions of cross-section. Area.	Width	Perimeter.	Max. depth	Mean veloc'y	Slope.	Authority.
				Sq. ft.	*Feet.*	*Feet.*	*Feet.*	*Feet.*		
1	Mississippi river,	Carrolton.	High w. 1851	193,968	2653	2693	136	5.9288	0.00002051	DeltaSurv'y
2	" "	"	" "	195,349	2656	2696	136	5·8869	0.00001713	" "
3	" "	"	May 31, 1851	180,968	2421	2461	131	4·0338	0.00000342	" "
4	" "	"	June 3, 1851	183,663	2429	2469	132	3·9775	0.00000384	" "
5	" "	Columbus.	May 15, 1858	148,042	2214	2247	88	6·9575	0·00006800	" "
6	" "	Vicksburg.	June 7, 1858	178,137	2729	2779	100	6·9496	0·00006379	" "
7	" "	"	H. w. 1858	179,502	2732	2782	101	6·8245	0·00004365	" "
8	" "	"	Nov. 6, 1858	78,828	2507	2530	63	3·5234	0·00002227	" "
9	" "	"	Dec. 18, 1858	134,942	2556	2589	83	5·5580	0·00003029	" "
10	" "	"	Dec. 24, 1858	150,354	2580	2621	90	6·3186	0·00004811	" "
11	Bayou Plaquemine,	Near Upper mouth.	Mar. 12, 1851	5,560	292	303	28	5.1979	0·00020644	Mr. C. Ellet.
12	" "	" " "	Jan. 16, 1859	4,259	268	278	24	3·9589	0·00014372	DeltaSurv'y
13	" La Fourche,	" " "	H. w. 1851	3,738	223	238	27	3·0765	0·00004468	" "
14	" "	" " "	May 6, 1851	3,025	223	232	24	2·8430	0·00003731	" "
15	" "	" " "	May 7, 1851	2,957	223	231	24	2·8069	0·00003655	" "
16	" "	" " "	May 8, 1851	2,868	223	230	23	2·7894	0·00004384	" "
17	C. & O. canal feeder	Near Georget'n, D.C.	Nov. 26, 1859	121	23	32·7	7·6	3·0323	0·00069851	" "
18	" "	" " "	Nov. 28, 1859	119	23	32.5	7·5	2·7227	0·00069851	" "
19	Ohio River,	Point Pleasant.	Nov. 20, 1858	7,218	1073	1074	8	2·5152	0·00009334	Mr. C. Ellet.
20	River Haine,	France.	—— 1782	248·5	48	50·5	8 ?	2·4947	0·00016534	M. Dubuat.
21	" "	"	—— 1782	306·4	50·5	53·4	9 ?	2·5579	0·00015593	" "
22	Canal,	England.		50	18	20·6	4	1·1336	0·00006313	Mr. Watt.
23	River Rhine,	Byland.	June— 1812	19,135	1155	1163	20	3·5749	0·00009769	Krayenhoff.
24	" "	Pannerden.	" "	6,304	557	563	17 ?	3·2766	0·00009986	"
25	" Waal,	Upper mouth.	" "	14,782	1328	1334	17 ?	3·1648	0·00010438	"
26	" Rhine,	Below the Yssel.	" "	5,341	700	704	12 ?	2·9167	0·00011744	"
27	" Yssel,	Upper mouth.	" "	1,930	321	324	9 ?	2·7727	0·00011657	"
28	" Tiber,	Rome.	June— 1821	2,355	243	249	15	3·4132	0·00013061	M. Buffon.
29	" Neva,	Russia.	June— 18—	43,461	1218	1227	50	3·2296	0·00001389	M. Destrem.
30	" Great Nevka,	"	June— 18—	15,554	881	893	21	2·0486	0·00001487	" "

Tests of the several formulæ for mean velocity.

Number of observations.	Chezy's formula with coefficient of			Dubuat's formula.	Girard's formula.	De Prony's formula with coefficients.				Young's formula.	Dupuit's formula.	St. Venant's formula.	Ellet's formula.	New formula.
	Young.	Eytelwein	Downing and others.			For canals.	For pipes and canals.	By Eytelwein.	By Weisbach.					
	Feet.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
1 . . .	+2·6888	+2·3390	+2·0854	+3·1820	+1·1140	+2·2017	+2·2430	+2·3974	+2·3644	+2·6547	+1·0536	+2·4381	+2·8837	+0·0385
2 . . .	+2·9167	+2·5961	+2·3636	+3·4374	+1·5736	+2·4887	+2·5204	+2·6584	+2·6206	+2·9000	+1·4629	+2·7002	+3·1500	+0·2425
3 . . .	+2·6973	+2·5530	+2·4484	+3·3542	+2·6207	+2·6208	+2·5978	+2·5378	+2·5725	+2·7822	+2·3651	+2·6534	+2·9552	+0·2593
4 . . .	+2·5522	+2·3984	+2·2868	+3·2073	+2·4188	+2·4572	+2·4369	+2·4820	+2·4182	+2·6320	+2·1732	+2·5009	+2·8230	+0·0658
5 . . .	+1·3152	+0·7061	+0·2643	+1·5090	−2·3975	+0·3004	+0·4281	+0·7288	+0·7388	+1·1238	−3·0950	+0·7159	+2·0962	−0·8093
6 . . .	+1·5591	+0·9772	+0·5552	+2·7722	−1·9246	+0·5998	+0·7184	+1·0038	+1·0091	+1·3844	−0·0904	+0·9996	+1·8883	−0·4602
7 . . .	+2·3506	+1·8677	+1·5174	+2·6733	−0·3106	+1·5929	+1·6784	+1·9078	+1·8968	+2·2359	−0·4739	+1·9298	+2·8054	+0·0713
8 . . .	+1·3028	+1·0631	+0·8893	+1·6224	+0·5812	+1·0377	+1·0434	+1·1361	+1·0853	+1·3343	+0·8166	+1·1737	+1·4852	−0·3978
9 . . .	+2·2085	+1·8469	+1·5847	+2·5718	+0·5389	+1·6975	+1·7427	+1·9037	+1·8726	+2·1690	+0·8041	+1·9437	+2·6953	+0·0430
10 . . .	+1·8902	+1·4121	+1·0654	+2·1721	−0·7304	+1·1425	+1·2263	+1·4530	+1·4411	+1·7785	−0·6254	+2·0143	+2·3442	−0·1982
11 . . .	+0·0094	−0·5507	−0·9569	−0·1965	−3·2932	−0·9055	−0·7942	−0·5211	−0·5194	−0·1556	−1·4981	−0·5187	+0·4408	−0·0448
12 . . .	+0·0033	−0·4238	−0·7334	−0·0583	−2·1974	−0·6407	−0·5738	−0·3760	−0·3963	−0·0801	−0·8769	−0·3435	+0·4993	−0·3871
13 . . .	+0·8434	+0·6023	+0·4275	+1·0077	+0·1116	+0·5756	+0·5817	+0·6751	+0·6245	+0·8751	+0·4812	+0·7129	+1·2065	−0·2076
14 . . .	+0·9837	+0·7829	+0·6374	+1·1533	+0·5428	+0·7964	+0·7900	+0·8069	+0·8040	+1·0392	+0·7353	+0·8921	+1·2623	+0·0213
15 . . .	+0·9835	+0·7866	+0·6439	+1·1535	+0·5695	+0·8039	+0·7963	+0 8651	+0·8076	+1·0613	+0·7447	+0·8954	+1·2442	+0·0304
16 . . .	+0·8184	+0·6056	+0·4513	+0·9686	+0·2926	+0·6072	+0·6044	+0·6821	+0·7270	+0·8657	+0·5499	+0·7156	+1·0997	−0 0768
17 . . .	−1·2535	−1·7162	−2·0517	−1·7040	−3·7470	−1·9699	−1·8912	−1·6733	−1·6876	−1·3746	−1·9100	−1·6470	−1·4773	−0·0709
18 . . .	−1·5406	−2·0008	−2·3346	−1·9857	−4·0141	−2·2520	−2·1741	−1·9576	−1·9723	−1·6603	−2·1895	−1·8953	−1·7497	−0·3594
19 . . .	+0·4038	+0·1759	+0·0106	+0·6998	−0·2313	+0·1623	+0·1643	+0·2504	+0·1977	+0·4406	+0·1520	+0·2864	+1·0867	+0·0298
20 . . .	+0·0901	−0·1694	−0·3577	+0·0453	−0·7802	−0·2148	−0·2028	−0·0990	−0·1467	+0·1054	−0·2003	−0·0593	+0·5240	+0·0257
21 . . .	+0·0364	−0·2358	−0·4333	−0·0038	−0·9301	−0·2940	−0·2779	−0·1671	−0·2128	+0·0447	−0·2847	−0·1265	+0·5194	−0·0886
22 . . .	+0·0901	−0·0226	−0·1043	+0·1478	+0·1788	+0·0737	+0·0425	+0·0655	−0·0040	+0·1871	+0·0257	+0·0684	+0·3413	−0·1965
23 . . .	+0·1952	−0·1696	−0·4342	+0·2315	−1·5005	−0·3224	−0·2761	−0·1133	−0·1439	+0·1506	−0·5043	−0 0736	+1·1067	−0·4809
24 . . .	+0·4577	+0·1534	−0·0673	+0·4877	−0·7571	+0·0627	+0·0891	+0·2179	+0·1774	+0·4492	+0·0070	+0·2597	+1·0020	−0·0544
25 . . .	+0·2978	−0·0117	−0·2361	−0·2478	−0·9577	−0·1078	−0·0797	−0·0520	+0·0125	+0·2862	−0·1620	+0·2895	+0·8310	−0·1808
26 . . .	+0·4004	+0·1288	−0·0681	+0·4037	−0·5617	+0·0712	+0·0871	+0·1976	+0·1518	+0·4107	+0·0629	+0·2382	+0·8733	+0·0973
27 . . .	+0·5513	+0·3115	+0·1376	+0·5584	−0·1711	+0·2860	+0·2918	+0·3845	+0·3337	+0·5803	+0·2855	+0·4221	+1·0048	+0·2993
28 . . .	+0·4503	+0·1305	−0 1015	+0·4294	−0·8867	+0·0238	+0·0553	+0·1928	+0·1587	+0·4315	−0·0107	+0·2347	+0·9410	+0·1378
29 . . .	+1·3598	+1·1579	+0·0115	+1·4974	+0·9109	+1·1702	+1·1641	+1·2357	+1·1790	+1·4133	+0·9731	+1·2671	+1·8573	−0·4976
30 . . .	+0·6919	+0·5455	+0·4393	+0·9601	+0·6026	+0·6112	+0·5888	+0·6300	+0·5649	+0·7771	+0·5348	+0·6463	+1·1609	−0·5191
Sum.	32·9420	28·4411	26·6988	40·4417	37·4472	28·0905	28·1506	29·5258	28·8412	33·3834	25·1488	33.6619	45·3547	6·3920

In order to understand the signs prefixed to the numbers in the foregoing table, it must be observed that the authors have tabulated the differences as *corrections*, not as *errors*. That is, each number must be applied to the result of the particular computation to which it relates, with the sign as written. Thus, the first number in the Chezy-Young formula, viz: +2·6888, indicates that this amount must be added to the value of v which the formula gives, in order to make it equal to the observed velocity, 5·9288. The formula gives 3·2400, and 3·2400+2·6888 =5·9288. This mode of exhibiting results, though it makes the comparative error striking, fails to convey an adequate impression of the *comparative approach to truth*, which is a different, and practically more important thing. Let us take, for illustration, the first four examples, with the results by several of the old formulæ and the new.

	1.	2.	3.	4.
Vel. observed,	5·9288	5·8869	4·0338	3·9775
Chezy-Young,	3·2400	2·9702	1·3365	1·4253
Dubuat,	2·7468	2·4495	0·6796	0·7702
Girard,	4·8148	4·3133	1·4131	1·5587
Prony-Eytelwein,	3·5314	3·2285	1·3960	1·4955
Prony-Weisbach,	3·5644	3·2663	1·4613	1·5593
Young,	3·2741	2·9869	1·2516	1·3455
St. Venant,	3·5907	3·1867	1·3804	1·4766
Ellet,	3·0451	2·7369	1·0786	1·1545
Humphreys and Abbot,	5·8903	5·6444	3·7745	3·9117

Thus, by comparing the actual velocities obtained by the different methods, it will be seen that most of the results are so far from the truth as to make them of little practical value; while the approach by the new formula is so near, that the difference is as likely to be due to errors of observation as to those of method.

There is another particular in which the table will not fail to attract notice. It is, that the old formulæ give better results upon rivers of moderate size, as upon the bayous, the Haine, the Rhine, the Tiber, &c., than upon the Mississippi; though upon the Mississippi itself, their results show great discrepancies. There is, however, one curious exception in the case of small streams. Numbers 17 and 18 are examples upon the feeder of the Chesapeake and Ohio Canal near Washington. The following are the results:

	1.	2.		1.	2.
Vel. observed,	3·0323	2·7227	Prony-Weisbach,	4·7199	4·7050
Chezy-Young,	4·2858	4·2633	Young,	4·4069	4·3830
Dubuat,	4·7363	4·7084	St. Venant,	4·6793	4·6180
Girard,	6·7793	6·7368	Ellet,	4·5096	4·4724
Prony-Eytelwein,	4·7056	4·6803	Humphreys and Abbot,	3·1032	3·0821

The old formulæ all give here a velocity largely in excess; whereas in large streams they are almost invariably in deficiency. The new formula represents these cases with as close an approach to observation as any others. The explanation of the anomaly is not obvious. The example of nearest general agreement of results, appears to be the small river Haine, No. 20, which gives the following:—

Vel. observed,	2·4947	Prony-Weisbach,	2·6414
Chezy-Young,	2·4046	Young,	2·3893
Dubuat,	2·4494	St. Venant,	2·5540
Girard,	3·2749	Ellet,	1·9707
Prony-Eytelwein,	2·5937	Humphreys and Abbot,	2·4690

If we examine the numerical ratio between the sums of the errors of the several formulæ in these thirty cases taken without regard to sign, as given in the table, to the sum of the observed velocities (115·4847), we shall find it to vary from twenty-two per cent for the formula of Dupuit, to thirty-nine per cent for that of Ellet. The formula of Humphreys and Abbot gives five and a half per cent. If we take the algebraic sum of these errors, this last ratio is reduced to three per cent; which is the tendency, as shown by this table toward excess. Examining the other formulæ in the same way, we shall see that they are all in deficiency, with the exception of Girard, who leans on the side of excess to the extent of eleven and a half per cent. The Chezy-Eytelwein formula gives a ratio of twenty-five per cent when the arithmetical sum of the errors is compared with the sum of the velocities; the Chezy-Downing formula gives twenty-three per cent on the same comparison. In these cases the algebraic sum of the errors shows a tendency to deficiency of fifteen and a half per cent for the first, and nine and a half for the second. Of all the old formulæ, the Dupuit appears to be the best; for the arithmetical sum of its errors bears the least ratio of all of them to the sum of the velocities; and the opposite errors, in these examples at least, almost exactly balance.

The second method employed by the authors of the report, to test the accuracy of their formulæ, consisted in computing the differences of level between points of the river distant from each other, in regard to which this difference had been ascertained by measurement. The same computation was made by Mr. Ellet's formula also, the results being introduced into the table along with those derived from the new formulæ, for the purpose of comparison. No computations were made in this case from the other formulæ, their large errors already showing their inapplicability to natural streams. An exception was made in favor of Mr. Ellet's, because it had been expressly designed for rivers. The present test applies equally to the bend formula, and to that for mean velocity. The following table embraces both data and

Tests of the formulæ for slope.

Stream.	Level-stations.	Date.	Area.	Width.	Perimeter.	Maximum depth.	Discharge.	Measured between level-stations: $\sin^2 d$.	Measured between level-stations: Distance.	Measured between level-stations: Fall.	Ellet's formula: Computed fall.	Ellet's formula: Error.	New formula: $h_{,}$	New formula: $h_{,,}$	New formula: Computed fall, or $h_{,}+h_{,,}$	New formula: Error.
			Sq. ft.	*Ft.*	*Ft.*	*Ft.*	*Cu. Ft.*	*Ft.*	*Miles.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*
Mississippi R.	Ft. St. Philip and B. La Fourche.	H. w. 1851	199,000	2,470	2,510	129	1,150,000	21·60	156·00	20·7	50·9	- 20·2	12·9	5·4	18·3	+ 2·4
" "	" " " " "	L. w. "	163,000	2,250	2,290	114	250,000	21·60	156·00	0·9	6·2	- 5·3	0·1	0·4	0·5	+ 0·4
" "	B. La Fourche and Red river.	H. w. "	200,000	3,000	3,035	113	1,200,000	15·39	122·60	23·7	47·7	- 24·0	17·0	4·1	21·1	+ 2·6
" "	" " " " "	L. w. "	100,000	2,750	2,770	78	250,000	15·39	122·60	3·7	16·6	- 12·9	1·8	0·7	5·5	+ 1·2
" "	Red river and Arkansas river.	H. w. 1858	199,000	4,080	4,115	96	1,200,000	56·50	373·00	112·0	172·1	- 60·1	98·4	15·3	113·7	- 1·7
" "	" " " " "	L. w. "	54,000	3,060	3,070	56	200,000	56·50	373·00	114·0	92·4	+ 21·6	117·7	5·8	123·5	- 9·5
" "	Arkansas river and Ohio river.	H. w. "	191,000	4,470	4,510	87	1,175,000	47·33	408·00	160·0	214·2	- 54·2	151·7	13·4	165·1	- 5·1
" "	" " " " "	L. w. "	45,000	3,400	3,410	49	150,000	47·33	408·00	159·0	121·2	+ 37·8	141·1	3·8	144·9	+ 14·1
B. Plaquemine.	Plaquemine and Indian Village.	H. w. 1850	5,500	300	318	31	33,500	8·63	8·33	20·0	12·1	+ 7·9	18·0	2·4	20·4	- ·04
" "	" " " "	March 12, '51	5,120	300	315	30	29,000	8·63	8·00	17·9	10·7	+ 7·2	14·9	2·1	17·0	+ 0·9
" "	" " " "	H. w. "	5,700	300	320	32	35,000	8·63	8·33	20·0	11·9	+ 8·1	17·4	2·4	19·8	+ 0·2
B. La Fourche.	Donaldsonville and Lockport.	" "	3,640	230	245	25	11,500	11·77	55·50	11·9	27·7	- 15·8	12·6	0·9	13·5	- 1·6
" "	" " "	April 11, '58	3,567	221	237	26	9,700	11·77	55·50	8·0	25·7	- 17·7	6·8	0·7	7·5	+ 0·5
Sum,										671·8	800·4	302·8			367·8	40·6

results. The data were not in all the cases known with equal degrees of exactness; but the small ratio of the errors to the distances on which the computations severally depend is not only satisfactory, but even surprising. In the last example but one, the error is regarded by the authors, as having been probably in great measure occasioned by the occurrence of crevasses between the points observed. The error which is largest in absolute amount is that between the Arkansas and Ohio rivers at low water, which is regarded as possibly due to sand bars.

The final test, and, as it seems to us, the most satisfactory of all, consists in the application of the new formula to the solution of the important question, how much will the level of a river be raised at a given locality, at which the cross-section and discharge are known, by any given definite increase of the discharge? In investigating this question, it is commonly assumed that the slope of the river is unaltered by the increased volume of discharge. But, as this assumption is not true, the results which are deduced from it are equally erroneous. In order to introduce the variation of slope, or the new slope produced by the addition to the volume, as an element in the computation, it was necessary to ascertain, if possible, by observation, the law which regulates the change.

The level of the water at the mouth of a river is not sensibly affected by a flood. For a certain distance up the course of the stream, the effect upon the slope produced by a rise in the river of a definite amount, will be equal to the total rise divided by the distance to the mouth. But, in general, an addition to the volume of waters produces a swell which passes down the stream like a great wave, so that the level may be actually falling near the head of the valley, when, at points lower down, it has not yet begun to rise. It is therefore evident that the same stand of the river is not always accompanied by the same slope, at any given point of observation, unless it be near the mouth.

From gauge observations, it appears that the form of the wave is tolerably regular, and that the daily *change of slope* is nearly the same for the same stand of the river in rising and falling. It is evident that observations on the passage of the great flood waves may be best conducted in the upper parts of the valley; inasmuch as the wave in its progress down the river tends, from the greater slope on the lower side, to spread itself over a wider and wider base, and loses therefore in the degree of its convexity. Columbus, Kentuky, was, on this account, first selected for study. The cross-section, perimeter, width, gauge-level and discharge of the river were determined for different dates during the progress of each of six marked rises of the river, and the corresponding slopes computed from the formula for those dates.

The *differences of slope* measured, of course, the change produced by the increased volume of discharge.

In the endeavor to ascertain the law of change, the slopes were first plotted as abscissæ, to the gauge-readings as ordinates; and straight lines were drawn connecting the points representing the top and bottom of each rise. These lines were not parallel, showing that the rate of increase of slope varies for different rises. In the further study of their relations, it was discovered that the difference of slope divided by the rise is the abscissa of a curve sensibly parabolic, in which the gauge-reading at the top of the rise, measured from low water mark, is the corresponding ordinate. Or, if x denote the rise, e the primitive gauge-reading, and $e+x$ the gauge-reading at flood; also, if $s_{,}$ and $s_{,,}$ represent the primitive slope and the slope at flood, then the following equation will be true:—

$$\frac{s_{,,}-s_{,}}{x}=\frac{1}{2\mathrm{P}}(e+x)^2, \quad \text{or} \quad s_{,,}-s_{,}=\frac{1}{2}\mathrm{P}(e+x)^2x.$$

The value of $\frac{1}{2\mathrm{P}}$ is to be determined by dividing $s_{,,}-s_{,}$ (of both which slopes the values are deduced, as just stated, by the formula, after the observations have determined the cross-section, discharge, perimeter, and rise of the river) by $(e+x)^2x$. For the same locality it is found to be constant; but it is different at different points in the length of the river.

If now we put $a_{,}$ $\mathrm{Q}_{,}$ $p_{,}$ $\mathrm{W}_{,}$ $v_{,}$ for the cross-section, discharge, perimeter, width, and mean velocity of the river in the primitive stage, and $a_{,,}$ $\mathrm{Q}_{,,}$ $p_{,,}$ $\mathrm{W}_{,,}$ and $v_{,,}$ for the same quantities after the rise; and if, in estimating the increased perimeter of the river occasioned by the rise, we neglect, as we may safely do for a large stream, the inclination of the banks, the new perimeter will be equal to the primitive perimeter increased by $2x$, and we shall have

$$\frac{a_{,,}}{p_{,,}+\mathrm{W}_{,,}}=\frac{a_{,}+\mathrm{W}_{,}x}{p_{,}+\mathrm{W}_{,}+2x}$$

Also, as these denominators are equal and numerators also, we shall have

$$a_{,,}=a_{,}+\mathrm{W}_{,}x; \quad \text{or} \quad a_{,,}v_{,,}=\mathrm{Q}_{,,}=a_{,}v_{,,}+\mathrm{W}_{,}v_{,,}x$$

and
$$x=\frac{\mathrm{Q}_{,,}-a_{,}v_{,,}}{\mathrm{W}_{,}v_{,,}}$$

Now, if the quantities $a_{,}$, $\mathrm{Q}_{,}$, $p_{,}$, $\mathrm{W}_{,}$, $v_{,}$ (with $z_{,}$ which is its function) be given, and it be required to know how much the river will rise if $\mathrm{Q}_{,}$ be made $\mathrm{Q}_{,,}$, the problem may be solved, and higher equations avoided, by an easy process of trial and error. Let $s_{,}$ be first computed from the formula

$$s_{,}=\left(\frac{(p_{,}+\mathrm{W}_{,})z_{,}^2}{195a_{,}}\right)^2.$$

Then, assuming some definite value for x, obtain the numerical values of $\frac{a_{\prime\prime}}{p_{\prime\prime}+W_{\prime\prime}}$ and $s_{\prime\prime}$; the former from the equation given just above, and the latter from the equation,

$$s_{\prime\prime}=s_{\prime}+\frac{1}{2P}(e+x)^2x,$$

in which the reciprocal of the parameter has the value belonging to the locality. This being done, $v_{\prime\prime}$ may be obtained from the equation for mean velocity already given, viz:

$$v_{\prime\prime}=\left(0{\cdot}0388-\left(225s^{\frac{1}{2}}\frac{a_{\prime\prime}}{p_{\prime\prime}+W_{\prime\prime}}\right)^{\frac{1}{4}}\right)^2;$$

and with this value of $v_{\prime\prime}$ a value of x may be formed from the equation just found;

$$x=\frac{Q_{\prime\prime}-a_{\prime}v_{\prime\prime}}{W_{\prime}v_{\prime\prime}}.$$

If this last value agrees with the assumed value, the problem is solved. If not, a new supposition must be made. But, as the true value always lies between the two erroneous values—that is, between the assumed one and the computed one—the approximation will be rapid. This method has been applied by the authors of the report to the calculation of many rises in the river, of which the particulars are given in the following table. The results are compared with calculations for the same rises from the formula of Mr. Ellet. The symbols $\Delta_{\prime}$ and L in the table belong to Mr. Ellet's formula, the manner of employing which it is not necessary here to explain.

The only criterion by which it is possible to judge of the value of hydraulic formulæ, is the degree of their accordance with direct observation. We have no principles of positive science, to which, in forming such estimates, we can confidently or safely trust. Were it otherwise, we should long since have had formulæ, concerning the truth of which there would be no room for doubt. But science is not in possession of the material for the construction of such expressions. It can only indicate certain variables which must enter into them; as to the manner in which they shall enter, or whether they are all that affect the case, it is silent. We do not know the physical law of resistance opposed to the movements of a fluid by the surfaces which confine it, nor does it yet appear how we can know it. And so long as it is a fact that all the postulates of theory, and all the resources of analysis, are powerless to tell us what amount of force will be consumed in driving a liquid, with a given velocity, into the mouth of a tube, or through the simplest orifice that can be made in the side of the containing vessel, we may well regard a problem affected by all the complex conditions which

Tests for the formulæ for oscillation caused by variation in discharge.

Locality.	Date.	e	$\Delta_{,}$	L	$a_{,}$	$W_{,}$	$p_{,}$	$Q_{,}$	$Q_{,,}—Q_{,}$	True oscillation. Kind.	True oscillation. Am't.	Computed oscillation. Ellet's formula. x	Computed oscillation. Ellet's formula. Error.	Computed oscillation. New formula. x	Computed oscillation. New formula. Error.
		Feet.	*Feet.*	*Miles.*	*Sq. feet.*	*Feet.*	*Feet.*	*Cubic feet.*	*Cubic feet.*		*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
Columbus - - -	Dec. 3 to Dec. 21, 1857	12·7	62	1076	92,730	2102	2115	220,000	+940,000	Rise.	25·3	72·9	-47·6	27·0	-1·7
	Dec. 11 to Dec. 16, "	25·8	75	1076	120,660	2160	2204	691,630	+445,970	Rise.	12·2	20·8	- 8·6	13·8	-1·6
	Mar. 17 to Mar. 28, 1858	25·4	75	1076	119,780	2157	2200	590,000	+530,000	Rise.	15·0	27·6	-12·6	15·0	0·0
	April 11 to April 26, "	24·4	74	1076	117,630	2157	2200	570,000	+690,000	Rise.	18·7	35·1	-16·4	18·7	0·0
	May 7 to June 22, "	32·0	81	1076	134,180	2199	2242	776,550	+606,530	Rise.	14·6	26·6	-12·0	14·7	-0.1
	June 3 to June 22, "	42·3	92	1076	156,500	2218	2252	1,160,970	+222,110	Rise.	4·3	8·2	- 3·9	5·2	-0.9
	July 17 to July 27, "	25·3	75	1076	119,580	2157	2200	424,530	+240,900	Rise.	6·6	17·9	-11·3	6·6	0·0
	Oct. 30 to Nov. 8, "	9·8	59	1076	86,670	2073	2086	143,710	+298,280	Rise.	11·9	37.8	-25·9	11·6	+0·3
Vicksburg - - -	March 6 to March 30, "	29·1	82	487	127,630	2522	2545	670,550	+438,880	Rise.	13·8	18·8	- 5·0	13·3	+0·5
	Mar. 10 to Mar. 23, "	30·9	84	487	132,200	2535	2560	748,200	+199,260	Rise.	6·1	8·5	- 2·4	6·3	-0·2
	Mar. 20 to Mar. 30, "	34·5	87	487	141,350	2558	2585	841,570	+267,860	Rise.	8·4	10·5	- 2·1	7·4	+1·0
	April 19 to June 26, "	45·4	98	487	169,500	2660	2698	1,105,000	+125,900	Rise.	2·9	4·4	- 1·5	2·8	+0·1
	Aug. 6 to Aug. 26, "	45·0	98	487	168,600	2658	2698	1,086,400	-372,340	Fall.	11·6	15·0	- 3·4	11·2	+0·4
	Aug. 26 to Sept. 1, "	33·4	86	487	138,530	2550	2575	714,060	-173,140	Fall.	6·7	8·9	- 2·2	6·7	0·0
	Nov. 3 to Nov. 15, "	8·7	61	487	77,360	2420	2430	236,000	+359,000	Rise.	16·8	25·7	- 8·9	17·6	-0·8
	Dec. 10 to Dec. 13, "	17·0	70	487	97,580	2455	2475	399,920	+117,700	Rise.	5·6	7·3	- 1·7	5·9	-0·3
Carrollton - - -	Feb. 17 to Feb. 23, 1851	6·3	128	121	164,170	2324	2355	534,780	+335,220	Rise.	5·2	9·8	- 4·6	4·5	+0·7
	Feb. 19 to March 1, "	8·2	130	121	168,840	2338	2368	630,000	+382,570	Rise.	5.2	11·0	- 5·8	4·7	+0·5
	Feb. 20 to Feb. 25, "	9·0	131	121	170,800	2344	2374	670,770	+239,130	Rise.	3·0	7·0	- 4·0	3·1	-0·1
	Feb. 25 to March 21, "	12·0	134	121	177,900	2364	2398	909,900	+229,800	Rise.	3·1	6·2	- 3·1	3·0	+0·1
	April 17 to May 9, "	14·8	136	121	183,800	2378	2416	1,065,000	-181,000	Fall.	1·8	5·1	- 3·3	3·2	-1·4
	May 27 to July 25, "	9·9	132	121	173,000	2350	2380	652,330	+222,670	Rise.	3·0	6·5	- 3·5	2·5	+0·5
	July 31 to Aug. 15, "	12·4	134	121	178,810	2367	2401	845,000	-175,610	Fall.	2·6	5·3	- 2·7	2·9	-0·3
	Jan. 7 to Jan. 17, 1852	0·8	122	121	152,000	2287	2312	310,000	+220,000	Rise.	4·3	7·2	- 2·9	4·4	-0·1
										Sum.	208·7	404·1	195·4	212·1	11·6

modify the flow of water in a natural channel, as practically beyond their reach. Hydraulic formulæ must, accordingly, from the nature of the case, be to a great extent empirical; and the highest degree of theoretic plausibility which such a formula may bring to recommend it, can at best only serve as an encouragement to us to try it, in order that we may ascertain how far it may truly represent nature. The experience gathered in such past trials has not, however, been of a nature to render the encouragement a very solid ground of hope for a favorable result.

The test then of actual trial is that to which we must bring at last all theorems in hydraulics; and our judgments of their merits will be regulated by the manner in which they stand this test. This is a principle which the authors of the report before us seem to have fully recognized; and the thoroughness with which they have applied it to their own formulæ is without any past example in the history of such investigations. We think them, therefore, fully justified in the modest claim with which they conclude this part of their labor, viz., that these formulæ are "entitled to the confidence of practical men."