

ART. IX.—*On a Determination of the Force of Gravity at the Summit of Fujiyama, Japan*; by T. C. MENDENHALL.

AN excursion to the summit of Fujiyama was made during the first ten days in August of the present year (1880), for the purpose of making a determination of the force of gravity at that point. In addition to four special students in physics from the Imperial University, the writer was accompanied by W. S. Chaplin, Esq., Professor of Civil Engineering, who determined the rate of the chronometer, and rendered much other valuable assistance. The preliminary experiments at the University in Tokio, as well as those upon the mountain, were mainly carried out, under the direction of the writer, by Messrs. Tanakadate and Tanaka, who had already gained much valuable experience in the determination of the force of gravity at Tokio.*

As stated in the previous paper, it was decided to make use of some form of the so-called "invariable" pendulum, and to compare a series of vibrations made at Tokio and at the top of the mountain. Owing to the difficulty of getting anything in the way of knife-edges made with sufficient accuracy in this country, nothing better could be done than to make use of a Kater's pendulum by Negretti and Zambra, after removing one of the knife-edges, the "tail-pieces" and all of the unnecessary parts. In the condition in which it was used, it consisted of a flat bar of brass 134 cm. long, 38 mm. wide and 4 mm. thick, with a knife-edge at a distance of about 15 cm. from one extremity. At nearly the lowest point was fixed the flat cylindrical weight, 10 cm. in diameter and 19 mm. thick, which was also of brass. On the short piece projecting above the knife-edge was an adjusting slide-piece which, as well as the cylindrical weight, was fixed in such a way that accidental movement was rendered impossible. In order to prevent any possibility of entire loss of results by means of the accidental injury of this pendulum, while being carried to and from the mountain, another of nearly the same vibrating period was prepared, the flat bar in this case being of well-seasoned wood, and the cylindrical weight being of somewhat different form. The knife-edge used was that which had been removed from the Kater's pendulum. During the month of July, both pendulums were vibrated in the Physical Laboratory of the University at Tokio in the same room and from the same pier referred to in the previous paper. Besides these two pendulums, the appliances consisted, in the main, of a Negus break-circuit chronometer, a chronograph and a portable transit instrument

* This Journal for August, 1880.

for rating the chronometer. Fortunately, all of the apparatus reached the top of the mountain in safety. The weather was exceptionally favorable during our stay, the nights being clear and the wind much less violent than is usual. The party occupied a small tent upon the summit of the mountain within a few feet of the edge of the crater, but it was evident that it could not be made a suitable place for carrying on the experiments. Fujiyama is a "holy mountain," and its ascent is accomplished yearly by thousands of pilgrims from all parts of Japan. This has resulted in the erection upon the summit of several small stone huts, which are used as temples. They are built of large and regularly-shaped pieces of lava, piled together so that the shrines within are perfectly protected from wind and weather. The priest in charge of one of these kindly permitted us to take possession of it during the few days of our stay, and it proved to be well adapted to our needs. In twenty-four hours after our arrival the pendulums were ready for work, and vibrations were continued during the greater portion of two days. Both pendulums were swung and the mode of recording the time was similar to that used in the previous determination at Tokio. The length of time in each series was, in general, thirty minutes. Temperature and barometric observations were made during the time of the experiments. Immediately after the pendulums were returned to Tokio they were again vibrated at the University, in order to discover if any change had taken place during their absence. With the results to be considered the brass pendulum alone is concerned. The two pendulums agree well with each other, but it is impossible to introduce any correction for the influence of moisture upon the wooden pendulum. Experiments made with it since the return from Fujiyama, under different degrees of humidity, show that such a correction would be necessary, so that it has not been introduced into the calculations, although it has served a useful purpose as a check. Many groups of vibrations were recorded both upon the mountain and at Tokio before and after the mountain work. Without quoting the individual results, it will be sufficient to remark that they agree among themselves slightly better than those given in the previous paper on the Tokio determination.

The vibrations at Tokio were all made under nearly the same conditions, and for convenience they were reduced to the common temperature of $23^{\circ}5$ and barometer 30 inches. The time of vibration of the pendulum at Tokio, under these conditions, the mean of all the results, was:

$$t_1 = .999834 \text{ seconds.}$$

On the summit of the mountain the barometric pressure was tolerably constant at 19.5 inches and the temperature $8^{\circ}5$, and

to these conditions they were all reduced after being corrected for arc, chronometer rate, &c. Finally the mean of all is reduced to the Tokio conditions as to temperature and pressure. The coefficient of expansion of the bar has not been determined, but it has been assumed to be 0.0000187 for 1° C. This is a commonly accepted coefficient for brass, and a comparison of the vibration periods of the pendulum, under different temperatures, indicates that it cannot be far from correct. The correction for difference of barometric height is the most difficult to determine. Were it possible to vibrate the pendulum at the same place under pressures widely differing, it might be determined experimentally. Lacking this, I was, fortunately, able to refer to a recent elaborate and exhaustive discussion of the whole subject, from an experimental as well as from a theoretical standpoint, by C. S. Peirce, Esq., of the U. S. Coast Survey.* In this valuable memoir Mr. Peirce gives a graphical representation of the periods of vibration of his pendulum, under various pressures, from 30 inches of mercury down to what is practically a vacuum. By interpolation the period for any pressure between these limits can be very closely ascertained, as also the correction in going from one pressure to another. There are important differences between the pendulum used by Mr. Peirce and that in use here, the principal one being the difference in shape of the cylindrical weights, and the fact that in our pendulum only one cylinder was carried. Nevertheless, a fair approximation to the correction may be taken from his curve showing the results with "heavy end down," and observing that the differences in the two pendulums are such as to make the correction for our pendulums considerably less than that of the Coast Survey. In this way, by considering these differences the correction used in the reduction was determined. After it had been established, I was fortunate in getting possession of the complete memoir of Mr. Bailey, published in 1832, in which his elaborate experiments to determine this correction are given in full. Among the many pendulums which he used was one "No. 22," which, in form and dimensions, resembles ours quite closely, and the results of his experiments with it confirm the accuracy of the assumption made. It will be remembered that, as all results are reduced to the Tokio conditions, the correction is to be made for only about one-third of an atmosphere, so that, although important, it is less so than if the reduction had been to a vacuum. It is thought, therefore, that the correction applied is not far wrong. The corrected time, then, appears to be as follows :

* Measurements of Gravity at Initial Stations in America and Europe. Appendix No. 15, U. S. Coast Survey. Report for 1876.

Time on Summit—temperature, $8^{\circ}5$; barometer, 19.5 inches;

$$t_2 = 1.000146$$

Correction for temperature

$$\text{to reduce to } 23^{\circ}5 \quad . \quad . \quad . \quad .000140$$

$$\text{Air correction} \quad . \quad . \quad . \quad .000050$$

Time reduced to Tokio conditions.

$$t_2 = 1.000336$$

Assuming the value of the force of gravity at Tokio to be as previously determined,

$$g_1 = 9.7984$$

it follows that on the summit of Fujiyama,

$$g_2 = 9.7886$$

Mr. Peirce has introduced an important correction to the time of vibration of a pendulum which depends upon the flexure of the support. I have carefully examined the support used in these experiments, attached to the massive stone pier, as described in a previous paper, and I can find no perceptible flexure. No means of making a minute examination were at hand on the summit of the mountain, but from the nature of the mounting there I am convinced that no considerable amount of flexure was possible.

The question at once suggests itself, whether it is possible to make use of this result in a determination of the density of the earth. While many of the circumstances are extremely favorable to this end, many of the data are, unfortunately, quite uncertain. It was originally planned to undertake at the same time a complete trigonometrical survey of the mountain, in order to obtain the necessary data as accurately as possible. This, however, we were obliged to defer, but it is hoped that it may be made at some future time. The following is offered as, perhaps, the nearest solution of the problem possible under the circumstances.

Fujiyama is an extinct volcano, whose height is known to be 2.35 miles very closely. It is renowned for its almost perfect symmetry of form, and for the fact that it rises solitary and alone out of a plain of considerable extent. Thus there is not much to consider except the attraction of the mountain itself. To determine this is, of course, a matter of considerable difficulty, but it is believed that a result not far out of the way is reached by making the following assumptions. Without any great error the mountain may be assumed to be a cone. The summit angle of this cone has been obtained by making careful measurements upon a large number of photographs of the mountain, taken from many different points of view. The mean of many measurements, which do not differ greatly among themselves, gives for this angle 138° . Another point of vital importance is the mean density of the mountain. The rock, as far as can

be discovered, is quite uniform in its composition throughout. It is a part of Japanese tradition, for it can hardly be called history, that the mountain was produced in a single night in the year B. C. 286. Many geologists are of opinion that it is mainly the result of a single eruption. A number of specimens from the surface have been examined, and it is found that when the air is retained in the pores the density is about 1.75, but when it is ground to a powder and the air excluded, it is 2.5. These facts were communicated to five geologists, at present employed in Japan, most of whom have considerable knowledge of the mountain from personal examination. They were requested to give an opinion as to what was the most probable mean density of the mountain. The mean of these results was 2.12, which is assumed to be correct in computing the result, and it also happens to be almost exactly the mean of the two densities given above. The time of vibration of the pendulum at the level of the sea at Tokio has been corrected for the difference of latitude between Tokio and Fujiyama, which is about 19', by means of the well known formula. When this is done it becomes

$$t_3 = .999847$$

From these times of vibration the part of the attraction on the summit, which is due to the mountain itself, is easily computed, and then the attraction of the mountain in terms of the density of the earth. The mountain is assumed to be a cone, whose semi-vertical angle is 69° and density 2.12, and its attraction on a particle at its vertex is computed. Equating these results the density of the earth results as follows:

$$D = 5.77$$

This result is somewhat greater than the generally accepted density, but when the uncertainty of some of the data is considered it must be regarded as remarkably close.

It is believed that the density of the mountain is the most uncertain of all the factors, and it will be of interest to reverse the problem and, assuming the density of the earth to be 5.67, as determined by Baily, find, by combining this with the pendulum experiments, the density of the mountain. When this is done the result is:

$$d = 2.08$$

It seems to me that these experiments establish, with considerable certainty, the fact that the mountain is, for some reason, deficient in attraction, which leads to many questions of great interest concerning the possible or probable structure of the mountain. It is possible, therefore, that these results may be of some value to geologists who are interested in the structure of volcanoes.