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MATHEMATICS OF TRACER MIXING IN SEDIMENTS: I. SPATIALLY-DEPENDENT, DIFFUSIVE MIXING

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ABSTRACT. Two conditions must be fulfilled before biological mixing of a tracer in sediments can be treated as a diffusive process. Firstly, the frequency of mixing events must be much greater than the rate of disappearance of the tracer. Secondly, the scale of material exchange must be smaller than the scale of the tracer profile and the thickness of the mixed-layer, L. Given these conditions, then time-averaging and a mixing length theory can be employed to derive, from the governing stochastic equation, the standard tracer conservation equation, where bioturbation appears as a diffusive transport term. Two endmember forms of biodiffusion can be recognized: interphase mixing where porosity and total solids are intermixed, and intraphase mixing where they are not.

The distribution of benthic infaunal organisms responsible for bioturbation suggests that the biodiffusion coefficient, $D_B(x)$, should decrease with depth. Steady-state tracer profiles produced with various forms of $D_B(x)$ differ from the profile generated with constant biodiffusivity only if $D_B(x)$ decreases faster than a quadratic decay and only when $wL/D_B(0) \leqslant 1$ and $0.03 \leqslant \lambda L^2/D_B(0) \leqslant 3$ (where $w \equiv$ sediment burial velocity and $\lambda \equiv$ tracer decay constant).

The possibility of determining the depth dependence of bioturbation from a single tracer profile is remote. The analysis suggests that an isotope characterized by $0.3 \leqslant \lambda L^2/D_B(0) \leqslant 3$ may be used to verify the mixing function suggested by the infauna distribution.

The evolution of an impulse source of tracer is remarkably similar for vastly different mixing functions. For this form of input, transient tracers are no more useful than steady-state tracers in distinguishing the form of $D_B(x)$.

INTRODUCTION

Geoscientists have long recognized that biological activity can modify the fabric, texture, structure, and composition of sediments (for example, Davison, 1891; Crozier, 1918; Watson, 1927; Moore, 1931; Dapples, 1942). The reviews by Rhoads (1974), Rhoads and Boyer (1982), Rowe (1974), Swartz and Lee (1980), Lee and Swartz (1980), Carney (1981), McCall and Tevesz (1982), and Aller (1982) present a fairly complete portrait of our present understanding of the phenomena involved. The displacement of

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sediment grains as a result of the activity of benthic and infaunal organisms is known as bioturbation (Richter, 1952). In the presence of compositional gradients, this displacement leads to net mass transfer.

The quantification of bioturbational rates and fluxes in sediments has become a prime concern to a large number of sedimentary geochemists. The aim of this effort has been to establish the contribution of bioturbational transport relative to advection and the effects of chemical or radioactive reaction on the distribution of various chemical species in sediments.

The standard method for determining biological mixing rates has been through the use of tracers. Tracers are compositionally or isotopically distinct species of sparse concentration that follow and mark the sediment motion without affecting this motion in any way. Useful tracers may be conservative-transient or steady-state reactive with a known rate of decay. Mixing information is extracted from tracer profiles by statistically fitting the solutions of conservation (transport) equations to the observed tracer distributions. The solutions contain certain arbitrary parameters which become defined by the data regression and are intended to characterize the mixing intensity and length.

This paper deals with the most popular of the currently used models for bioturbation, the diffusion analogy. Goldberg and Koide (1962) are responsible for the introduction of this formulation for biological mixing. They hypothesized that burrowing organisms should cause mixing of sediment in such a way as to create a net flow of sediment tracer (that is, in their case ^{230}Th) from "a richer to a poorer part of the deposit"; therefore, they envisioned a down-gradient transport analogous to molecular or "eddy"-diffusion,

$$J = - D_B \frac{\partial C}{\partial x} \qquad (1)$$

where J is the bioturbational flux, C is the concentration of the tracer in some appropriate units, x the depth in the sediment relative to the sediment-water interface, and D_B is a proportionality constant, later called the biodiffusion coefficient. Eq (1) is entirely phenomenological; consequently, D_B is not known *a priori* but must be deduced from observation.

The diffusion model was essentially ignored for a decade until Guinasso and Schink (1975) generalized the earlier work to include transient tracers and obtained the first calculated values of the biodiffusion coefficient.

Thereafter, there appeared a flood of papers reporting D_B values for deep-sea sediments (Nozaki and others, 1977; Peng, Broecker, and Berger, 1979; Dayal and others, 1979; Cochran and Krishnaswami, 1980; DeMaster and Cochran, 1982; Officer and Lynch, 1983a; Aller and DeMaster, 1984; Kadko and Heath, 1984) in near-shore sediments (Benninger and others, 1979; Cochran and Aller, 1979; Santschi and others, 1980; Aller, Benninger, and Cochran, 1980; Krishnaswami and others, 1980; Olsen and others, 1981; Carpenter, Peterson, and Bennett, 1982; Nittrouer and others, 1983/84), and in lacustrine sediments (Robbins, Krezoski, and Mozley, 1977; Robbins, 1978; Robbins and others, 1979; Krishnaswami and others, 1980; Sickle, Weimer, and Larsen, 1983; Officer and Lynch, 1982; Bukata and Bobba, 1984).

The simple diffusive formulation for bioturbation has been the subject of some criticism. This model describes inadequately situations where biological activity causes sediment to move advectively over relatively large distances, for example, the conveyor-belt like transport affected by head-down deposit feeders (Fisher and others, 1980; Carney, 1981; Aller, 1982). Because infaunal abundance and activity vary with depth (for example, Myers, 1977; Robbins, Krezoski, and Mozley, 1977; Jumars, 1978; Robbins, 1978; McCall and Tevesz, 1982; Germano, ms), Guinasso and Schink (1975), Schink and Guinasso (1977), Jumars (1978), Carney (1981), and Aller (1982) argue that the biodiffusion coefficient should be depth dependent.

Models with depth dependent biodiffusion coefficients have been proposed and solved by Schink and Guinasso (1977), Nozaki (1977), Peng, Broecker, and Berger (1979), Santschi and others (1980), Olsen and others (1981), Christensen (1982), and Kadko and Heath (1984). It is not apparent that these spatially-dependent D_B-models produce significantly different fits to observed tracer distributions.

The diffusion-model is often utilized without consideration of the conditions for its validity, with the result that it has been misapplied. This problem results from the fact that the necessary conditions for this model have not been explicitly defined. In addition, the analogy between (eddy) diffusion and bioturbation remains nebulous and formally unstated. This lack of definition is at the root of the disagreement on the correct form of the tracer conservation equation as expressed in the discussions of Christensen (1983) and Officer and Lynch (1983b).

This paper is intended to clarify the definition of biodiffusion, provide explicit criteria for the validity of the diffusion analogy, establish the correct forms of the tracer conservation equations, and explore the effect and need of a spatially-dependent biodiffusion coefficient.

TIME AND SPACE CONTINUA, AVERAGING, AND THE GOVERNING EQUATIONS

On the microscopic scale, sediments are a mixture of discrete grains of varying size, shape, and composition. The sediment exhibits marked discontinuities in its properties at this scale; yet, geochemical studies treat sediments as an idealized system of superimposed continua capable of occupying each point of the sediment simultaneously. This permits us to speak of both the porosity and the concentration of a tracer at a given depth.

What conditions permit the adoption of a spatial continuum for the actual discrete fields in sediments? Furthermore, on what scale should a sediment be sampled so that the values reflect this hypothetical continuum? These problems have been examined by hydrologists and fluid dynamicists dealing with multi-phase flows (for example, Whitaker, 1969;

Bear, 1972; Gray, 1975, 1983; Cushman, 1983; Baveye and Sposito, 1984). They conclude that a condition sufficient for the validity of the spatial-continuum representation is that the scale of the microscopic variations, l_m, must be much smaller than the scale of the macroscopic variations, L, of the properties of interest, and that the sampling must be performed on an intermediate scale, l_o, that is,

$$l_m << l_o << L \tag{2}$$

In the present context, l_m is the average sediment grain size, and L is the length (depth interval) over which the tracer changes significantly in concentration (or activity) or the total mixing depth, whichever is smaller. The values of l_m, l_o, and L are not absolutes but depend on environmental conditions such as sediment type and mixing behavior of the benthic organisms and on the decay rate or distribution of the tracer. For example, a sand would have l_m of about 0.5 mm. If a tracer is distributed over the top 10 cm of the sediment, that is, L, then a continuum will approximate samples taken on intervals of 0.5 to 2 cm. For a mud, the lower bound is greatly reduced and considerably smaller samples can be utilized in connection with the continuum representation.

Unlike the real tracer field, the continuum field is infinitely divisible without losing any of its defining properties, possesses spatial partial derivatives to any order desired, and can be subject to limiting operations, that is, limit $x \rightarrow 0$ (Truesdell and Toupin, 1960). The adoption of a continuum model does not preclude the existence of macroscopic discontinuities in the sediment property fields (for example, contacts between different sediment types). These discontinuities must be finite in number, and the value of the sediment properties and their derivatives must be defined in the limit as the discontinuity is approached. (We can thus speak of piecewise continuous properties in a continuum sediment.)

The modelling effort can be further simplified, because most diagenetic changes in sediments are directed in the vertical direction. This has encouraged the use of a uni-dimensional continuum model (Berner, 1980). This treatment is valid, if the horizontal scale, l_h, of variations is much larger than the scale of the vertical change, L, that is,

$$l_h >> L \tag{3}$$

When biological patchiness is intense, eq (3) will fail.

Consider now mixing in our hypothetical continuum sediment while drawing on experience gained from observation of real sediment. Macrofauna and perhaps meiofauna rework and displace this sediment. If we could continuously monitor a tracer at a correct sampling scale, we would observe that the concentration of a tracer at a given point could change temporally in a quite sudden manner, unrelated to decay and simple burial. These sudden changes are due to biological activity.

As an organism moves past or occupies a point in the sediment, it may remove material to another depth or deposit material taken from another point within the sediment column. Because the newly introduced sediment has a different tracer concentration than the original sediment, an abrupt concentration change is recorded. As organisms can move in three dimensions, their movements appear more-or-less random in one dimension (let's exclude head-down deposit feeders), and as all points in the sediment cannot be continuously occupied by infauna, the changes in the tracer concentration appear as random fluctuations.

The time-average of this fluctuating tracer field may not be random and, in fact, may constitute a smooth, continuous profile. In this situation, it may be possible to describe the mixing activity of the infauna as a diffusive process. We now examine those mathematical and physical conditions that must be met to permit averaging and use of the diffusion analogy.

As stated above, bioturbation results from the movement of sediment grains by organisms from one depth to another. Figure 1 illustrates a hypothetical, instantaneous tracer distribution, $\tilde{C}(x,t)$, in a sediment. If the biological transportation of material from an arbitrary depth, say x_0, is to neighboring points, for example, x_1 and x_2, then the mixing is termed local, that is, the transport distance is small compared to the tracer profile scale. As shown below, diffusional mixing is a limiting case of this class of mixing. If mixing causes the exchange of material between widely

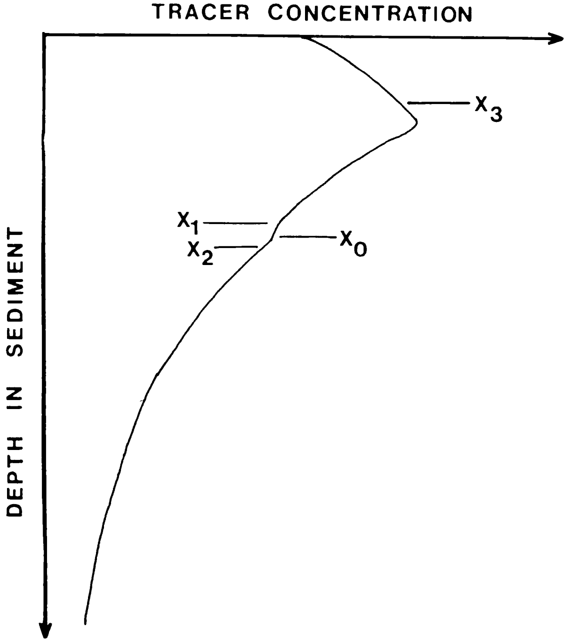

Fig. 1. A schematic, hypothetical tracer distribution in sediments. The axes are arbitrary, but the tracer concentration or activity increases to the right and depth increases downward. The x-axis is the sediment-water interface. Mixing between neighboring points such as x_0 and x_1 or x_2 are local exchanges, while any exchange between these points and x_3 is term nonlocal.

separated points on the tracer profile, for example, between x_0 and x_3, then it is called nonlocal (Imboden, ms). This latter form cannot be described as diffusive and is explicitly excluded from consideration.

The constraint of local mixing imposes another length scale requirement,

$$l_b << l_o << L \tag{4}$$

where l_b is the typical distance over which organisms exchange or transport sediment. (There is no restriction on the relative sizes of l_b and l_m.)

The other restrictions on the diffusion analogy are best discovered by developing a conservation equation for the instantaneous (fluctuating) tracer field and averaging to obtain an equation for a temporally smooth field subject to diffusion. To begin the derivation, consider a small interval Δx centered about some arbitrary depth x in the sediment. (Note $\Delta x < l_o$.) Over a short period of time, Δt, sediment and tracer would be seen to enter or leave at the top and/or the bottom of a box of unit cross-sectional area and thickness Δx due to burial and biological transport. In addition a small amount of the tracer could decay and/or be produced in the box over the period Δt. These processes could combine to cause a net increase or decrease in the amount of sediment and tracer in the box, that is

$$(\pm)\text{ change} = \text{flow in} - \text{flow out} - \text{decay} + \text{production} \tag{5}$$

On a short time scale, the local bioturbational mixing appears as a stochastic contribution to the advection, in addition to the regular burial component.

The decay term in eq (5) is assumed to be linear in the amount of tracer. This assumption is not too restrictive as the vast majority of nonconservative tracers employed in geology are of this type. Tracers with nonlinear decay can be accommodated with a moderate increase in mathematical complexity. The production term in this same equation is assumed independent of the tracer concentration but explicitly dependent on x.

Approximate mathematical expressions of the terms in eq (5) are given in table 1. These are approximate, because certain quantities that vary with time have been treated as constant, for example, $\tilde{C}(x,t)$ in the decay term. Use of these approximations causes no lasting problems. Substituting, the balance is given by

$$[\tilde{\phi}_s(x,t+\Delta t)\tilde{C}(x,t+\Delta t) - \tilde{\phi}_s(x,t)\tilde{C}(x,t)]\Delta x \approx$$

$$\Big[\tilde{\phi}_s\left(x-\frac{\Delta x}{2},t\right)\tilde{w}\left(x-\frac{\Delta x}{2},t\right)\tilde{C}\left(x-\frac{\Delta x}{2},t\right)$$

$$-\tilde{\phi}_s\left(x+\frac{\Delta x}{2},t\right)\tilde{w}\left(x+\frac{\Delta x}{2},t\right)\tilde{C}\left(x+\frac{\Delta x}{2},t\right)$$

$$-\tilde{\phi}_s(x,t)\,k(x)\,\tilde{C}(x,t)\Delta x + \tilde{P}(x,t)\Delta x\Big]\Delta t \tag{11}$$

TABLE 1

Approximate mathematical forms of the terms in the instantaneous tracer conservation equation (eq 5)

$$\text{Accumulation (change)} \approx [\tilde{\phi}_s(x,t+\Delta t)\tilde{C}(x,t+\Delta t) - \tilde{\phi}_s(x,t)\tilde{C}(x,t)]\,\Delta x \tag{6}$$

$$\text{flow in} \approx \left[\tilde{\phi}_s\left(x-\frac{\Delta x}{2},t\right)\tilde{w}\left(x-\frac{\Delta x}{2},t\right)\tilde{C}\left(x-\frac{\Delta x}{2},t\right)\right]\Delta t \tag{7}$$

$$\text{flow out} \approx \left[\tilde{\phi}_s\left(x+\frac{\Delta x}{2},t\right)\tilde{w}\left(x+\frac{\Delta x}{2},t\right)\tilde{C}\left(x+\frac{\Delta x}{2},t\right)\right]\Delta t \tag{8}$$

$$\text{decay} \approx \tilde{\phi}_s(x,t)\,k(x)\,\tilde{C}(x,t)\,\Delta t\,\Delta x \tag{9}$$

$$\text{production} \approx \tilde{P}(x,t)\,\Delta t\,\Delta x \tag{10}$$

where

$\tilde{C}$ ≡ instantaneous amount (activity) of tracer per unit volume of solids

$\tilde{\phi}_s$ ≡ solid volume fraction

$\tilde{w}$ ≡ local advection velocity of the solids

$k(x)$ ≡ rate constant for tracer disappearance

$\tilde{P}$ ≡ rate of production of the tracer per unit volume per unit time

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where $\phi_s \equiv$ solid volume fraction, that is, $1 - \phi$ where $\phi \equiv$ porosity, and the overbar ~ indicates a quantity subject to stochastic fluctuations.

The fluxes at the top and bottom of the box can be expressed in terms of the value at the center by using the truncated Taylor series,

$$j(x-\Delta x/2,t) \approx j(x,t) - \frac{\partial j(x,t)}{\partial x}\frac{\Delta x}{2} \tag{12}$$

$$j(x+\Delta x/2,t) \approx j(x,t) + \frac{\partial j(x,t)}{\partial x}\frac{\Delta x}{2} \tag{13}$$

where j represents an advective flux. Similarly, the future value at $t + \Delta t$ can be expressed in terms of the value at t,

$$\tilde{\phi}_s\tilde{C}(x,t+\Delta t) \approx \tilde{\phi}_s\tilde{C}(x,t) + \frac{\partial\tilde{\phi}_s\tilde{C}(x,t)}{\partial t}\Delta t \tag{14}$$

If these series are substituted into eq (11), then after dividing this equation by Δx and Δt and taking the limit as $\Delta \rightarrow 0$, there results the equality

$$\frac{\partial\tilde{\phi}_s\tilde{C}}{\partial t} = -\frac{\partial\tilde{\phi}_s\,\tilde{w}\,\tilde{C}}{\partial x} - \tilde{\phi}_s\,k(x)\,\tilde{C} + \tilde{P}(x,t) \tag{15}$$

Eq (15) is the instantaneous, differential conservation equation for the tracer. Eq (15) includes the effects of local mixing, yet no biodiffusion term appears in this equation. On the other hand, the dependent variables in eq (15) are not of the mathematical type found in the usual diagenetic equations. (Eq 15 is a stochastic differential equation.)

The time averaging operation employed here is the standard time average (Hinze, 1975),

$$<p> = \frac{1}{\Delta t_{av}} \int_{t-\Delta t_{av}/2}^{t+\Delta t_{av}/2} p \, dt_0 \tag{16}$$

where t_0 is a dummy variable, Δt_{av} is the averaging period, and p any property, for example, $\tilde{\phi}_s$, $\tilde{C}$, et cetera.

Limits must now be placed on the relative magnitude of Δt_{av}. The averaging period must embrace a fairly large number of stochastic mixing events. Therefore, if Δt_b is an estimate of the period between mixing events,

$$\Delta t_b << \Delta t_{av} \tag{17}$$

On the other hand, the averaging period must be smaller than the relaxation time, Δt_r, of the sediment/tracer system. The relaxation time is the lesser of the decay time for the tracer (half-life divided by 0.693) or the time to bury the mixed layer, that is, L/w_0, where w_0 is the maximum burial velocity. Thus,

$$\Delta t_b << \Delta t_{av} << \Delta t_r \tag{18}$$

In a transient case, there is an introduction time associated with the tracer input function (excluding a Dirac delta function) which must also be considered in estimating Δt_r.

The diffusional formulation for bioturbation assumes that eq (18) is valid at all depths. Despite the possibility of mid-depth maxima, infaunal population densities ultimately decrease with depth, and so should mixing frequencies. Although there may be a sufficient number of mixing events to satisfy eq (18) near the sediment-water interface, a rapid decrease in biological perturbations may lead to severe violation of this requirement at depth.

To introduce some quantitative appreciation for the limits imposed by eq (18), consider ^{210}Pb, which has a characteristic decay period of about 32 yrs (half-life/0.693), and is commonly used to determine mixing rates in deep-sea sediments. A $\Delta t_r \cong 32$ yr requires that the period of mixing events should be no more than about 1 yr, that is, $\Delta t_b \leqslant \Delta t_r/25$. Another isotope employed to extract particle reworking rates in near shore sediments is ^{234}Th. With a decay period of 35 days, a continuum model would demand a maximum mixing period of 1 day at every depth. Observations of mixing frequencies (not just feeding rates) are clearly needed in order to check that the minimum values are achieved.

Given that an averaging time does exist, then as a first step the stochastic variables are written as the sum of a non-fluctuating quantity and a biologically-induced fluctuation in a manner reminiscent of the treatment of fluctuations in turbulent fluid flows (Reynolds, 1895), for example

$$\tilde{\phi}_s \equiv \phi_s + \phi_s' \tag{19A}$$

$$\tilde{C} \equiv C + C' \tag{19B}$$

$$\tilde{w} \equiv w + w' \tag{19C}$$

where the non-primed quantities are the time-smooth quantities familiar to diagenetic models, and the primed quantities are superimposed fluctuation. (In an earlier derivation, Boudreau, ms, assumed that $(\tilde{\phi}_s\tilde{C})$ must be treated as a single variable. Although that approach may still be valid, I present below a derivation that does not rely on this assumption.)

The averaging produces different results depending on the ability of organisms to intermix total solids and fluids. There are two endmember cases. If bioturbation does not mix fluid against total solids, then this form of mixing is called intraphase mixing (Berner, 1980). The word phase is not used in its thermodynamic sense. In this case, $\phi_s' = 0$. The name interphase mixing is used to describe mixing that affects porosity ($\phi_s' \neq 0$). As shown below, the failure to recognize these two fundamentally different modes of mixing is at the root of the disagreement between Christensen (1983) and Officer and Lynch (1983b). There are no data on the prevalence of these two endmember forms of mixing.

Substitution of definitions (19A, B, C) into eq (15) produces, for interphase mixing,

$$\frac{\partial\phi_s C}{\partial t} + \frac{\partial\phi_s' C}{\partial t} + \frac{\partial\phi_s C'}{\partial t} = -\frac{\partial\phi_s w^* C}{\partial x} - \frac{\partial}{\partial x}\Big[\phi'_s w^* C + \phi_s(w^*)'C$$

$$+ \phi_s w^* C' + \phi_s'(w^*)'C + \phi_s' w^* C' + \phi_s(w^*)'C' + \phi_s'(w^*)'C'\Big]$$

$$- k(x)\,[\phi_s C + \phi_s' C + \phi_s C' + \phi_s' C'] + P(x,t) + P'(x,t) \qquad (20)$$

and for intraphase mixing,

$$\frac{\partial\phi_s C}{\partial t} + \frac{\partial\phi_s C'}{\partial t} = -\frac{\partial\phi_s w C}{\partial x} - \frac{\partial}{\partial x}\Big[\phi_s w' C + \phi_s w C'$$

$$+ \phi_s w' C'\Big] - \phi_s\,k(x)\,[C + C'] + P(x,t) + P'(x,t) \qquad (21)$$

Note that the burial velocity for interphase mixing is adorned with a superscript *. This is done to distinguish it from the velocity during intraphase mixing. Such discrimination is necessary because these two quantities are not generally equal (as will be shown later).

Eqs (20) and (21) are now time-averaged by utilizing the operator given by eq (16) along with the so-called Reynolds averaging properties associated with this operator (Hinze, 1975, p. 6-7)

$$<p> = p \qquad (22A)$$

$$<p'> = 0 \qquad (22B)$$

$$<p\,p'> = 0 \qquad (22C)$$

$$<p_1'\,p_2'> \neq 0 \qquad (22D)$$

$$<\frac{\partial p'}{\partial t}> = 0 \qquad (22E)$$

where p_1' and p_2' are two arbitrary fluctuating quantities. It is further assumed that the order of temporal integration and spatial differentiation can be inverted as needed.

The averaging procedure generates the following conservation equations. For interphase mixing,

$$\frac{\partial \phi_s C}{\partial t} = -\frac{\partial}{\partial x}\Big[C<(w^*)'\,\phi_s'> + \phi_s<(w^*)'\,C'> + w^*<\phi_s'C'> + <\phi_s'(w^*)'C'> \Big] - \frac{\partial \phi_s w^* C}{\partial x} - k(x)\,[\phi_s C + <\phi_s'C'>] + P(x,t) \quad (23)$$

and for intraphase mixing,

$$\frac{\partial \phi_s C}{\partial t} = -\frac{\partial}{\partial x}\Big[\phi_s<C'w'> \Big] - \frac{\partial \phi_s w C}{\partial x} - \phi_s\, k(x)\, C + P(x,t) \quad (24)$$

Notice that the time average of the product of fluctuations is not zero (see Hinze, 1975).

Further simplification of eq (23) is necessary. The fluctuating quantities ϕ_s' and C' must be small compared to the non-fluctuating means, ϕ_s and C. If this were not the case, mixing would have to occur on a scale comparable to the scale of the tracer profile, and the mixing would be nonlocal. Because this scale of exchange is specifically excluded, it follows that

$$\phi_s C >> <\phi_s'C'> \quad (25)$$

and the second half of the decay term in eq (23) can be neglected.

Similarly, the terms contained within the large brackets in eq (23) do not contribute equally to mixing. Over the mixing distance l_b, eq (25) implies that $\partial\phi_s/\partial x$ and $\partial C/\partial x$ are nearly constants, such that

$$\phi'_s \cong \frac{1}{2}\, l_b \frac{\partial \phi_s}{\partial x} \quad (26)$$

and

$$C' \cong \frac{1}{2}\, l_b \frac{\partial C}{\partial x} \quad (27)$$

Conversely, the fluctuations in the burial velocity, $(w^*)'$, are not necessarily small compared to the mean w. During local mixing, infauna cause intense bioturbation by moving sediment very fast over short distances. In fact, diffusion is obtained as the limiting case

$$(w^*)' \rightarrow \infty \quad (28)$$

and

$$l_b \rightarrow 0 \quad (29)$$

but in such a way that the product

$$(w^*)'\, l_b \rightarrow \text{finite and non-zero} \quad (30)$$

In reality, infaunal activity cannot be expected to fulfill such extreme conditions. The diffusive representation is thus a limiting case that only approximates bioturbation.

Using the approximations given by eqs (26) and (27), we obtain

$$<\phi_s'(w^*)'> \cong \frac{1}{2} <(w^*)'l_b> \frac{\partial \phi_s}{\partial x} \tag{31}$$

$$<C'(w^*)'> \cong \frac{1}{2} <(w^*)'l_b> \frac{\partial C}{\partial x} \tag{32}$$

$$<\phi_s'C'> \cong \frac{1}{4} <l_b^2> \frac{\partial \phi_s}{\partial x} \frac{\partial C}{\partial x} \tag{33}$$

$$<\phi_s'(w^*)'C'> \cong \frac{1}{4} <(w^*)'l_b^2> \frac{\partial \phi_s}{\partial x} \frac{\partial C}{\partial x} \tag{34}$$

In the limit given by eqs (28) through (30), the right hand sides of eqs (33) and (34) approach zero, while eqs (31) and (32) remain finite and non-zero. In the diffusive limit, eq (23) reduces to,

$$\frac{\partial \phi_s C}{\partial t} = -\frac{\partial}{\partial x}\left[\frac{1}{2} <(w^*)'l_b> \frac{\partial \phi_s C}{\partial x} + \phi_s w^* C\right] - k(x)\,\phi_s C + P(x,t) \tag{35}$$

where the chain rule for differentiation has been employed.

Eq (27) can also be substituted into the equation for intraphase mixing, eq (24), to arrive at

$$\frac{\partial \phi_s C}{\partial t} = -\frac{\partial}{\partial x}\left[\phi_s\left[\frac{1}{2} <w'l_b> \frac{\partial C}{\partial x} + wC\right]\right] - k(x)\,\phi_s C + P(x,t) \tag{36}$$

The terms $<(w^*)'l_b>$ and $<w'l_b>$ both have units of length squared over time, which are those of diffusion coefficients. If we are correct in our supposition that these terms are responsible for moving tracer and/or sediment down gradient, then we can equate these coefficients to the negative of biodiffusion coefficients,

$$\frac{1}{2} <(w^*)'l_b> = -D_B^*(x,t) \tag{37A}$$

$$\frac{1}{2} <w'l_b> = -D_B(x,t) \tag{37B}$$

Numerically, D_B^* and D_B may be equal, but the distinction is retained to emphasize the two endmembers. The definition of the eddy-diffusion coefficient that appears in Prandtl's mixing-length theory is similar to eq (37). However, in developing the turbulent flow theory, Prandtl assumed that $w' << w$ (Hinze, 1975), which is not necessarily the case with biodiffusion. The analogy between eddy-diffusion and biodiffusion is not perfect.

Substitution of eqs (37A, B) into eq (35) generates the final form of the tracer conservation equations for interphase mixing,

$$\frac{\partial \phi_s C}{\partial t} = \frac{\partial}{\partial x}\left[D_B^*(x,t)\frac{\partial \phi_s C}{\partial x} - \phi_s w^* C \right] - k(x)\,\phi_s C + P(x,t) \qquad (38)$$

and into eq (36) for intraphase mixing,

$$\frac{\partial \phi_s C}{\partial t} = \frac{\partial}{\partial x}\left[\phi_s\, D_B(x,t)\frac{\partial C}{\partial x} - \phi_s\, w\, C \right] - k(x)\,\phi_s C + P(x,t) \qquad (39)$$

The additional component of the diffusive flux in eq (38), that is, $D_B(x,t)$ $C\,\partial\phi_s/\partial x$, may contribute substantially if there is a pronounced ϕ_s gradient. This situation is typical of surficial sediment.

By introducing the new tracer concentration per unit mass of total solids, that is, $s \equiv C/\rho_s$, (where ρ_s is the density of the solids), and setting $P(x,t) = 0$, eq (38) can be converted to eq (2) of Christensen (1983), and eq (39) can be converted to eq (6) of Officer and Lynch (1983b).

It is now possible to show that w^* and w are not, in general, identical. To prove this, let us compare eqs (38) and (39) in a situation where all other parameters and variables are the same, that is, $D_B(x,t) = D_B^*(x,t)$, et cetera. Subtracting eq (39) from eq (38) produces

$$\frac{\partial}{\partial x}\left[D_B(x,t)\, C\frac{\partial \phi_s}{\partial x} - \phi_s C(w^* - w) \right] = 0 \qquad (40)$$

Excluding the trivial cases where $C = 0$ and $\phi_s = 0$, and the pathological case where the diffusive term is the same constant for all depths and times, eq (40) states that w^* can equal w only if porosity is constant with depth. Note that w is identical to the burial velocity without mixing, while w^* is a calculated quantity dependent on the intensity of mixing.

As a final point, the solutions to eqs (38) and (39) govern the time averaged behavior of the dependent variables, for example, tracer activity. This fact has important consequences to any comparisons made between model predicted tracer profiles and observational data. Sampling is almost invariably of short duration, Δt_s, if not instantaneous. It is reasonable to expect that the sampling time is considerably shorter than the period for mixing fluctuations, particularly in deep-sea sediments. The observed picture of the sediment can differ from the time-averaged prediction. A profile sampled before a mixing event will look different from a sample immediately after this event. Sampled profiles may have small kinks in them caused by a mixing event. When time-averaged, these kinks disappear. Parameter values extracted from a single observed profile may not be equal of time-averaged values.

A related problem concerns the presence of a steady-state. A steady-state exists if and only if there is an averaging time, Δt_{ss}, such that the integral,

$$C(x) = \frac{1}{\Delta t_{ss}} \int_{t-\Delta t_{ss}/2}^{t+\Delta t_{ss}/2} C(x,t_0)\, dt_0 \qquad (41)$$

yields the same function $C(x)$ for all possible choices of Δt_{ss} within the range

$$\Delta t_b << \Delta t_{av} \leqslant \Delta t_{ss} << \Delta t_r \tag{42}$$

This makes it difficult, if not impossible, to sample the steady-state directly. The validity of eq (41) implies that

$$\frac{\partial C}{\partial t} = 0 \tag{43}$$

which is the traditional definition of the steady-state (Berner, 1980).

TRACER PROFILES PRODUCED BY SPATIALLY-DEPENDENT BIODIFFUSION COEFFICIENTS

The decrease in macrofaunal and meiofaunal abundance with depth in sediments is believed by many to be reflected in a proportional fall in the intensity of bioturbation. Christensen (1982) claims that the population data of Robbins, Krezoski, and Mozley (1977) are sufficiently gaussian that a mixing coefficient of this functionality is appropriate. These data are not unambiguously characterized by a gaussian distribution, and other functions may serve equally well. The population data presented by Jumars (1978), Jumars and Eckman (1983), Thiel (1983), and Germano (ms) could be described using a wide range of mathematical expressions. Not enough data have accumulated to make any definitive statement on the distributional character of infaunal populations, much less on the appropriate form of the bioturbational mixing function.

Even as abundance data accumulate, other factors will have to be considered in formulating the link between biology and mixing functions. The relationship between faunal density and bioturbational intensity may not be a direct one. The dominant activity, that is, feeding, burrowing, resting, et cetera, certainly plays a role in establishing the form and rate of biological reworking at each sediment depth. The relative proportions of these activities are also known to vary with depth (for example, McCall and Tevesz, 1982).

Some depth dependence of the mixing function can, nevertheless, be expected. The prime question is what degree (strength) of depth dependence of $D_B(x)$ is needed to produce tracer profiles appreciably different from those predicted with a constant coefficient, which is almost exclusively assumed. This query may be answered directly by solving the tracer conservation equations for various choices of $D_B(x)$ and comparing the resulting profiles.

The influence of the form of $D_B(x)$ is examined by studying the solution of the tracer conservation equation for a radioisotope ($k(x) = \lambda$) with no *in situ* production, that is, $P(x,t) = 0$, in a sediment with constant porosity and treating D_B as a constant with respect to time. Of these assumptions, that of constant porosity is the least justified. The object of this investigation is, however, the effect of the form of D_B not that of porosity. If only for this reason, this perhaps important complicating factor is ignored. (Note that for intraphase mixing, the porosity could be removed via the transformation proposed by Officer, 1982.) As a result of this assumption, the difference between interphase and intraphase mixing evaporates. The time independence of D_B would be violated in sediments subject to strong seasonal forcings (Aller, 1980), but the need to keep this investigation to a reasonable size precludes consideration of this complicating factor. As a result of these assumptions, the tracer conservation equation becomes

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left[D_B(x)\frac{\partial C}{\partial x} - w\,C \right] - \lambda\,C \tag{44}$$

The analysis of eq (44) is greatly simplified, if the governing equations are first made nondimensional. The nondimensional approach reduces the amount of work necessary to complete the analysis, highlights which combination of parameters truly control the solution and can identify dynamically similar situations. All these points will be demonstrated as the analysis unfolds.

To make eq (44) dimensionless, we note that the depth is expressed in terms of fractions of the mixed layer, $\xi \equiv x/L$. Time is referenced relative to the characteristic diffusion time, that is, $\tau \equiv tD_B(0)/L^2$. A dimensionless concentration, Θ, is defined by dividing C by the initial concentration at the interface for the case where D_B is a constant, designated $C^{\dagger}(0,0)$. The governing equation now reads,

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial}{\partial \xi}\left[f(\xi)\frac{\partial \Theta}{\partial \xi} - Pe\,\Theta \right] - Da(I)\,\Theta \tag{45}$$

valid for $0 \leqslant \xi \leqslant 1$ and where

$$Pe \equiv \frac{wL}{D_B(0)} \tag{46A}$$

$$Da(I) \equiv \frac{\lambda L^2}{D_B(0)} \tag{46B}$$

$$f(\xi) \equiv \frac{D_B(x)}{D_B(0)} \tag{46C}$$

Four parameters, $D_B(0)$, w, λ, and L, are associated with eq (44) while eq (45) contains only two, that is, Pe and Da(I)! Investigation of the influence of the form of $D_B(x)$ while varying four parameters in the dimensional case is a vastly greater effort than studying these effects in nondimensional form with only two parameters. (About n^2 less calculations, if n is the number of parameter values tested.)

Secondly, the dimensional concentration, C, is not a function of the four individual parameters of eq (44). It is, in fact, a function of the two nondimensional parameters groupings Pe and Da(I). To illustrate this point with a simple example, the steady-state solution to eq (44) with $D_B(x) = D_B$ (a constant) is

$$C = A_1\, e^{a_1 x} + A_2\, e^{a_2 x} \tag{47}$$

where A_1 and A_2 are arbitrary constants and

$$\alpha_{1,2} \equiv \frac{w \pm (w^2 + 4D_B\lambda)^{1/2}}{2D_B} \tag{48A}$$

But by dividing D_B into the numerator and multiplying by L, we find that

$$\alpha_{1,2} \equiv \frac{Pe \pm (Pe^2 + 4Da(I))^{1/2}}{2} \cdot \frac{1}{L} \tag{48B}$$

Only the parameter groupings of the nondimensional equation appear in the solution to the dimensional equation. This can be shown to be the case for any analytical form of $D_B(x)$ and also for the time dependent case.

Thirdly, the parameters Pe and Da(I) have special meanings beyond that of the individual dimensional parameters. The parameter Pe is a type of Peclet number (Bear, 1972, p. 600; Parker, Boggs, and Blick, 1974, p. 138; Dominico, 1977). The Peclet number is a measure of the relative influences of burial and biodiffusion in the system under consideration, at least in the vicinity of the sediment-water interface. Large values imply burial dominance, and low values biodiffusional (mixing) dominance. Officer and Lynch (1982) remark that this grouping can be identified as the inverse of the parameter G employed by Guinasso and Schink (1975).

The grouping labeled Da(I) is the bioturbational equivalent to the First Damkohler number of engineering and nonequilibrium thermodynamics (Dominico, 1977). The First Damkohler number is a measure of the influence of decay over the time scale of mixing. Large Da(I) values indicate substantial decay over a time period needed to mix the tracer across a layer of sediment the thickness of the mixed zone. Low values characterize the opposite.

To give the reader some feeling for these parameters, table 2 lists some values calculated from literature cited values of D_B, L, and λ. Note that on average, Pe tends to be smaller than one (mixing dominates over burial), particularly in deep-sea sediments, and Da(I) is usually larger than 1 for the tabulated isotopes.

The nondimensional form of eq (44) is not unique. Eq (45) is just one of at least three possibilities. For example, Guinasso and Schink (1975) use the nondimensional time $T \equiv tw/L$ and obtain the form

$$\frac{\partial\Theta}{\partial T} = \frac{\partial}{\partial\xi}\left[\frac{f(\xi)}{Pe}\frac{\partial\Theta}{\partial\xi} - \Theta\right] - Da(II)\,\Theta \tag{49}$$

where

$$Da(II) \equiv \frac{\lambda L}{w} \tag{50}$$

The Second Damkohler number, Da(II) (see Dominico, 1977), relates the influence of decay on the tracer distribution over a time period needed to deposit (or bury) a layer of sediment the thickness of the mixed zone. A large value indicates substantial decay, and a small number the converse.

TABLE 2

Values of dimensionless parameters for various natural environments

Location	Source	Isotope	Pe	Da(I)	Da(II)
Long Island Sound	Aller, Benninger, and Cochran (1980)	^{234}Th	0.01-5.0	5-878	262
	Krishnaswami and others (1980)	^{7}Be	0.03-0.1	5-14	71.6
	Benninger and others (1979)	^{210}Pb	0.33-7.5	0.3-12	1.4
Naragansett Bay	Santschi and others (1980)	^{234}Th	0.002-0.018	9-171	7375
		^{210}Pb	0.12-0.95	11-87	96
New York Bight (cores SBC-1 & SBC-5)	Santschi and others (1980)	^{234}Th	0.007	55.4	7914
		^{210}Pb	0.15	14.2	95
		^{234}Th	0.26	148	57
		^{210}Pb	3.3	297	90
NW Pacific Shelf	Carpenter, Peterson, and Bennett (1982)	^{210}Pb	0.07-7.0	0.07-13	1.0-3.0
North Atlantic	Turekian, Cochran, and DeMaster (1978)	^{210}Pb	0.016	16.6	1037
Antarctic	(same)	^{210}Pb	0.0013-0.27	8-67	360
North Pacific	(same)	^{210}Pb	0.004	4.6	1150
Western Pacific	Peng, Broecker, and Berger (1979)	^{210}Pb	0.067	17	251
Equatorial Pacific	DeMaster and Cochran (1982)	^{210}Pb	0.005	7	1400
		^{32}Si	0.023	5	213
Antarctic	(same)	^{210}Pb	0.025	79	3152
Deep-Sea Carbonate Sediment	(composite)	^{14}C	$\sim$0.01	$\sim$0.1	$\sim$1

This non-uniqueness should not be disturbing, because the two approaches shown here are not independent. The three parameter groupings Pe, Da(I), and Da(II) are interrelated,

$$\mathrm{Da(II)} = \frac{\mathrm{Da(I)}}{\mathrm{Pe}} \tag{51}$$

The first treatment is probably better, if the system is mixing dominated, that is, low Pe values.

The influence of $f(\xi)$ on the solution of eq (45) is now examined for both steady-state and transient conditions.

Steady-state solutions.—The boundary condition at the top of the sediments ($\xi = 0$) is a fixed flux condition.

$$\mathrm{Pe}\,\Theta - \frac{d\Theta}{d\xi} = \Psi \tag{52}$$

where Ψ is the dimensionless input flux of the tracer at the sediment-water interface, that is, $f_c L/D_B(0)/(1 - \phi(0))/C^{\dagger}(0,0)$ where f_c is the (dimensional) input. In practice, Ψ is set equal to 1 in all cases. This particular choice does not affect the findings of this study and is quite convenient.

The boundary condition at the base of the mixed layer depends on the behavior of $f(\xi)$. If $f(\xi) \neq 0$, then the appropriate condition is (Danckwerts, 1953; Wehner and Wilhelm, 1956; Pearson, 1959; Bischoff, 1961)

$$\frac{d\Theta}{d\xi} = 0 \tag{53}$$

and if $f(\xi) = 0$ then,

$$\Theta = \text{finite as } \xi \rightarrow 1 \tag{54}$$

The steady-state form of eq (45),

$$\frac{d}{d\xi}\left[f(\xi)\frac{d\Theta}{d\xi} - \text{Pe}\,\Theta \right] - \text{Da(I)}\,\Theta = 0 \tag{55}$$

was solved analytically for five different forms of $f(\xi)$, that is, $f(\xi) = 1$, $1 - \xi^2$, $1 - \xi$, $(1 - \xi)^2$, and $\exp(-(\xi/(1 - \xi))^2)$, where this last function is called a pseudo-gaussian.

Although natural biodiffusion is undoubtedly more complex, the test functions constitute a mathematically well-defined sequence of progressively stronger decaying functions. In principle, it is possible to determine if observed mixing intensities decay faster or slower than the tested forms.

Analytical solutions of eq (55) for the first three functions of this sequence are displayed in table 3. (The solution methods are detailed in Boudreau (ms). A solution for $f(x) = e^{-ax}$ on the interval $0 \leqslant x < \infty$ which includes the burial term omitted by Nozaki (1977) is also reported by Boudreau, ms.)

The more exotic depth functionalities require the use of numerical methods of solution. Consequently, a finite-difference scheme to solve eq (55) was devised for an arbitrary $f(\xi)$. The scheme is detailed in the appendix. It has two noteworthy features. First, it contains a parameter that allows the scheme to switch smoothly from a centered-difference formulation when diffusion is dominant ($f(\xi) \sim 1$ and $\text{Pe} < 1$) to an upstream-difference formulation when advection is dominant ($f(\xi) \sim O(0)$ or $\text{Pe} > 1$). Secondly, unlike the methods of Peng, Broecker, and Berger (1979), Santschi and others (1980), Olsen and others (1981), and Kadko and Heath (1984), the scheme solves the two-point boundary value problem, presented by eq (55), directly and not as the long time solution to an initial value problem, a process that may be unstable if $f(\xi)$ is sufficiently small. The scheme used here offers a rather substantial savings in computational effort and appears stable.

Figures 2 through 4 illustrate the tracer profiles generated by the mixing functions stated above for $\text{Pe} = 0.001$, 0.1, 10, and $\text{Da(I)} = 0.1$ to 1000. (Four other intervening values of Pe were also considered, but the

TABLE 3

Mixing function	Solution to eq (55)	Definition of constants and functions	Sources of information and calculation algorithms
Constant	$\Theta = A_1\, e^{a\xi} + A_2\, e^{b\xi}$	$a \equiv (Pe - (Pe + 4Da)^{1/2})/2$ $b \equiv (Pe + (Pe + 4Da)^{1/2})/2$ $A_1 = \Psi[Pe(1 - ae^{(a-b)}/b) - a(1 - e^{(a-b)})]^{-1}$ $A_2 = -(A_1 ae^{(a-b)})/b$	See any standard text
$1 - \xi$	$\Theta = \dfrac{\Psi\, \eta^{-Pe/2}\, I_{Pe}\, (2(Da\, \eta)^{1/2})}{(Da)^{1/2}\, I_{Pe-1}\, (2Da^{1/2})}$	$\eta \equiv 1 - \xi$ $I_{Pe} \equiv$ modified Bessel function of the first kind of order Pe	Olver (1972) for Bessel function Boudreau (ms) for derivation Luke (1977), Cody (1983), or Campbell (1981) for algorithms
$(1 - \xi)\,(1 + s\xi)$ $s > 0$	$\Theta = A_3F\, (a;\, b;\, c;\, \epsilon)$	$\epsilon \equiv s(1 - \xi)/(1 + s)$ $\epsilon_0 \equiv s/(1 + s)$ $F\, (a;\, b;\, c;\, \epsilon) \equiv$ Gauss' hypergeometric function † $a \equiv (1 - (1 - 4Da/s)^{1/2})/2$ † $b \equiv Da/s/a$ † $c \equiv (Pe + 1 - s)/(1 + s)$ $A_3 = \Psi\, [Pe\, F(a;\, b;\, c;\, \epsilon_0) + \epsilon_0 ab/c\, F(a + 1;\, b + 1;\, c + 1;\, \epsilon_0)]^{-1}$	Erdelyi (1953), Lebedev (1965), Oberhettinger (1972) for the properties of $F(a;\, b;\, c;\, \epsilon)$ Boudreau (ms) for derivation Luke (1977) for an algorithm † These constants often have imaginary values. Luke's algorithm allows for this. Boudreau (ms) suggests a simple scheme to avoid complex arithmetic.
$(1 - \xi)\,(1 - s\xi)$ $0 < s < 0.5$	$\Theta = A_4F\, (a';\, b';\, c';\, \epsilon')$	$\epsilon' \equiv -s(1 - \xi)/(1 - s)$ $a' \equiv (1 + (1 + 4Da/s)^{1/2})/2$ $b' \equiv -Da/s/a'$ $c' \equiv (Pe + 1 - s)/(1 - s)$ $A_4 = A_3$ w/primed variables	See the references given in previous case
$(1 - \xi)\,(1 - s\xi)$ $0.5 < s < 1$	(A complex solution involving hypergeometric functions.)		See Boudreau (ms)

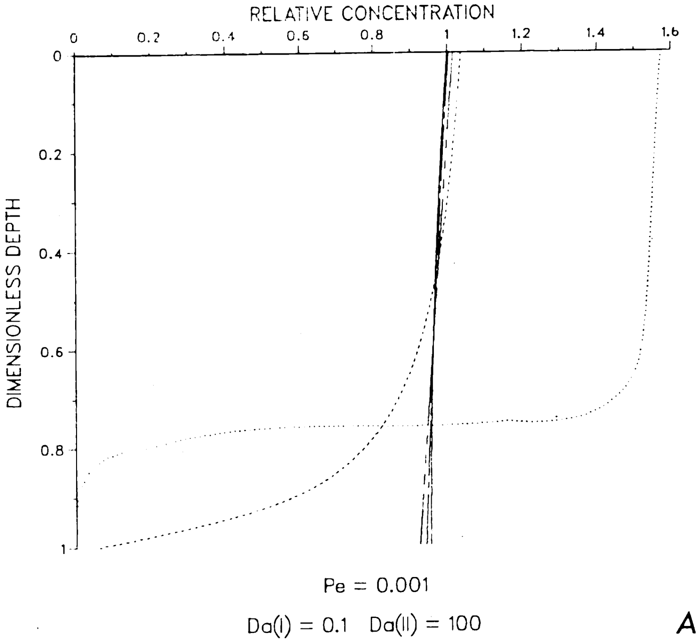
general trends are adequately displayed in the figures presented here.) For the majority of the plots, there is a great similarity of the curves regardless of the form of f(ξ) and particularly for small and large Da(I) values and large Peclet numbers. The pseudo-gaussian based curves with Da(I) $\leqslant$ 10 and Pe = 0.001 and 0.1 do display appreciable deviations at depth from the trends set by the other curves. (Plots with Da(I) less than 0.1 are not shown because these profiles are essentially superimposed subvertical lines.)

Profiles with large Da(I) values (isotopes that are short-lived on the time scale of mixing) are virtually identical, because the tracer disappears at shallow depths before it can "feel" f(ξ) decay. For small Da(I) values (that is, long lived isotopes compared to the mixing time), the profiles are insensitive to the decrease in the intensity of mixing. So little of the long lived isotopes decays that even the small amounts of mixing near the base of the mixed layer are sufficient to keep the sediment essentially homogenized with respect to these isotopes. Profiles with large Pe values also exhibit little dependence on the form of the mixing function as biodiffusion is relatively less important than burial in dictating the shapes of these profiles. Therefore, there exists only a narrow window of parameter values, that is, Pe < 1.0, 0.3 $\leqslant$ Da(I) $\leqslant$ 3, in which the profiles exhibit any deviation from a common behavior, and this is restricted to profiles generated by mixing functions that decay faster than $(1 - \xi)^2$.

Identification of the form of the mixing function solely from observed tracer profiles will prove to be difficult. In the above calculations, the mixed layer depth, L, is assumed known. This is usually an additional unknown. It has been suggested that long lived isotopes, or more precisely small Da(I)-valued isotopes, should give this information. However, figure 2 shows that a small Da(I)-valued isotope would suggest a shallower total mixing depth than the actual value of L, if the unknown mixing function decays as steeply as a pseudo-gaussian, that is, $\xi = 0.8$ rather than 1.0. (If the mixing depth is assumed shallower than the true value, the assumed mixing function need not decrease as strongly as the real f(ξ). From table 2, interpretation of ^{14}C is susceptible to this error.)

Another practical problem with identification of the mixing function is the quality and quantity of data. The errors associated with radiochemical measurements and the inaccuracies and coarseness of the sampling are often such that almost any function could fit the data. Unless the quality of the data warrants more effort, the assumption of constant D_B over the mixing depth is all that can truly be justified.

Perhaps the best approach in extracting mixing functions is to utilize information on the depth distribution of biological populations and activities to infer L and f(x), then to verify and quantify these choices by comparing the predicted and observed profile of a number of isotopes with at least one in the range 0.3 $\leqslant$ Da(I) $\leqslant$ 3.0. A shorter lived isotope would be useful in determining the value of $D_B(0)$.



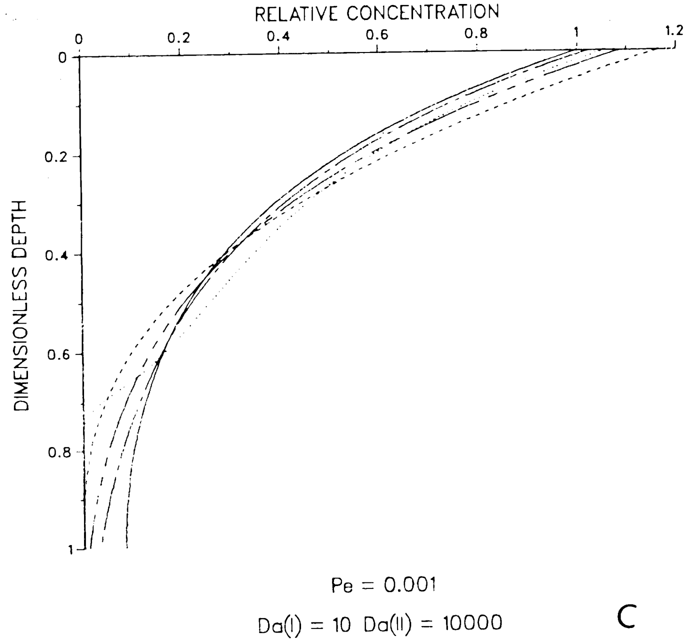
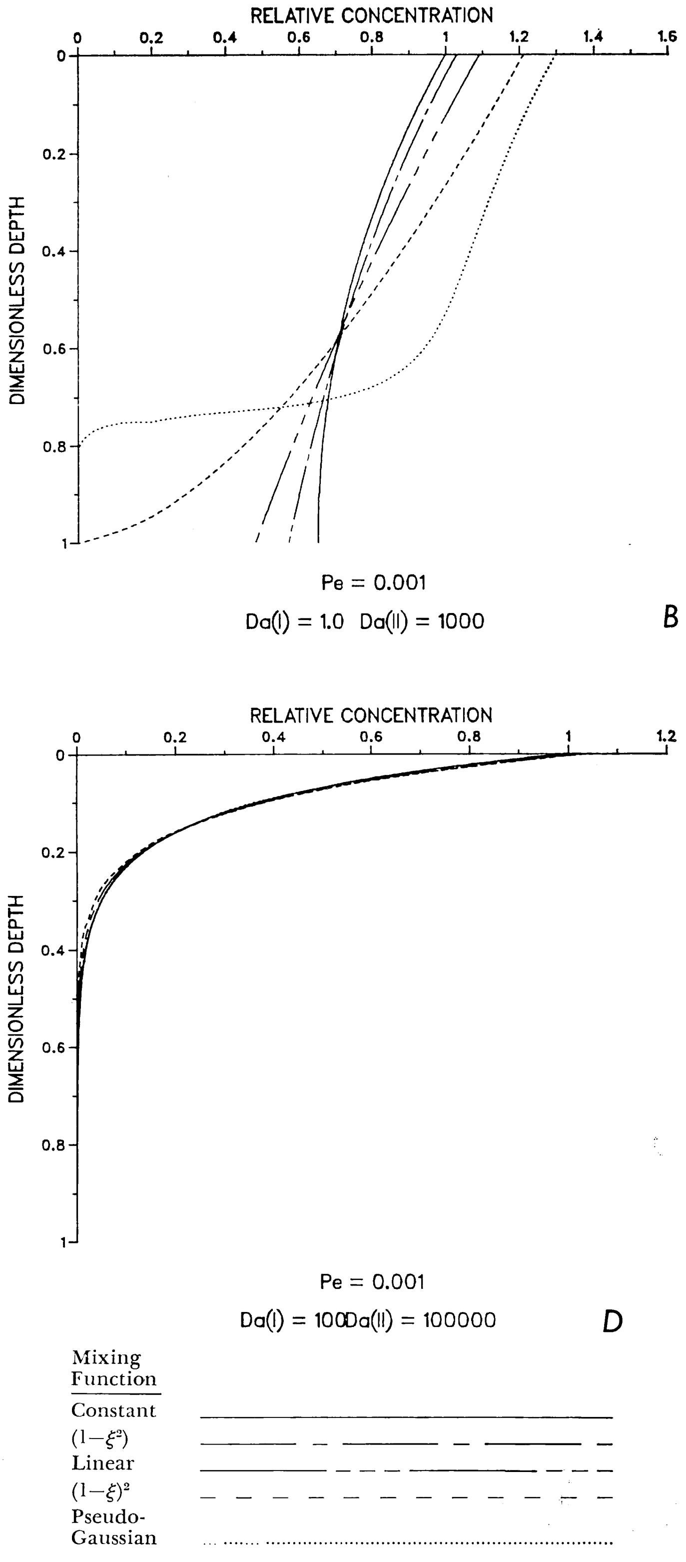
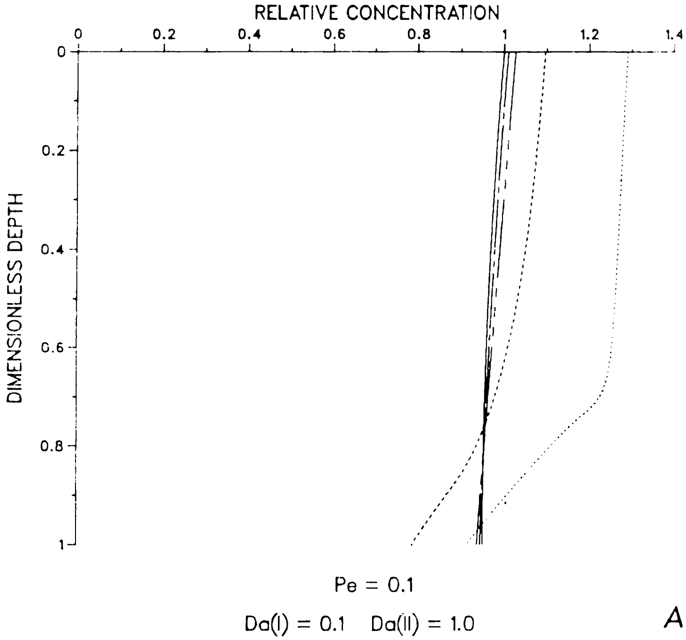


Fig. 2. Diagrams A, B, C, and D. Dimensionless tracer profiles produced by various depth-dependent diffusivities for the conditions Pe = 0.001 (diffusion dominates strongly over advection, at least near the sediment-water interface) and Da(I) values ranging from 0.1 (slowly decaying tracer) to 100 (rapidly decaying tracer). The corresponding values of Da(II) are also shown. (Please ignore the small kinks in the pseudo-gaussian curve on diagrams A and B. They are unfortunate artifacts of the plotting technique.) Key: (ξ = depth fraction in mixed layer).


RELATIVE CONCENTRATION
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
DIMENSIONLESS DEPTH
0
0.2
0.4
0.6
0.8
1
Pe = 0.001
Da(I) = 1.0 Da(II) = 1000
B
RELATIVE CONCENTRATION
0
0.2
0.4
0.6
0.8
1
1.2
DIMENSIONLESS DEPTH
0
0.2
0.4
0.6
0.8
1
Pe = 0.001
Da(I) = 100Da(II) = 100000
D
Mixing Function
Constant
(1−ξ²)
Linear
(1−ξ)²
Pseudo-Gaussian



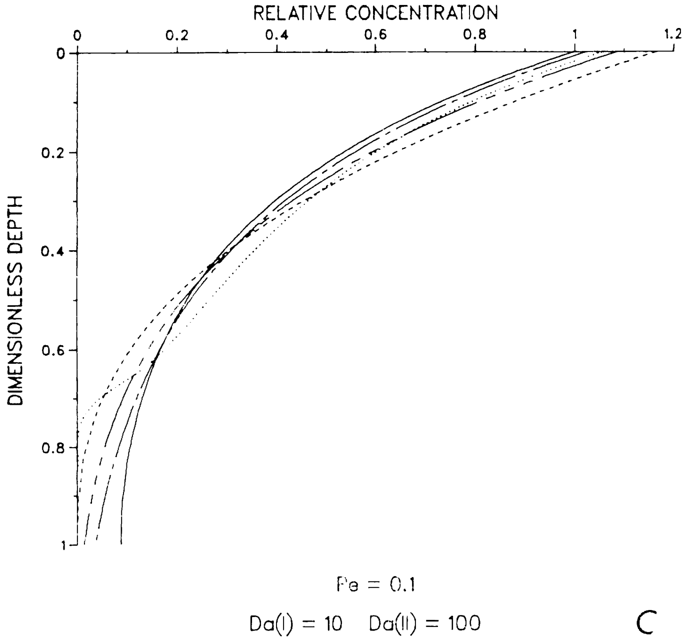
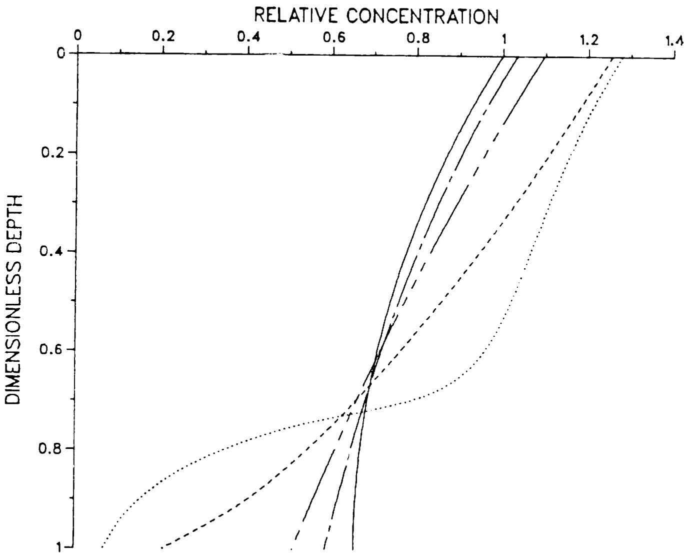


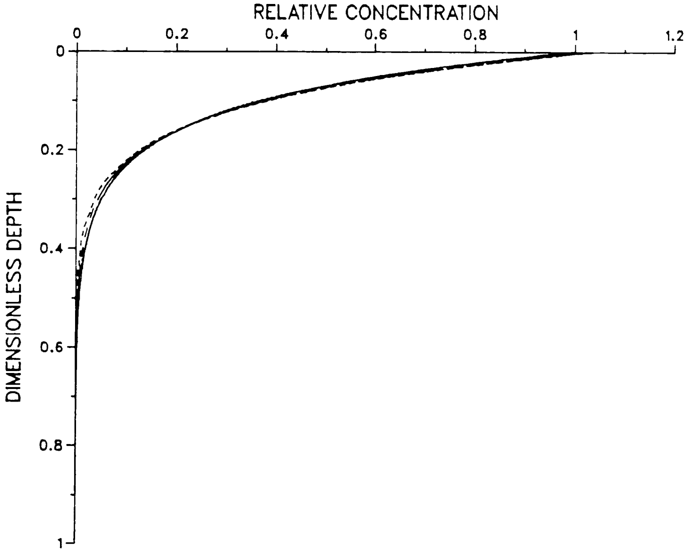
Fig. 3. Diagrams A, B, C, and D. Dimensionless tracer profiles produced by various depth-dependent diffusivities for the conditions Pe = 0.1 (diffusion dominates moderately over advection, at least near the sediment-water interface) and Da(I) = 0.1 to 100 (with corresponding values of Da(II).). Key: (ξ = depth fraction in mixed layer).



Pe = 0.1

Da(I) = 1.0 Da(II) = 10

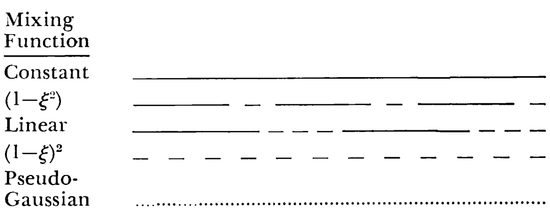
B

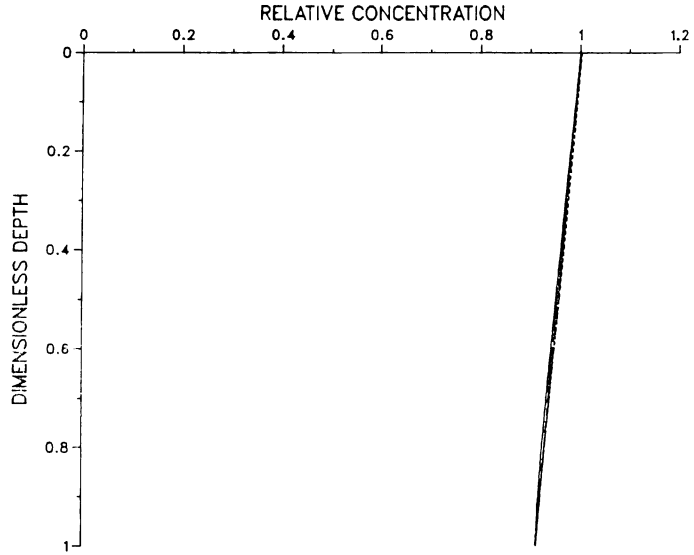


Pe = 0.1

Da(I) = 100 Da(II) = 1000

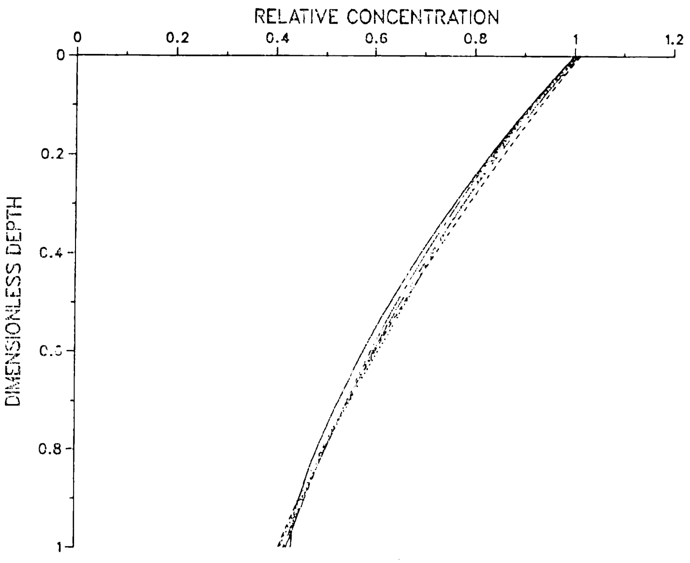
D





Pe = 10

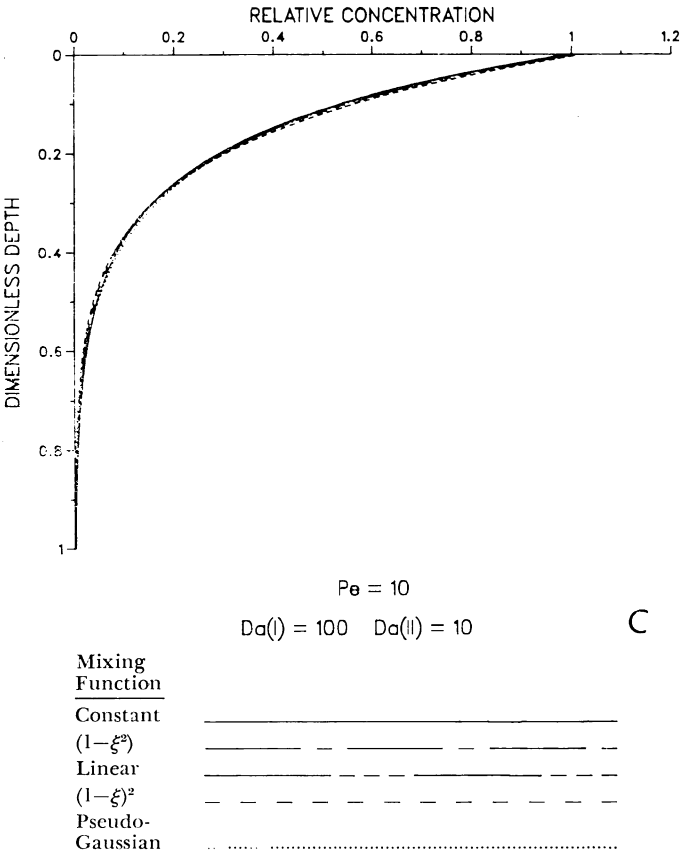
Da(I) = 1.0 Da(II) = 0.1 **A**



Pe = 10

Da(I) = 10 Da(II) = 1.0 **B**

Fig. 4. Diagrams A, B, and C. Dimensionless tracer profiles produced by various depth-dependent diffusivities for the conditions Pe = 10 (advection dominates over diffusion) and Da(I) = 1.0 to 100 (with corresponding Da(II) values). Key: (ξ = depth fraction in mixed layer).



No isotope is inherently included in the range of sensitive Da(I)-values, 0.3 to 3.0. This is a dynamic condition that is a function of the environmental parameters L (mixing length) and the initial intensity of mixing, $D_B(0)$, and the decay constant of the isotope, λ. To illustrate this point, let us consider three examples. The first concerns a near shore sediment with $w = 0.1$ cm/yr, $L = 10$ cm, and $D_B(0) = 1.0$ to 10 cm²/yr, that is, $Pe = 0.1$ to 1.0. (This hypothetical situation is not unlike the FOAM Site (Goldhaber and others (1977).) If the optimum isotope has a Da(I) value of 1, then λ must be between 10^{-2} and 10^{-1}/yr. Silicon-32 and ^{210}Pb are both useful in this situation.

As a second example, another near shore sediment has $w = 0.01$ cm/yr, $L = 10$ cm, and $D_B(0) = 30$ cm²/yr, that is, $Pe = 0.003$ (Benninger and others, 1979). The optimum isotope should have a decay constant between 0.1 and 1.0/yr. These values bracket ^{228}Th ($\lambda = 0.361$/yr, Koide, Bruland, and Goldberg, 1973), although ^{210}Pb may still be useful. Benninger and others (1979) did not measure ^{228}Th, but they did obtain a ^{210}Pb profile. Their sampling interval is probably too large to provide any detailed information on the mixing function, but they did find that a two step mixing function provided a better fit to their data than a constant.

Finally, consider a deep-sea sediment with $w = 0.001$ cm/yr, $L = 10$ cm, and $D_B(0) = 0.1$ cm^2/yr, that is, $Pe = 0.1$, which is typical of carbonate sediments. The optimum isotope has a λ of 1×10^{-3}/yr or about a 700 yr half-life. There is no known isotope of this half-life that meets the requirements of particle reactivity and steady input necessary to be modelled in the manner used in this section. Silicon-32 may exhibit some depth dependent behavior, but this may not be diagnostic.

One comforting thought in these results is that, at least at steady-state, the dissimilitudes produced by different mixing functions are not so great as to discredit the result of past studies. For many purposes, the differences between the mixing functions are within the errors in the measurements.

Transient tracers.—The aim in this section is to investigate the effect of the depth dependence of the diffusivity on the time evolution of an impulse of tracer as studied by Guinasso and Schink (1975). The dimensionless conservation equation in this case reads,

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial}{\partial \xi}\left[f(\xi) \frac{\partial \Theta}{\partial \xi} - \mathrm{Pe}\, \Theta \right] \tag{56}$$

with the initial condition (that is, tracer distribution),

$$\Theta(\xi, 0) = \delta(0) \tag{57}$$

where $\delta(0)$ is a Dirac delta function, that is, an impulse (see Arfken, 1970, or Guinasso and Schink, 1975, for some details on the properties and the interpretation of this function).

The boundary condition at $\xi = 0$ is

$$\mathrm{Pe}\, \Theta - \frac{\partial \Theta}{\partial \xi} = 0 \tag{58}$$

and we again have either eq (53) in partial derivative form (Wooding, 1972) or eq (54), dictated by the form of $f(\xi)$.

A small set of mixing functions is presented here. Specifically, they are the constant, the linear decay, and the pseudo-gaussian function. The solution with $f(\xi) = 1$ is detailed in Guinasso and Schink (1975), but as pointed out by Officer and Lynch (1982), it was previously available in Carslaw and Jaeger (1959). Because the solution is readily available, it is not repeated here.

The solution with $f(\xi) = 1 - \xi$ is not generally available to this writer's knowledge. A separation of variable methods of solution is presented in Boudreau (ms)[1] with a final result given by

$$\Theta = (1 - \xi)^{-\mathrm{Pe}/2} \sum_{n=1}^{\infty} A_n\, J_{\mathrm{Pe}}\left(\mu_n (1 - \xi)^{1/2}\right) e^{-\beta_n^2 \tau} \tag{59}$$

---

[1] Note that eq (E28b) of Boudreau (ms) has a typographical error. The bar in front of the r.h.s. is not a negative, but the fraction bar for 1/2. The numerator and denominator were inadvertently omitted.

where $J_{Pe}(\ ) \equiv$ Bessel function of the first kind of order Pe (for an algorithm, see Campbell, 1979), $\mu_n \equiv$ n-th root of $J_{Pe-1}(\mu)$ (see Olver (1960), Hall (1967), and Piessens (1984) for methods of calculating μ_n),

$$A_n = \frac{1}{J_{Pe}(\mu_n)} \tag{60}$$

and

$$\beta_n^2 \equiv \frac{\mu_n^2}{4} + Da(I) \tag{61}$$

The pseudo-gaussian again requires the use of numerical methods of solution. A modification of the steady-state, finite-difference scheme produces a formulation akin to the famed Crank-Nicholson scheme (Crank and Nicholson, 1947). The details are provided in the appendix.

Figure 5 compares a time sequence of tracer profiles produced with the constant (solid lines) and linear (dotted lines) diffusivities for three different Peclet numbers. Counting along the x-axis, there are five labelled curves on the first plot. The numbers increase right to left and correspond to dimensionless times of $\tau = 0.001$, 0.004, 0.01, 0.05, and 0.1. The plot for $Pe = 0.1$ has a sixth curve on the far left hand side corresponding to $\tau = 0.5$. To get a feel for the times involved: for deep-sea conditions, $D_B = 0.1$ cm²/yr and $L \approx 10$ cm, so that a $\tau = 0.1$ is equivalent to 100 yrs.

Early time profiles ($\tau \leqslant 0.01$) are essentially identical as the pulse has not penetrated deep enough in this short time to detect a drop-off in $f(\xi)$. Even for longer time the deviation is not great and probably below analytical detection. As in the steady-state case, a linear depth decay in $f(\xi)$ is simply not strong enough to be differentiated from a constant mixing function. Investigation with other mixing functions, though not shown here, indicates that the decay must be stronger than quadratic before the tracer profiles diverge (as in the steady-state case).

Figure 6 compares the constant diffusivity (solid lines) and the pseudo-gaussian (dotted lines). As in the previous case, early time profiles are indistinguishable. This unfortunately is also the time when the profiles are easiest to analyze. For longer times, that is, $\tau > 0.01$, two types of behavior can be discriminated. For large Pe numbers (strong advection compared to mixing), the near coincidence continues until the pulse has left the mixed layer. For small Pe values (weak advection compared to mixing), a divergence develops which is greater than in the linear case. The disappearance of diffusive transport as $\xi \rightarrow 1$ and the weak advection combine to trap the tracer in the mixed layer. Perhaps the integrated amount of tracer in the mixed layer or other statistical properties of the tracer profile both within and below the mixed layer can be used to distinguish mixing functions for small Pe. Again, it will probably be necessary to use biological information to infer L and f(x), if the method is to be successful.

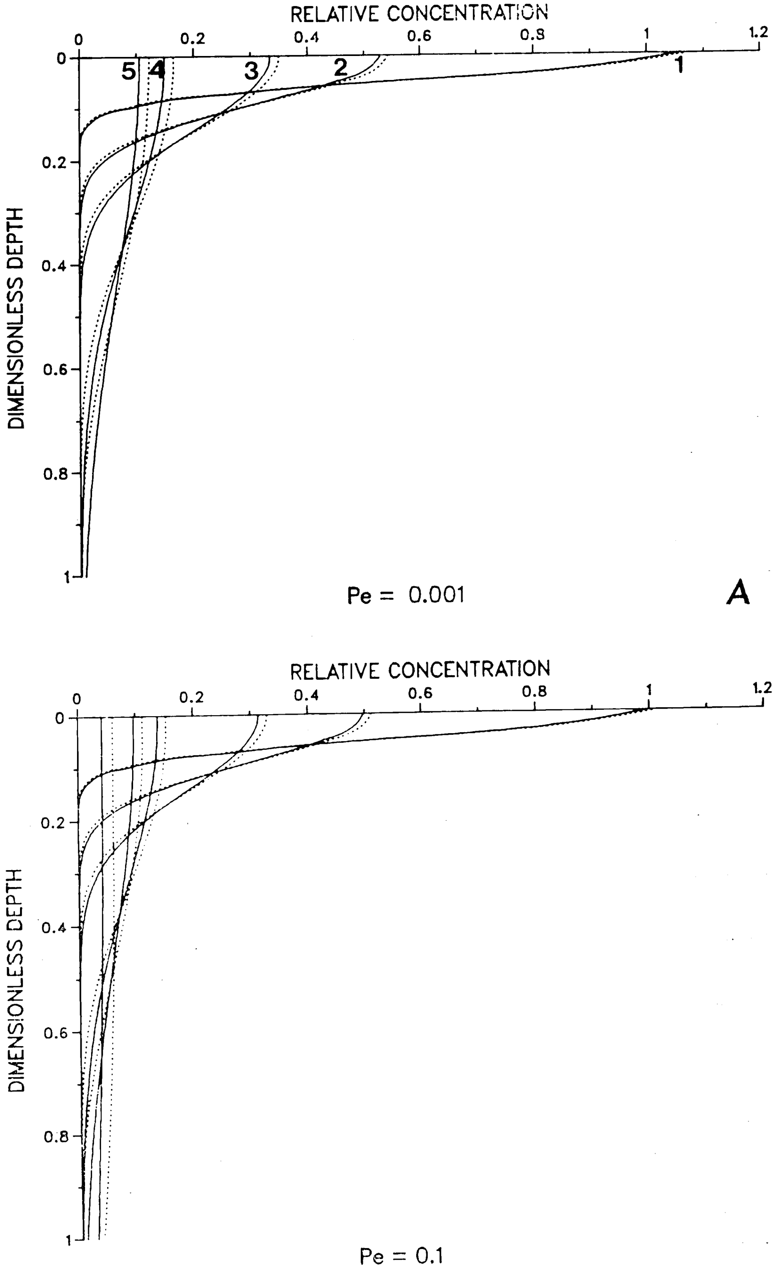


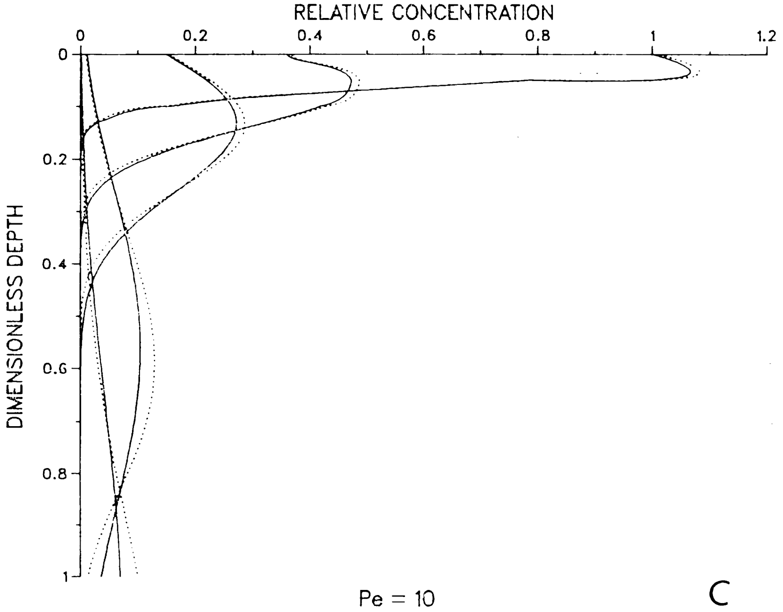
Fig. 5. Diagrams A, B, and C. Transient tracer profiles produced by constant (solid lines) and linear (dotted lines) diffusivities responding to an impulse source. The diagrams are for different Peclet numbers, with low Pe values for diffusion domination (at least in the vicinity of the sediment-water interface) and high values indicative of ad-

CONCLUSIONS

The use of a diffusion-type model for bioturbation implicitly assumes both a spatial and temporal continuum for variables and processes (parameters). If these continua are to be realistic representations of the actual fields, then the length and time scale conditions given by eqs (4) and (18), respectively, must be fulfilled. The validity of these inequalities permits spatial and time averaging which produces the canonical forms of the conservation equations for a tracer in marine sediments.

The observed decrease of the benthic macrofaunal population with depth in the sediment strongly suggests that the biodiffusion coefficient should be a function of depth. Calculations of steady-state tracer profiles for a variety of decaying mixing functions subject to a wide range of burial velocities and mixing intensities show that these profiles become sensitive to the decrease in mixing only if the mixing function diminishes faster than a quadratic and the tracer fulfills the condition that $0.3 \leqslant \mathrm{Da(I)} \leqslant 3.0$, with $\mathrm{Pe} \leqslant 1.0$.

The time evolution of an impulse source of a conservative tracer in a sediment is sensitive to the mixing function only if $\mathrm{Pe} < 10$, f(x) decreases at a rate faster than a quadratic, and only for times greater than $0.01 \cdot L^2/D_B(0)$.



vection domination. In the first diagram, the curves are labelled 1 to 5 to indicate the dimensionless time, that is, $\tau = 0.001$ for curve 1, $\tau = 0.004$ for curve 2, $\tau = 0.01$ for curve 3, $\tau = 0.05$ for curve 4, and $\tau = 0.1$ for curve 5. Diagram B has an extra curve on the far left corresponding to $\tau = 0.5$.

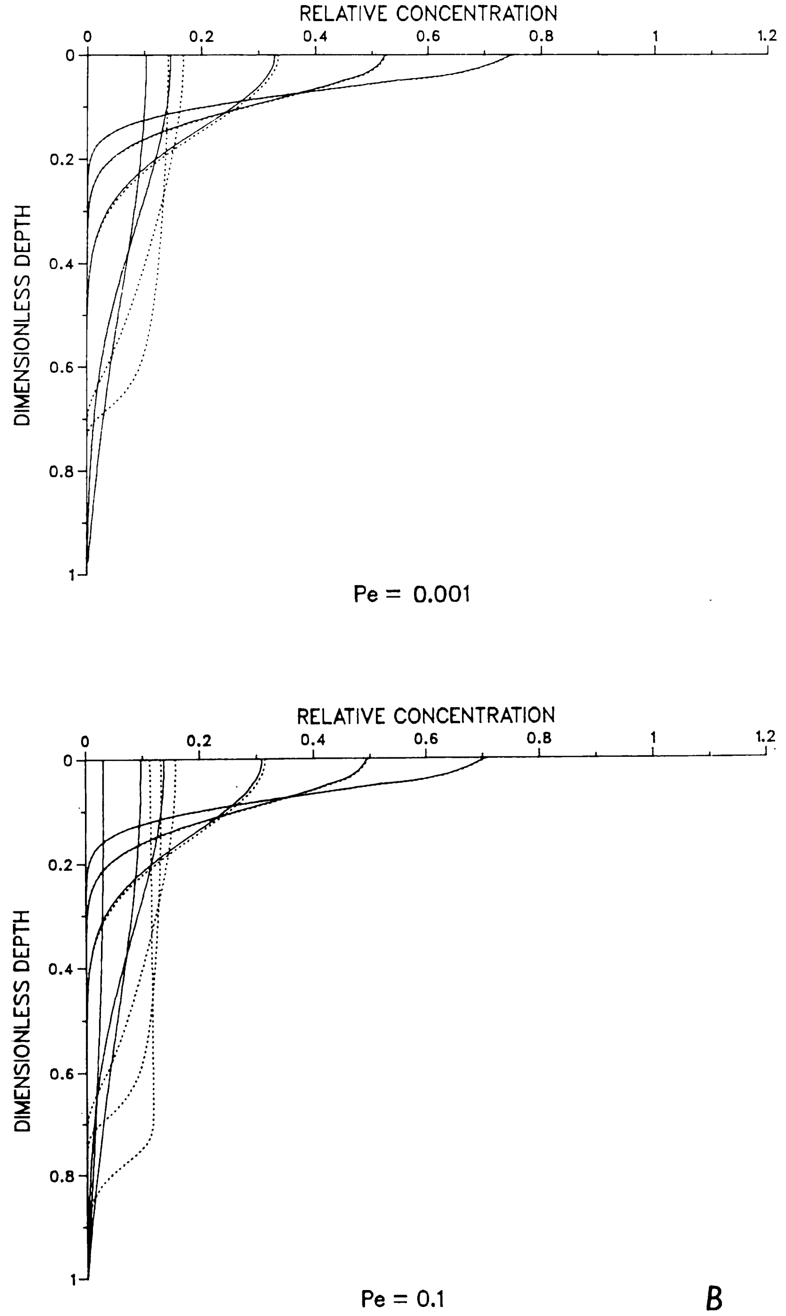
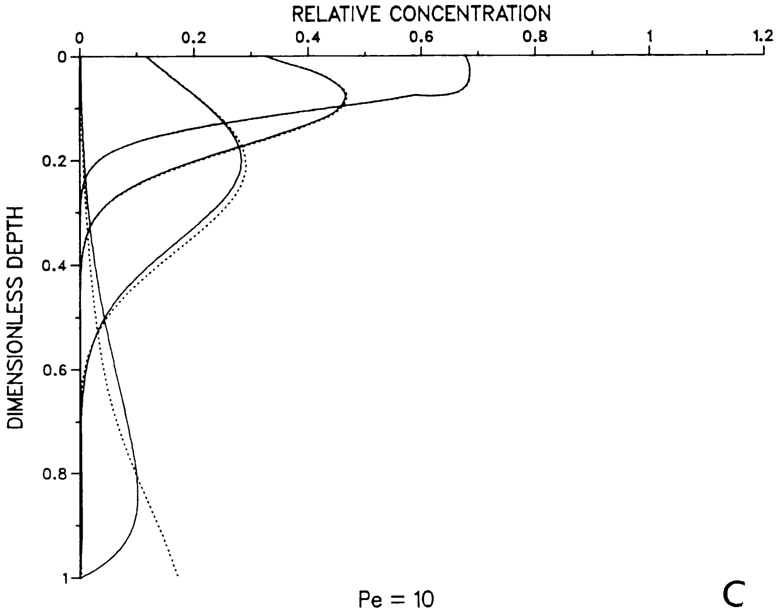


Fig. 6. Diagrams A, B, and C. Same as figure 5, but comparing the constant (solid lines) and pseudo-gaussian (dotted lines).



Tracer distributions are probably most useful in determining and quantifying mixing functions if the form and mixing depth can first be inferred from biological data.

The similarities in the tracer profiles produced by different depth dependent mixing functions suggest that a constant diffusivity or a two layer model with differing but constant diffusivities may be all that can be justified in light of the coarse spacing and large errors associated with past radiochemical data. For isotopes outside the window of sensitive parameter values, the data may never be of sufficient accuracy to warrant the use of realistic but more complex mixing functions.

Note added in press: After submission of the final version of this manuscript, the author discovered the excellent paper by Corrsin (1974). This paper deals with the limitations of the eddy-diffusion model in modelling turbulent fluxes. Many of the points I have discussed can also be found in Corrsin's work.

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Appendix

Finite-difference schemes

A (1) Steady-state.—The form of the tracer conservation equation "discretized" by the finite-difference scheme is

$$f(\xi)\frac{d^2\Theta}{d\xi^2}+\left[\frac{df(\xi)}{d\xi}-\mathrm{Pe}\right]\frac{d\Theta}{d\xi}-\mathrm{Da}\,\Theta=0 \tag{A1}$$

where, for simplicity, $\mathrm{Da} \equiv \mathrm{Da(I)}$. This may not be the best choice of forms, but it appears adequate as numerical and analytical solutions for test cases usually agree to far better than 3 percent.

Because diffusion is a symmetric process (Fiadeiro and Veronis, 1977), the diffusion term is approximated by the central-difference formula

$$\frac{d^2\Theta}{d\xi^2}\cong\frac{\Theta_{i-1}-2\Theta_i+\Theta_{i+1}}{h^2} \tag{A2}$$

where the subscript on C indicates the node position on the finite-difference grid and h is the distance between nodes.

Eq (A1) also contains a first derivative term. The finite-difference formula for the first derivative which formally has the same order of accuracy as eq (A2) is the central-difference formula (Mitchell and Griffiths, 1980; Ozisik, 1980)

$$\frac{d\Theta}{d\xi}=\frac{\Theta_{i+1}-\Theta_{i-1}}{2h} \tag{A3}$$

Spalding (1972), Barrett (1974), Chien (1977), Fiadeiro and Veronis (1977), Heinrich, Huyakorn, and Zienkiewicz (1977) all observe that, for large Peclet numbers, advection is a strong directional process, and that the central-difference scheme, eq (A3), becomes inaccurate and can even be instable. They note that for large Pe, the upstream or backward-difference formula produces good results (Ozisik, 1980),

$$\frac{d\Theta}{d\xi}=\frac{\Theta_i-\Theta_{i-1}}{h} \tag{A4}$$

but that it is formally of accuracy O(h) (see also Gupta, 1983; Shyy, 1985; Patel, Markatos, and Cross, 1985).

The diffusivity, $f(\xi)$, goes from unity to zero on the interval [0,1]. The local Peclet number

$$\epsilon=\frac{wh}{D_B(0)\,f(\xi)} \tag{A5}$$

also varies on the same interval from some finite value at $\xi = 0$ to an infinitely large value at $\xi = 1$. The central-difference formulation becomes unstable under these conditions (Heinrich, Huyakorn, and Zienkiewicz, 1977; Siemieniuch and Gladwell, 1978; Morton, 1980).

To overcome these difficulties, a weighted differencing scheme (Fiadeiro and Veronis, 1977) is used that generates the central-difference when ϵ is small and the upwind-difference when ϵ is large, that is,

$$\frac{d\Theta}{d\xi}=\frac{(1-\sigma_i)\Theta_{i+1}+2\sigma_i\Theta_i-(1+\sigma_i)\Theta_{i-1}}{2h} \tag{A6}$$

where

$$\sigma_i\equiv\operatorname{cotanh}(\epsilon/2)-\epsilon/2 \tag{A7}$$

The weighing parameter, σ_i, approaches zero as ϵ approaches zero and one when ϵ approaches infinity.

Combining these results, the finite-difference equivalent to eq (A1) reads,

$$\begin{aligned}&\left[\frac{f(\xi_i)}{h}+\left[\frac{df(\xi)}{d\xi}\bigg|_{\xi_i}-\mathrm{Pe}\right]\frac{(1-\sigma_i)}{2}\right]\Theta_{i+1}\\&-\left[\frac{2f(\xi_i)}{h}-\left[\frac{df(\xi)}{d\xi}\bigg|_{\xi_i}-\mathrm{Pe}\right]\sigma_i-h\,\mathrm{Da}\right]\Theta_i\\&+\left[\frac{f(\xi_i)}{h}-\left[\frac{df(\xi)}{d\xi}\bigg|_{\xi_i}-\mathrm{Pe}\right]\frac{(1+\sigma_i)}{2}\right]\Theta_{i-1}=0\end{aligned} \tag{A8}$$

In practice, forty nodes were employed.

The boundary condition at $\xi = 0$, eq (52), must also be made discrete. A central-difference for the derivative involves the concentration at the exterior point $i = -1$, that is,

$$\mathrm{Pe}\,\Theta_0 - \frac{1}{2h}(\Theta_1 - \Theta_{-1}) = \Psi \tag{A9}$$

This value at the exterior point can be eliminated quite easily (for example, Nogotov, 1978, p. 72-75). If we apply eq (A8) at $\xi = 0$, that is, $i = 0$, then we obtain,

$$\Theta_{-1} = \left[\frac{1}{h^2} - \left[\left.\frac{df(\xi)}{d\xi}\right|_0 - \mathrm{Pe}\right]\frac{(1+\sigma_0)}{2h}\right]^{-1} \cdot$$

$$\left[\left[\frac{2}{h^2} - \left[\left.\frac{df(\xi)}{d\xi}\right|_0 - \mathrm{Pe}\right]\frac{\sigma_0}{h} + \mathrm{Da}\right]\Theta_0\right.$$

$$\left. - \left[\frac{1}{h^2} + \left[\left.\frac{df(\xi)}{d\xi}\right|_0 - \mathrm{Pe}\right]\frac{(1-\sigma_0)}{2h}\right]\Theta_1\right] \tag{A10}$$

This equation can be substituted into eq (A9) to eliminate Θ_{-1} and produce a difference formula of $O(h^2)$ accuracy.

No special precautions need be taken at the point $\xi = 1$, and the condition there is simply the governing equation itself.

A (2) Transient state.—In the transient-state, both space and time derivatives must be approximated. The approach used here has been to treat the spatial derivatives in the same way as in the steady-state case and to approximate the time derivative by the forward-difference formula,

$$\frac{\partial\Theta}{\partial\tau} = \frac{{}^{n+1}\Theta_i - {}^{n}\Theta_i}{k} \tag{A11}$$

where the superscripts n and n+1 indicate the n'th and n'th plus one time steps, respectively, and k is the size of the time step.

To avoid severe restrictions on the magnitude of k associated with fully explicit schemes, the method of Crank and Nicholson (1947) is invoked to make the scheme semi-implicit. The right hand side of eq (45) becomes,

$$\frac{1}{2}\left[f(\xi_i)\left[\frac{{}^{n+1}\Theta_{i-1} - 2\,{}^{n+1}\Theta_i + {}^{n+1}\Theta_{i+1}}{h^2} + \frac{{}^{n}\Theta_{i-1} - 2\,{}^{n}\Theta_i + {}^{n}\Theta_{i+1}}{h^2}\right]\right.$$

$$+ \left[\left.\frac{\partial f(\xi)}{\partial\xi}\right|_{\xi_i} - \mathrm{Pe}\right]\left[\frac{(1-\sigma_i)\,{}^{n+1}\Theta_{i+1} + 2\sigma_i\,{}^{n+1}\Theta_i - (1+\sigma_i)\,{}^{n+1}\Theta_{i-1}}{2h}\right.$$

$$\left.\left. + \frac{(1-\sigma_i)\,{}^{n}\Theta_{i+1} + 2\sigma_i\,{}^{n}\Theta_i - (1+\sigma_i)\,{}^{n}\Theta_{i-1}}{2h}\right] - \mathrm{Da}\,({}^{n+1}\Theta_i + {}^{n}\Theta_i)\right] \tag{A12}$$

where σ_i is again defined by eq (A7).

Veronis (personal commun.) has recently pointed out that the use of the weighing function for the steady-state, that is, eq (A7), is not appropriate for transient cases and leads to distortion of solution. (Chien, 1977, has suggested that an additional weighing function for the time derivative is needed, but it is not clear that his assumption of local steady-state is appropriate). Until the analysis for the transient case is successfully completed, use of eq (A7) must continue by necessity. In practice, it was found that the analytical and numerical solutions with $f(\xi) = 1 - \xi$ differed by less than a few percent.

The boundary condition at $\xi = 0$ reads,

$$\frac{\mathrm{Pe}}{2}({}^{n+1}\Theta_0 + {}^{n}\Theta_0) - \frac{1}{2}\left[\frac{{}^{n+1}\Theta_1 - {}^{n+1}\Theta_{-1}}{2h} + \frac{{}^{n}\Theta_1 - {}^{n}\Theta_{-1}}{2h}\right] = \Psi \tag{A13A}$$

or

$$\frac{\mathrm{Pe}}{2}({}^{n+1}\Theta_0 + {}^{n}\Theta_0) - \frac{1}{2}\left[\frac{{}^{n+1}\Theta_1 + {}^{n}\Theta_1}{2h} - \frac{{}^{n+1}\Theta_{-1} + {}^{n}\Theta_{-1}}{2h}\right] = \Psi \tag{A13B}$$

The term on the left hand side of this last equation involving values at the exterior point must again be eliminated. The procedure is the same as before. Eqs (A11) and (A12) are applied to the point $\xi = 0$ to obtain,

$$\begin{aligned}
\frac{1}{2}\,[{}^{n+1}\Theta_{-1} + {}^{n}\Theta_{-1}] = \Big[\frac{1}{h^2} - \Big[\frac{\partial f(\xi)}{\partial \xi}\Big|_0 - Pe\Big]\frac{(1+\sigma_0)}{2h}\Big]^{-1} \bullet \\
\Big[\Big[\frac{1}{k} + \frac{1}{2}\Big[\frac{2}{h^2} - \Big[\frac{\partial f(\xi)}{\partial \xi}\Big|_0 - Pe\Big]\frac{\sigma_0}{h} + Da\Big]\Big]\,{}^{n+1}\Theta_0 \\
-\Big[\frac{1}{h^2} + \frac{1}{2}\Big[\frac{\partial f(\xi)}{\partial \xi}\Big|_0 - Pe\Big]\frac{(1-\sigma_0)}{2h}\Big]\,{}^{n+1}\Theta_1 \\
+\Big[-\frac{1}{k} + \frac{1}{2}\Big[\frac{2}{h^2} - \Big[\frac{\partial f(\xi)}{\partial \xi}\Big|_0 - Pe\Big]\frac{\sigma_0}{h} + Da\Big]\Big]\,{}^{n}\Theta_0 \\
-\Big[\frac{1}{h^2} + \frac{1}{2}\Big[\frac{\partial f(\xi)}{\partial \xi}\Big|_0 - Pe\Big]\frac{(1-\sigma_0)}{2h}\Big]\,{}^{n}\Theta_1\Big]
\end{aligned} \tag{A14}$$

This result can be substituted into eq (A13B) to eliminate the exterior point.

The boundary condition at $\xi = 1$ again needs no special consideration, but the initial condition does pose some problems. Approximation of the delta function proved to be difficult. The best results were obtained by using the tracer distribution predicted by the analytical solution for constant diffusivity at very short times ($\tau < 0.001$) as the initial condition rather than trying to deal with the delta function directly. This works because for short times the pulse does not penetrate far into the sediment and thus cannot feel that $f(\xi)$ is a function of depth, that is, the solution for nearly all possible $f(\xi)$'s looks the same at the beginning.

This scheme can easily be modified to account for a variable but known porosity function which would allow both interphase and intraphase mixing to be modelled.

REFERENCES

Aller, R. C., 1980, Diagenetic processes near the sediment-water interface of Long Island Sound. I. Decomposition and nutrient element geochemistry (S,N,P): Adv. in Geophysics, v. 22, p. 237-356.

——— 1982, The effects of macrobenthos on chemical properties of marine sediment and overlying water, *in* McCall, P. L., and Tevesz, M. J. S., eds., Animal-Sediment Relations: New York, Plenum Press, p. 53-102.

Aller, R. C., Benninger, L. K., and Cochran, J. K., 1980, Tracking particle-associated processes in nearshore environments by use of ^{234}Th/^{238}U disequilibrium: Earth Planetary Sci. Letters, v. 47, p. 161-175.

Aller, R. C., and DeMaster, D. J., 1984, Estimates of particle flux and reworking at the deep-sea floor using ^{234}Th/^{238}U disequilibrium: Earth Planetary Sci. Letters, v. 67, p. 308-318.

Arfken, G., 1970, Mathematical Methods for Physicists: New York, Academic Press, 815 p.

Barrett, K. E., 1974, The numerical solution of singular-perturbation boundary-value problems: Quart. Jour. Mech. and Applied Math., v. 27, p. 57-68.

Baveye, P., and Sposito, G., 1984, The operational significance of the continuum hypothesis in the theory of water movement through soils and aquifers: Water Resources Research, v. 20, p. 521-530.

Bear, J., 1972, Dynamics of Fluids in Porous Media: New York, Am. Elsevier, 764 p.

Benninger, L. K., Aller, R. C., Cochran, J. K., and Turekian, K. K., 1979, Effects of biological sediment mixing on the ^{210}Pb chronology and trace metal distribution in a Long Island Sound sediment core: Earth Planetary Sci. Letters, v. 43, p. 241-259.

Berner, R. A., 1980, Early Diagenesis: A Theoretical Approach: New Jersey, Princeton Univ. Press, 241 p.

Bischoff, K. B., 1961, A note on boundary conditions for flow reactors: Chem. Eng. Sci., v. 16, p. 131-133.

Boudreau, B. P., ms, 1985, Diagenetic Models of Biological Processes in Aquatic Sediments: Ph.D. dissert., Yale Univ., New Haven, Conn., 526 p. (University Microfilms, Ann Arbor, Mich.)

Bukata, R. P., and Bobba, A. G., 1984, Determination of diffusion coefficients associated with the transport of ^{210}Pb radionuclides in lake bed sediments: Environmental Geology, v. 5, p. 133-141.

Campbell, J. B., 1979, Bessel functions $J\upsilon(x)$ and $Y\upsilon(x)$ of real order and real argument: Computer Physics Commun., v. 18, p. 133-142.

——— 1981, Bessel Functions $I\upsilon(z)$ and $K\upsilon$ of real order and complex argument: Computer Physics Commun., v. 24, p. 97-105.

Carney, R. S., 1981, Bioturbation and biodeposition, *in* Boucot, A. J., ed., Principles of Benthic Marine Paleoecology: New York, Acad. Press, p. 357-399.

Carpenter, R., Peterson, M. L., and Bennett, J. T., 1982, ^{210}Pb-derived sediment accumulation and mixing rates for the Washington continental slope: Marine Geology, v. 48, p. 1155-1172.

Carslaw, H. S., and Jaeger, J. C., 1959, Conduction of Heat in Solids: London, Clarendon Press, 510 p.

Chien, J. C., 1977, A general finite-difference formulation with application to Navier-Stokes equations: Computers and Fluids, v. 5, p. 15-31.

Christensen, E. R., 1982, A model for radionuclides in sediments influenced by mixing and compaction: Jour. Geophys. Research, v. 87, p. 566-572.

——— 1983, Mixing, sedimentation rates and age dating for sediment cores—comment: Marine Geology, v. 52, p. 291-297.

Cochran, J. K., and Aller, R. C., 1979, Particle reworking in sediments from the New York Bight Apex: evidence from ^{234}Th/^{238}U disequilibrium: Estuarine Coastal Marine Sci., v. 9, p. 739-747.

Cochran, J. K., and Krishnaswami, S., 1980, Radium, thorium, uranium, and ^{210}Pb in deep-sea sediments and pore waters from the north equatorial Pacific: Am. Jour. Sci., v. 280, p. 849-889.

Cody, W. J., 1983, Algorithm 597, Sequence of modified Bessel functions of the first kind: Assoc. Computing Machinery, Trans. on Math. Software, v. 9, p. 242-245.

Corrsin, S., 1974, Limitations of gradient transport models in random walks and in turbulence: Adv. Geophysics, v. 18A, p. 25-60.

Crank, J., and Nicholson, P., 1947, A practical method for the numerical evaluation of solutions of partial differential equations of the heat conduction type: Cambridge Philos. Soc. Proc., v. 43, p. 50-67.

Crozier, W. J., 1918, The amount of bottom material ingested by Holothurians (Stichopus): Jour. Expt. Zoology, v. 26, p. 379-389.

Cushman, J. H., 1983, Multiphase transport equations: I. General equations for macroscopic statistical, local, space-time homogeneity: Transport Theory Statistical Physics, v. 12, p. 35-71.

Danckwerts, P. V., 1953, Continuous flow systems. Distribution of residence times: Chem. Eng. Sci., v. 2, p. 1-13.

Dapples, E. C., 1942, The effect of macro-organisms upon near-shore sediments: Jour. Sed. Pet., v. 12, p. 118-126.

Davison, C., 1891, On the amount of sand brought up by lob worms to the surface: Geol. Mag. [Great Britain], v. 8, p. 489-493.

Dayal, R., Okubo, A., Duedall, I. W., and Ramamoorthy, A., 1979, Radionuclide redistribution mechanisms at the 2800-m Atlantic nuclear disposal site: Deep-Sea Research, v. 26A, p. 1329-1345.

DeMaster, D. J., and Cochran, J. K., 1982, Particle mixing rates in deep-sea sediments determined from excess ^{210}Pb and ^{32}Si profiles: Earth Planetary Sci. Letters, v. 61, p. 257-271.

Dominico, P. A., 1977, Transport phenomena in chemical rate processes in sediments: Ann. Rev. Earth Planetary Sci., v. 5, p. 287-317.

Erdelyi, A., ed., 1953, Higher Transcendental Functions (The Bateman Manuscript Project), v. 1: New York, McGraw-Hill, 302 p.

Fiadeiro, M., and Veronis, G., 1977, On weighted-mean schemes for the finite-difference approximation to the advective-diffusion equation: Tellus, v. 29, p. 512-522.

Fisher, J. B., Lick, W. J., McCall, P. L., and Robbins, J. A., 1980, Vertical mixing of lake sediments by tubificid oligochaetes: Jour. Geophys. Research, v. 85, p. 3997-4006.

Germano, J. D., ms, 1983, Infaunal Succession in Long Island Sound: Animal-Sediment Interactions and the Effects of Predation: Ph.D. dissert., Yale Univ., New Haven, Conn., 142 p.

Goldberg, E. D., and Koide, M., 1962, Geochronological studies of deep sea sediments by the ionium/thorium method: Geochim. et Cosmochim. Acta, v. 26, p. 417-450.

Goldhaber, M. B., Aller, R. C., Cochran, J. K., Rosenfeld, J. K., Martens, C. S., and Berner, R. A., 1977, Sulfate reduction, diffusion, and bioturbation in Long Island Sound sediments: report of the FOAM group: Am. Jour. Sci., v. 277, p. 192-237.

Gray, W. G., 1975, A derivation of the equations for multiphase transport: Chem. Eng. Sci., v. 30, p. 229-233.

——— 1983, Local volume averaging of multiphase systems using a non-constant averaging volume: Internat. Jour. Multiphase Flow, v. 9, p. 755-761.

Guinasso, N. L., and Schink, D. R., 1975, Quantitative estimates of biological mixing rates in abyssal sediments: Jour. Geophys. Research, v. 80, p. 3032-3043.

Gupta, M. M., 1983, A survey of some second-order difference schemes for the steady-state convection-diffusion equation: Internat. Jour. Numerical Methods for Fluids, v. 3, p. 319-331.

Hall, L. S., 1967, Bessel function $J_\nu(z)$ of complex order and its zeroes. Cambridge Philos. Soc. Proc., v. 63, p. 141-146.

Heinrich, J. C., Huyakorn, P. S., and Zienkiewicz, O. C., 1977, An upwind finite element scheme for two-dimensional convective transport equation: Internat. Jour. Numerical Methods Eng., v. 11, p. 131-143.

Hildebrand, F. B., 1976, Advanced Calculus for Applications: Englewood Cliffs, N.J., Prentice-Hall, 646 p.

Hinze, J. O., 1975, Turbulence: New York, McGraw-Hill, 790 p.

Imboden, D. M., ms, 1981, Tracers and mixing in the aquatic environment: Habilitation thesis, Swiss Fed. Inst. Technology, Bern, Switzerland, 137 p.

Jumars, P. A., 1978, Spatial autocorrelation with RUM (Remote Underwater Manipulator): vertical and horizontal structure of a bathyal benthic community: Deep-Sea Research, v. 25, p. 589-604.

Jumars, P. A., and Eckman, J. E., 1983, Spatial structure within deep-sea benthic communities, *in* Rowe, G. T., ed., The Sea, Deep-Sea Biology, v. 8: New York, Wiley-Intersci., p. 399-451.

Kadko, D., and Heath, G. R., 1984, Models of depth-dependent bioturbation at Manop Site H in the eastern equatorial Pacific: Jour. Geophys. Research, v. 89, p. 6567-6570.

Koide, M., Bruland, K., and Goldberg, E. D., 1973, Th-228/Th-232 and Pb-210 geochronologies in marine and lake sediments: Geochim. et Cosmochim. Acta, v. 37, p. 1171-1187.

Krishnaswami, S., Benninger, L. K., Aller, R. C., and Von Damm, K. L., 1980, Atmospherically-derived radionuclides as tracers of sediment mixing and accumulation in near-shore marine and lake sediments: evidence from ^{7}Be, ^{210}Pb and ^{239,240}Pu: Earth Planetary Sci. Letters, v. 47, p. 307-318.

Lebedev, N. N., 1965, Special Functions and Their Applications (Silverman, R. A., translator): New York, Dover, 308 p.

Lee, H., and Swartz, R. C., 1980, Biological processes affecting the distribution of pollutants in marine sediments. Pt. II. Biodeposition and bioturbation, *in* Baker, R. A., ed., Contaminants and Sediments, v. 2: Ann Arbor, Mich., Ann Arbor Sci., p. 555-606.

Luke, Y. L., 1977, Algorithms for the Computation of Mathematical Functions: New York, Academic Press, 284 p.

McCall, P. L., and Tevesz, M. J. S., 1982, The effects of benthos on the physical properties of freshwater sediments, *in* McCall, P. L., and Tevesz, M. J. S., eds., Animal-Sediment Relations, New York, Plenum Press, p. 105-176.

Mitchell, A. R., and Griffiths, D. F., 1980, The Finite Difference Method in Partial Differential Equations: New York, John Wiley & Sons, 272 p.

Moore, H. B., 1931, The muds of the Clyde Sea area. III. Chemical and physical conditions; rate and nature of sedimentation; and infauna: Marine Biol. Assoc. United Kingdom Jour., v. 17, p. 325-358.

Morton, K. W., 1980, Stability of finite difference approximations to a diffusion-convection equation: Internat. Jour. Numerical Methods Eng., v. 15, p. 677-683.

Myers, A. C., 1977, Sediment processing in a marine subtidal sandy bottom community. II. Biological consequences: Jour. Marine Research, v. 35, p. 633-647.

Nittrouer, C. A., DeMaster, D. J., McKee, B. A., Cutshall, N. H., and Larsen, I. L., 1983/84, The effect of sediment mixing on Pb-210 accumulation rates for the Washington continental shelf: Marine Geology, v. 54, p. 201-221.

Nogotov, E. F., 1978, Applications of Numerical Heat Transfer: Paris, France, UNESCO, Hemisphere Publishers, 142 p.

Nozaki, Y., 1977, Distributions of natural radionuclides in sediments influenced by bioturbation (in Japanese): Geol. Soc. Japan Jour., v. 83, p. 699-706.

Nozaki, Y., Cochran, J. K., Turekian, K. K., and Keller, G., 1977, Radiocarbon and ^{210}Pb distribution in submersible-taken deep-sea cores from project FAMOUS: Earth Planetary Sci. Letters, v. 34, p. 167-173.

Oberhettinger, F., 1972, Hypergeometric functions, *in* Abramowitz, M., and Stegun, I. A., eds., Handbook of Mathematical Functions: New York, Dover Pub., p. 555-556.

Officer, C. B., 1982, Mixing, sedimentation rates and age dating for sediment cores: Marine Geology, v. 46, p. 261-278.

Officer, C. B., and Lynch, D. R., 1982, Interpretation procedures for the determination of sediment parameters from time-dependent flux inputs: Earth Planetary Sci. Letters, v. 61, p. 55-62.

——————— 1983a, Determination of mixing parameters from tracer distributions in deep-sea sediment cores: Marine Geology, v. 52, p. 59-74.

——————— 1983b, Mixing, sedimentation rates and age dating for sediment cores—Reply I.: Marine Geology, v. 52, p. 292-296.

Olver, F. W. J., 1960, Bessel Functions.—Part III. Zeros and Associated Values, *in* Royal Soc. Math. Tables 7: Cambridge, Cambridge Univ. Press, 79 p.

——————— 1972, Bessel functions of integer order, *in* Abramowitz, M., and Stegun, I. A., eds., Handbook of Mathematical Functions: New York, Dover Pub., p. 355-433.

Olsen, C. R., Simpson, H. J., Peng, T.-H., Bopp, R. F., and Trier, R. M., 1981, Sediment mixing and accumulation rate effects on radionuclide depth profiles in Hudson estuary sediments: Jour. Geophys. Research, v. 86, p. 11020-11028.

Ozisik, M. N., 1980, Heat Conduction: New York, John Wiley & Sons, 687 p.

Parker, J. D., Boggs, J. H., and Blick, E. F., 1974, Introduction to Fluid Mechanics and Heat Transfer: Reading, Mass., Addison-Wesley, 612 p.

Patel, M. K., Markatos, N. C., and Cross, M., 1985, A critical review of seven discretization schemes for convection-diffusion equations: Internat. Jour. Numerical Methods for Fluids, v. 5, p. 255-244.

Pearson, J. R. A., 1959, A note on the "Danckwerts" boundary conditions for continuous flow reactors: Chem. Eng. Sci., v. 10, p. 281-284.

Peng, T.-H., Broecker, W. S., and Berger, W. H., 1979, Rates of benthic mixing in deep-sea sediment as determined by radioactive tracers: Quarternary Research, v. 11, p. 141-149.

Piessens, R., 1984, Chebyshev series approximations for the zeros of the Bessel functions: Jour. Computational Physics, v. 53, p. 188-192.

Reynolds, O., 1895, On the dynamical theory of incompressible viscous fluids and the determination of the criterion: Royal Soc. [London] Philos. Trans., v. A186, p. 123-164.

Rhoads, D. C., 1974, Organism-sediment relations on the muddy sea floor: Oceanography and Marine Biology Ann. Rev., v. 12, p. 263-300.

Rhoads, D. C., and Boyer, L. F., 1982, The effects of marine benthos on the physical properties of sediments. A successional perspective, *in* McCall, P. L., and Tevesz, M. J. S., eds., Animal-Sediment Relations: New York, Plenum Press, p. 3-52.

Richter, R., 1952, Fluidal-texture, *in* Sediment-Gesteinen und uber Sedifluktion uberhaupt: Notizbl. Hess. L.-Amt. Bodenforsch., v. 3, p. 67-81.

Robbins, J. A., 1978, Geochemical and geographical applications of radioactive lead, *in* Nriagu, J. O., ed., Biogeochemistry of Lead: Amsterdam, Elsevier, p. 285-337.

Robbins, J. A., Krezoski, J. R., and Mozley, S. C., 1977, Radioactivity in sediments of the Great Lakes: Post-depositional redistribution by deposit-feeding organisms: Earth Planetary Sci. Letters, v. 36, p. 325-333.

Robbins, J. A., McCall, P. L., Fisher, J. B., and Krezoski, J. R., 1979, Effect of deposit feeders on migration of ^{137}Cs in lake sediments: Earth Planetary Sci. Letters, v. 42, p. 277-287.

Rowe, G. T., 1974, The effects of the benthic fauna on the physical properties of deep-sea sediments, *in* Inderbizen, A. L., ed., Deep-Sea Sediments: New York, Plenum Press, p. 381-400.

Santschi, P. H., Li, Y.-H., Bell, J. J., Trier, R. M., and Kawtaluk, K., 1980, Pu in coastal marine environments: Earth Planetary Sci. Letters, v. 51, p. 248-265.

Schink, D. R., and Guinasso, N. L., 1977, Effects of bioturbation on sediment-seawater interaction: Marine Geology, v. 23, p. 133-154.

Sickle, J. V., Weimer, W. C., and Larsen, D. P., 1983, Mixing rates in Shagawa Lake, Minnesota, sediments as determined from ^{103}Ru profiles: Geochim. et Cosmochim. Acta, v. 47, p. 2189-2197.

Siemieniuch, J. L., and Gladwell, I., 1978, Analysis of explicit difference methods for a diffusion-convection equation: Internat. Jour. Numerical Methods Eng., v. 12, p. 899-916.

Shyy, W., 1985, A study of finite difference approximations to steady-state, convection-dominated flow problems: Jour. Computational Physics, v. 57, p. 415-438.

Spalding, D. B., 1972, A novel finite difference formulation for differential expressions involving both first and second derivatives: Internat. Jour. Numerical Methods Eng., v. 4, p. 551-559.

Swartz, R. C., and Lee, H., 1980, Biological processes affecting the distribution of pollutants in marine sediments. Part I. Accumulation, trophic transfer, biodegradation and migration, *in* Baker, R. A., ed., Contaminants and Sediments: v. 2: Ann Arbor, Mich., Ann Arbor Sci., p. 533-554.

Thiel, H., 1983, Meiobenthos and Nanobenthos of the deep-sea, *in* Rowe, G. T., ed., The Sea, Deep-Sea Biology, v. 8: New York, Wiley-Intersci., p. 167-230.

Truesdell, C., and Toupin, R., 1960, The classical field theories, *in* Flugge, S., ed., Encyclopedia of Physics, v. III/1: Berlin, Springer-Verlag, p. 226-793.

Turekian, K. K., Cochran, J. K., and DeMaster, D. J., 1978, Bioturbation in deep-sea deposits: rates and consequences: Oceanus, v. 21, p. 34-41.

Watson, A. T., 1927, Observations on the habits and life history of *Pectinaria (Lagis) koreni*: Liverpool Biol. Soc. Proc. and Trans., v. 42, p. 25-60.

Wehner, J. F., and Wilhelm, R. H., 1956, Boundary conditions of flow reactor: Chem. Eng. Sci., v. 6, p. 89-93.

Whitaker, S., 1969, Advances in the theory of fluid motion in porous media: Indus. Eng. Chemistry, v. 61, p. 14-28.

Wooding, R. A., 1972, Perturbation analysis of the equation for the transport of dissolved solids through porous media. III. Influence of boundary conditions: Jour. Hydrology, v. 16, p. 241-245.