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PREPARATION OF BETA DIAGRAMS IN STRUCTURAL GEOLOGY BY A DIGITAL COMPUTER

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with the collaboration of
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ABSTRACT. In a region of folded rocks the attitudes of fold axes may be determined by finding the intersections of a group of measured bedding or bedding foliation planes. Such intersections have been termed "beta lineations" by Sander and may be compared with linear structures measured directly in the field. For any n planes there are n(n-1)/2 intersections. Graphical preparation of beta diagrams on a Schmidt net is practically limited to 25 or 30 planes, whereas a much larger sampling is desirable. A program has been developed in which a computer calculates the intersections and counts their distribution in a series of overlapping compartments of a hemisphere, each 1 percent of the area of the whole. In a group of examples from the Orange area, Massachusetts, the beta maxima correspond very closely with concentrations of measured linear structures and indicate that the latter are "b lineations". As many as 162 planes giving 13,041 intersections were run in one test of field data.

STATEMENT OF PROBLEM

One method of study of a region of folded rocks is to plot measured bedding or bedding foliation planes on the lower hemisphere of a Schmidt net (Lambert equiareal projection). The intersections of the planes then can be regarded as the attitudes of axes about which the bedding or bedding foliation is folded. Such intersections have been termed "beta lineations" by Sander (1942) and diagrams showing them. "beta diagrams". They may be compared directly with measured linear structures such as minor fold axes, fine wrinkles on micaceous surfaces, and long axes of prismatic minerals or mineral aggregates. Where intersections and measured linear structures coincide in attitude it may usually be assumed that the linear features are related to the folding and are "b lineations". A simple example of such a comparison is given in figure 1.

Considerable caution must be exercised in interpreting beta diagrams because in certain types of complex folding concentrations of intersections can appear that do not, in fact, represent a true direction of folding. However, it has been the author's experience that they may call attention to important structural features which might otherwise be overlooked, as well as give useful quantitative information on the attitude of fold axes. It is left to the reader to envision for himself the many different possible types of beta diagrams resulting from such structures as chevron folds, domes, conical folds, and folded folds.

For any n measured attitudes or planes under consideration the total number of intersections is n(n-1)/2. Each plane contributes (n-1) intersections to the total. The fraction each plane contributes to the total is:

$$\frac{(n-1)}{n(n-1)/2} = \frac{2}{n}, \qquad \text{where } n > 1,$$

which decreases as n increases.

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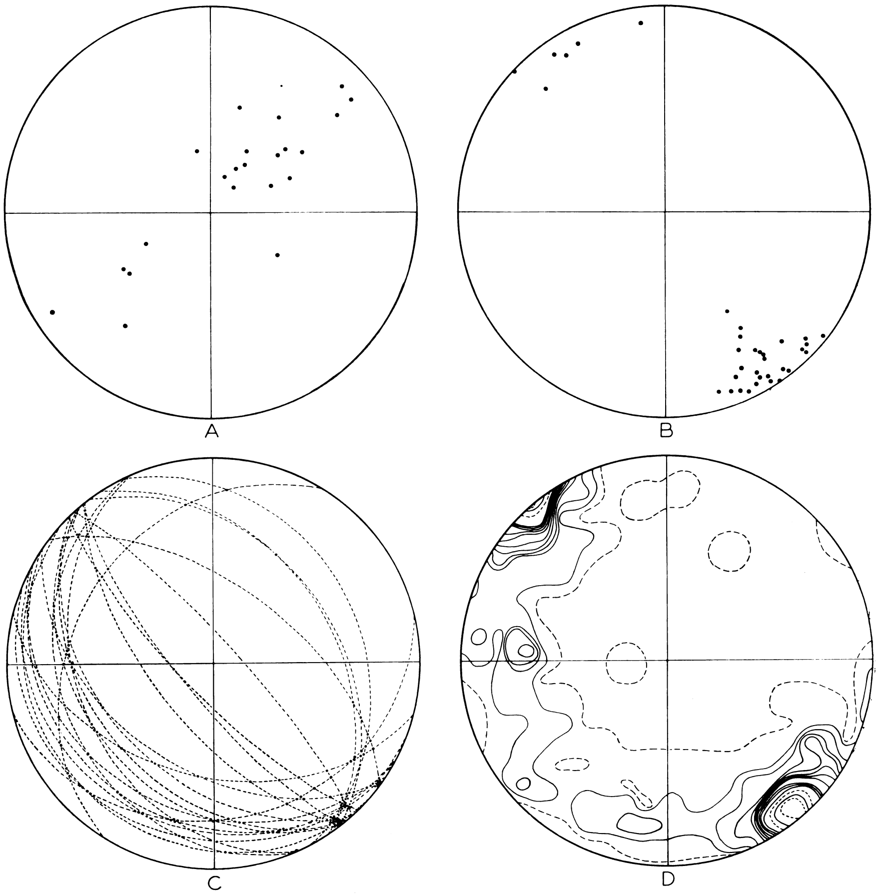

Fig. 1. Example of coincidence of beta maximum with lineations and minor fold axes in schists of the biotite zone, East Otago, New Zealand. Lower hemisphere projection; north at top.

A. Poles of 22 bedding and schistosity planes.
B. 34 lineations and minor fold axes.
C. 22 planes and their intersections shown directly.
D. Intersections of 22 planes contoured by circular point counter.
Contours per 1% area: 0% (dashed), 1-10%, 15% (dashed), 20%, 25% (dashed). 0% contour defined as the outline of areas containing the centers of 1% circles that contain no intersections.

The intersections of 25 or 30 planes can be found graphically on a 20 cm Schmidt net. This number can be extended by various procedures but no large number can be employed. Not only would the finding of intersections become extremely tedious, but the plotting and counting of many hundreds of points for contouring would be virtually impossible.

With 25 planes, each plane is responsible for 24 intersections or 8 percent of the total. In this case one anomalous or erroneous measurement, for example, one taken on a large float block mistaken for outcrop, would have a large influence out of all proportion to its importance. The use of a large number of planes not only gives a fairer sample of a sector under study, but should reduce anomalous or erroneous results, provided the erroneous measurements are not systematic, because the proportion contributed by each plane is reduced.

The finding of a series of intersections is a repeated task which is a natural operation for a computer. The problem was first presented to Professor Robin Robinson who derived the trigonometric relations and formulae needed for solution. The general method of approach to the computer was worked out in conference with Professor Thomas Kurtz, who made time available on the Dartmouth College Royal-McBee LGP-30 computer. Early work on the program was done by Cary Wyman, an undergraduate mathematics major, and Professor Robinson. The final and successful program, including several time-saving innovations, and the results were completed by Stephen J. Garland, also an undergraduate.

GENERAL PROCEDURE AND TRIGONOMETRIC SOLUTION

As finally used the computer was programmed to complete two main operations. First, it finds the intersections between every pair of planes using trigonometric formulae and previously established trigonometric subroutines. Second, it counts the intersections. The hemisphere of the projection is divided into overlapping areas, each 1 percent of the total area. Whenever an intersection falls into one of these 1 percent areas, it is counted. Output is in the form of a chart showing the total intersection count in each of the 1 percent areas.

A bedding plane is uniquely determined either by strike and dip readings, or by latitude and longitude angles of its pole on a unit sphere whose equatorial plane is taken as horizontal at the point of measurement, the pole being determined by the downward directed normal to the bedding plane at the sphere's center.

Let s = strike angle, measured clockwise from north. (Thus N 15° E means s = 15°, N 30° W means s = –30°.) Let d = dip angle. Let θ = longitude of pole measured clockwise from north, and let ϕ = latitude of pole on lower hemisphere. Then $\theta = s + 90°$ if dip is west, $\theta = s - 90°$ if dip is east, and $\phi = 90 - d$. The machine program was written to convert strike and dip to latitude and longitude of pole before proceeding to simultaneous solutions.

The plane whose pole is the point (x_1, y_1, z_1) on the sphere has the equation $x_1x + y_1y + z_1z = 0$. Since the sphere is given parametrically by $x = \cos\phi \cos\theta$, $y = \cos\phi \sin\theta$, $z = \sin\phi$, this equation becomes

$$\cos\phi_1 \cos\theta_1 \cos\phi \cos\theta + \cos\phi_1 \sin\theta_1 \cos\phi \sin\theta + \sin\phi_1 \sin\phi = 0,$$

or $\quad \cos\theta_1 \cos\theta + \sin\theta_1 \sin\theta = -\tan\phi_1 \tan\phi,$

or $\quad \cos(\theta - \theta_1) = -\tan\phi_1 \tan\phi.$

For a second plane, we have similarly

$$\cos\theta_2 \cos\theta + \sin\theta_2 \sin\theta = -\tan\phi_2 \tan\phi.$$

For the simultaneous solution, we have then

$$\frac{\cos\theta_1 + \sin\theta_1 \tan\theta}{\cos\theta_2 + \sin\theta_2 \tan\theta} = \frac{\tan\phi_1}{\tan\phi_2} = a.$$

The simplified formulae for the simultaneous solution may then be written

$$a = \frac{\tan \phi_1}{\tan \phi_2},$$

$$\tan \theta = \frac{a \cos \theta_2 - \cos \theta_1}{\sin \theta_1 - a \sin \theta_2},$$

$$\tan \phi = -\frac{\cos (\theta - \theta_1)}{\tan \phi_1}.$$

The machine is programmed to carry the operations through in this order, the sign of cos $(\theta - \theta_1)$ in the third equation indicating that θ must differ from θ_1 (and hence similarly from θ_2) by at least 90°, which allows the machine to select the true solution for θ from the second equation. The machine is also programmed to pursue an alternative routine whenever a denominator vanishes or a fraction is indeterminate.

Finally, the solution itself is not kept, but only the fact that it falls within certain compartments on the lower hemisphere, so that the final output of the machine is a matrix, showing how many solutions fall within each of these compartments. Each compartment contains exactly 1 percent of the area of the hemisphere, but they overlap each other, so that there are essentially 400 of them.

In principle, one compartment is centered at each point whose longitude is a multiple of 9° and the sine of whose latitude is a multiple of 0.1, its boundaries being the lines of longitude and latitude that pass through the next nearest compartmental centers on each side. The theory behind this compartmentalization is the geometric theorem that the spherical surface lying between two parallel planes is proportional to the distance ($z' - z = \sin \phi' - \sin \phi$) between the planes, independently of the position of the planes, so long as they both cut the sphere. In order to secure continuity across the equator, two semi-compartments bordering on the equator at diametrically opposite points are treated as a single compartment with center on the equator.

In practice, the compartments near the pole of the equatorial plane are so long and narrow that it has been found advisable to replace those with centers at latitude arcsin (0.9) by compartments twice as wide longitudinally and half as wide latitudinally. An additional ring of compartments of this modified type are centered at latitude arcsin (0.95), and a single circular compartment with center at the pole and reaching to latitude arcsin (0.99) takes care of points near the pole, where forty of these compartments come together. Altogether there are 441 compartments, each of those with center on the equator appearing twice in the matrix printed by the machine, while the single compartment centered at the pole is listed separately.

Figure 2 shows on a Lambert equiareal projection of the hemisphere all points at which compartments are centered (fig. 2C), a nonoverlapping portion of the compartments whose centers are at latitudes whose sines are 0.0, 0.2, 0.4, 0.6, 0.8, 0.95 (fig. 2B), and a nonoverlapping portion of the compartments whose centers are at latitudes whose sines are 0.1, 0.3, 0.5, 0.7, 0.9, 1.0 (fig. 2A). When the numbers from the 40 by 11 matrix produced by the machine are written at the locations in figure 2C, the resulting distribution may be con-

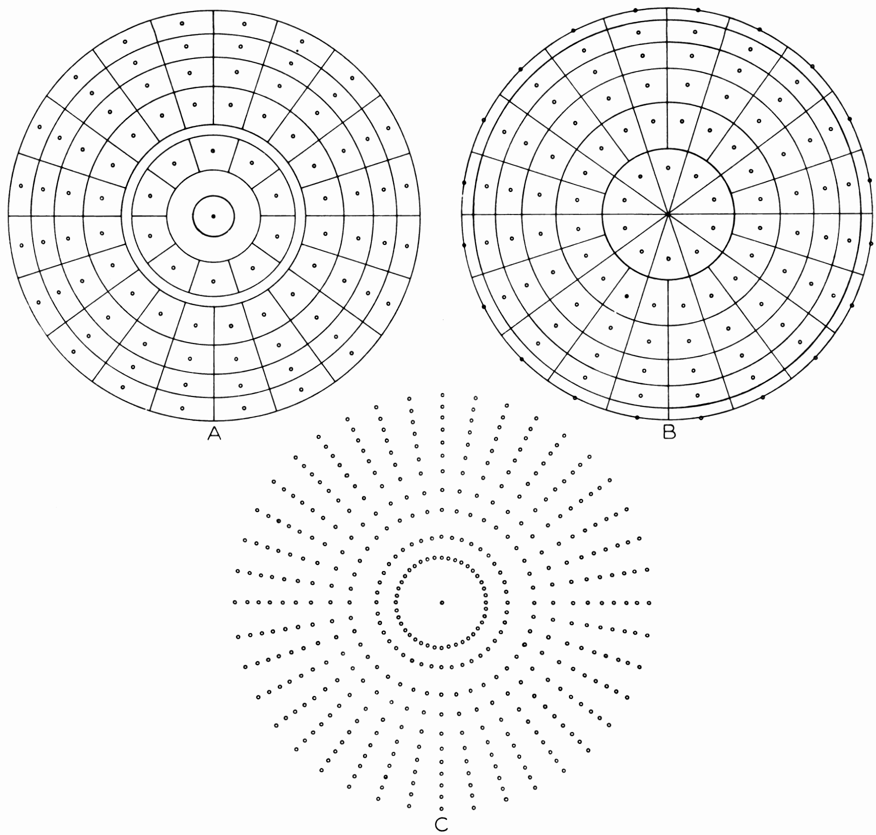

Fig. 2. Compartmentalization of lower hemisphere used in computer program shown on Lambert equiareal projection.

A. Non-overlapping portion of the compartments whose centers are at latitudes whose sines are 0.1, 0.3, 0.5, 0.7, 0.9, 1.0.

B. Non-overlapping portion of the compartments whose centers are at latitudes whose sines are 0.0, 0.2, 0.4, 0.6, 0.8, 0.95.

C. All points at which compartments are centered.

toured as on a Schmidt net. In order to make as close a comparison as possible between the machine results and measured linear structures, these same compartment charts (fig. 2A,B) can be used to contour the measured structures manually, instead of using the usual circular point counters.

DETAILS OF COMPUTER PROGRAM

Computation was carried out on the Royal-McBee LGP-30, a desk-size, stored program computer with a 4096-word magnetic drum memory.

The program itself was designed to accept up to 800 strike and dip readings of planes, convert these readings to latitude and longitude of their poles, compute the intersections of all pairs of these planes, and count the number of

TABLE 1

Format of input for sector 1

SECTOR 1

89′

18e′	63e′	42e′	37w′	1w′	50e′
34e′	79e′	23e′	32w′	24w′	13e′
19e′	77e′	30e′	22w′	1w′	87e′
5e′	40e′	19e′	89e′	1w′	89e′
18e′	86e′	8e′	35e′	18w′	29e′
17e′	61w′	3e′	74w′	8w′	67e′
4e′	73w′	61e′	41w′	34w′	27e′
20e′	40w′	28e′	34w′	47w′	24e′
27e′	47w′	17e′	29w′	10w′	53e′
45e′	18w′	44e′	14w′	41w′	18e′
14e′	41w′	59e′	21w′	71w′	19e′
38e′	57w′	31w′	21e′	63w′	13e′
14e′	46w′	26w′	50e′	36w′	43e′
25e′	31w′	36w′	58e′	13w′	30e′
6e′	70w′	40w′	41e′	77w′	26e′
15e′	66w′	7w′	50e′	24w′	23e′
9e′	50e′	32w′	34e′	33w′	64e′
10e′	25w′	1w′	44e′	36w′	28e′
6e′	71w′	7w′	45e′	9w′	19e′
6e′	68w′	6w′	39e′	17w′	31e′
3e′	66e′	24w′	75e′	41w′	38e′
68e′	27w′	19w′	30e′	23w′	38e′
31e′	21w′	10w′	34e′	7w′	59e′
33e′	19w′	4w′	45e′	6w′	34e′
4e′	87e′	22w′	24e′	82w′	25e′
5e′	66w′	16w′	52e′	21w′	42e′
4e′	87w′	3w′	71e′	3w′	65e′
24e′	20w′	23w′	47e′	2w′	65w′
3e′	76w′	82w′	22e′	6w′	32e′
14e′	54e′	83w′	15e′		

these intersections in each of 441 compartments. Input is accomplished via punched paper tape (see table 1 for format), while output is in the form of a 40 by 11 matrix, the columns indicating divisions of latitude and the rows indicating divisions of longitude (see table 2 for sample of output sheet), and a single compartment.

The details of the computation are as follows:[1]

(1) Storage for the output table T and compartment L12 is cleared to zero.

(2) n, the number of readings, is read in.

(3) n readings are then read in and converted from strike and dip to latitude (ϕ) and longitude (θ).

(4) The intersection of each pair of planes is computed.

  4.1. If $\phi_a = O$, a flag is set up to prevent an indeterminate fraction from occurring when θ and ϕ are computed.

  4.2. Counters are advanced to prepare for the computation of the next intersection.

[1] A copy of the computer program in Algol 60 computer language may be obtained from the author at the Department of Geology, University of Massachusetts, Amherst, Massachusetts.

TABLE 2

Matrix output for sector 1

SECTOR 1	11	10	9	8	7	6	5	4	3	2	1
1	1	2	12	26	42	200	416	710	1023	715	223
2	3	5	10	14	56	157	284	541	863	632	184
3	5	6	10	26	74	137	247	372	351	232	123
4	5	7	13	40	62	137	295	277	142	100	72
5	5	10	9	33	58	111	183	121	29	16	20
6	4	10	18	40	55	68	76	47	20	7	9
7	3	13	21	39	31	25	29	16	9	2	3
8	5	12	15	25	24	15	14	11	5	0	2
9	5	9	13	18	22	12	2	6	6	1	1
10	6	7	16	21	18	13	5	4	4	3	0
11	6	4	19	22	16	18	11	4	3	4	2
12	5	6	12	18	18	22	17	5	2	3	4
13	4	7	8	15	16	16	14	7	5	2	3
14	2	7	9	15	20	16	9	13	12	2	1
15	2	6	8	7	21	26	18	21	24	14	3
16	2	3	6	4	12	19	25	28	30	33	17
17	2	1	3	6	7	17	28	27	37	46	33
18	3	0	1	3	4	14	21	19	29	46	53
19	6	2	0	1	4	4	6	17	18	34	55
20	5	2	3	0	4	5	4	14	23	51	124
21	8	5	3	1	3	3	8	14	25	79	223
22	7	5	3	3	7	14	19	20	24	64	184
23	3	3	7	13	16	18	15	11	12	40	123
24	3	3	5	12	10	6	4	3	11	34	72
25	0	0	1	1	0	1	2	4	12	21	20
26	0	0	1	0	0	0	1	9	11	8	9
27	0	1	1	0	1	2	5	11	10	6	3
28	0	1	1	0	3	4	6	5	3	5	2
29	1	2	1	1	3	3	4	6	4	2	1
30	1	2	0	1	1	4	6	6	6	2	0
31	2	2	0	0	1	5	6	4	4	3	2
32	2	2	1	1	4	6	3	4	3	3	4
33	1	2	1	1	4	5	4	5	2	3	3
34	1	3	1	1	3	2	6	6	1	2	1
35	0	2	3	4	5	1	6	8	3	1	3
36	0	2	4	5	5	1	14	16	4	2	17
37	0	1	5	3	4	4	24	31	26	27	33
38	0	0	10	9	5	13	56	107	91	55	53
39	0	0	11	42	49	52	125	206	154	65	55
40	0	1	7	47	66	144	288	466	473	261	124
L12	6										

(5) After each intersection is computed, it is classified according to the compartments in which it falls. First, both its coordinates are made positive.

5.1. Matrix subscripts are computed, using 18° divisions of longitude and divisions of latitude whose sines differ by 0.2 except for columns 1, 10, and 11 in which the sines differ by 0.1.

5.2. If an intersection has latitude greater than arcsin (0.99), L12 is incremented.

5.3. The count of intersections in each of the overlapping compartments containing the computed intersection is incremented.

(6) After a delay to ready the typewriter for printing, the size of the compartments in columns 1, 10, and 11 is adjusted.

6.1. The revised number of intersections in a compartment in column 1 is determined by adding the number of intersections in that compartment to the number in the compartment 180° away. The revised count thus gives the number of intersections within arcsin (0.1) either side of the equator and within 18° longitude.

6.2. The revised number of intersections in a compartment in column 10 or 11 is determined by adding the number of intersections in two compartments on either side of the given compartment. The revised count thus gives the number of intersections within a segment 36° wide and with a sine of latitude ranging over an interval of 0.1.

(7) The output matrix is printed, followed by L12. After a delay, the computer is ready to accept a new set of data.

On the LGP-30, the program will read in coordinates at the rate of 125 planes per minute. Approximately 1000 intersections are computed per hour, while output takes 8 to 9 minutes. On larger machines, such as the IBM 704 or 709, computing time would be approximately 1/1000 that of the LGP-30, thus enabling much larger samples to be run in a reasonable amount of time.

RESULTS ON FIELD DATA

As a test for the program, a group of sectors was chosen from a portion of the Orange area, Massachusetts-New Hampshire, currently under study by the author (see figs. 3, 6, and 8). The area lies immediately east and southeast of the point where the states of Massachusetts, New Hampshire, and Vermont come together, 75 miles west-northwest of Boston. For the general geologic setting the reader is referred to the Geologic Map of Massachusetts and Rhode Island by Emerson (1917).

Apart from the fact that the results were immediately useful in the study of the area, the sectors chosen have several advantages. The stratigraphy of the

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Fig. 3. Geologic map of portion of the Orange area, Massachusetts, showing sectors 1 and 2 from which computer input was collected. Dip symbols show generalized attitude of both normal and inverted bedding. Millers River crosses map area from east to west.

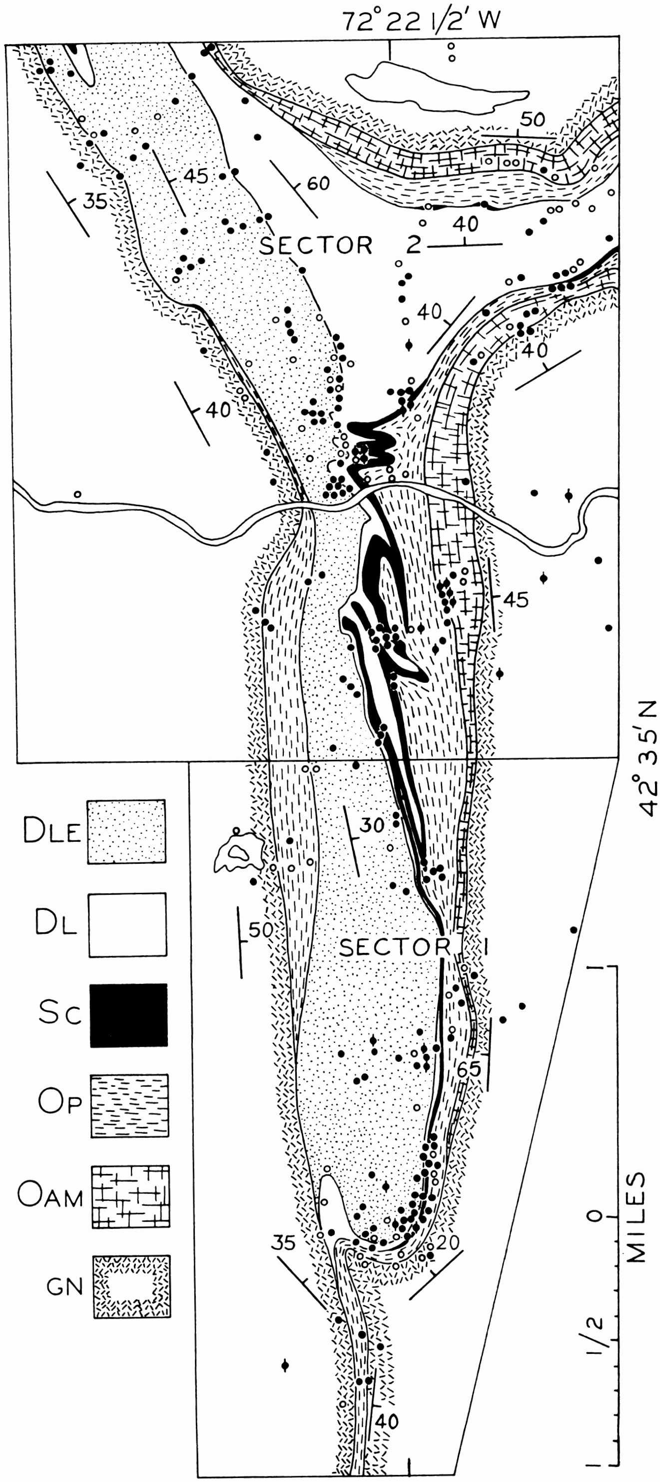
Open circle: Measurement of bedding or bedding foliation.

Closed circle: Measurement of bedding or bedding foliation, and associated lineation or minor fold axis.

Closed circle with cross bar: Measurement of lineation or minor fold axis.

Rock Units:

- DLE: Erving member of the Littleton formation—interbedded epidote amphibolite, quartz-plagioclase-biotite granulite, and mica schist with garnet, staurolite, and kyanite (lower Devonian).
- DL: Lower member of the Littleton formation—mica schist with garnet and staurolite (lower Devonian).
- SC: Clough formation—quartzite (lower Silurian).
- OP: Partridge formation—sulfidic mica schist and amphibolite (middle Ordovician).
- OAM: Ammonoosuc volcanics—amphibolite, quartz-feldspar gneiss, and quartz-plagioclase-anthophyllite gneiss (middle Ordovician).
- GN: Coarse-grained gneisses and related rocks below the Ammonoosuc and Partridge formations. To west, Pelham dome; to east, Kempfield anticline; at north corner, Warwick dome.


72° 22 1/2′ W
42° 35′ N
SECTOR 2
SECTOR 1
50
60
45
35
40
40
40
40
45
30
50
65
35
20
40
DLE
DL
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Op
OAM
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MILES
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1/2
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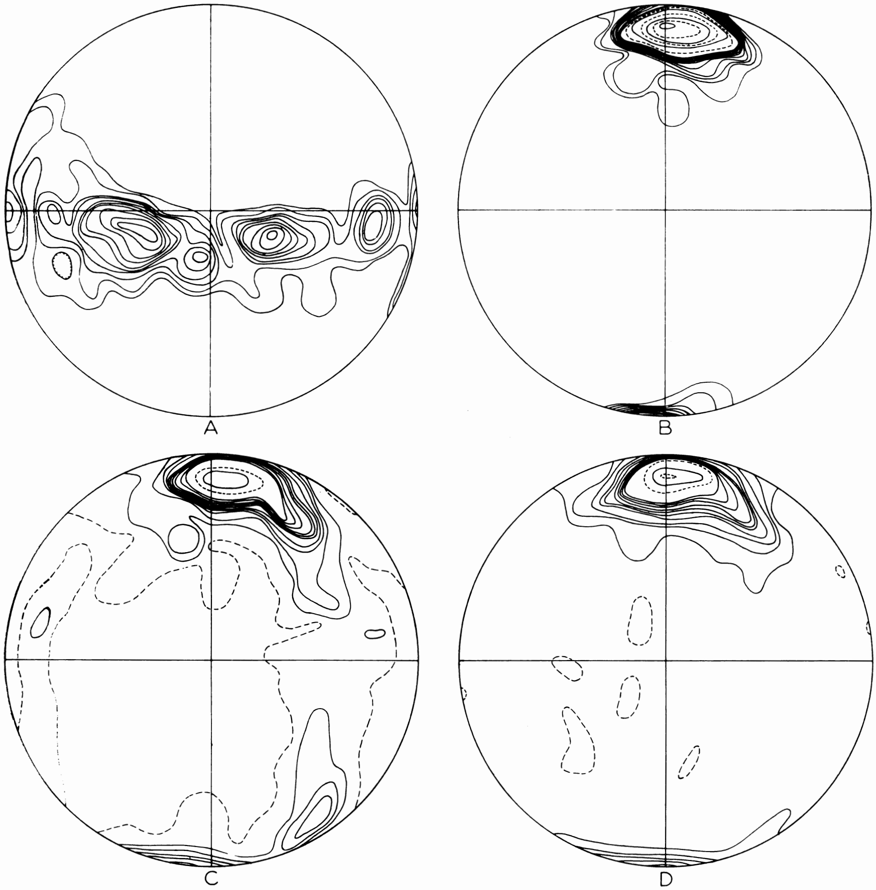

Fig. 4. Sector 1.
A. Poles of 89 bedding and bedding foliation planes. Contours: 1-10%.
B. 73 lineations and minor fold axes. Contours: 1-10%, 15% (dashed), 20%, 25% (dashed), 30%, 35% (dashed), 40%.
C. Computer-prepared beta diagram of 25 randomly selected planes. Contours: 0% (dashed), 1-10%, 15% (dashed), 20%.
D. Computer-prepared beta diagram of 89 planes. Contours: 0% (dashed), 1-10%, 15% (dashed), 20%, 25% (dashed).
0% contour defined as the outline of areas containing the centers of compartments which contain no points.

area is well defined, and the pattern of the folds easily seen. For the purpose of checking the program, folds could be chosen in which the result could be predicted ahead of time. In other sectors the results gave new information not otherwise obtainable. In spite of the obvious fold patterns on the map, the regional relationships of the structures have not been fully settled, and extended discussion will be postponed to a future publication.

The closest check of the program was made by running the same set of 25 planes both graphically and by computer. In this way several errors and omis-

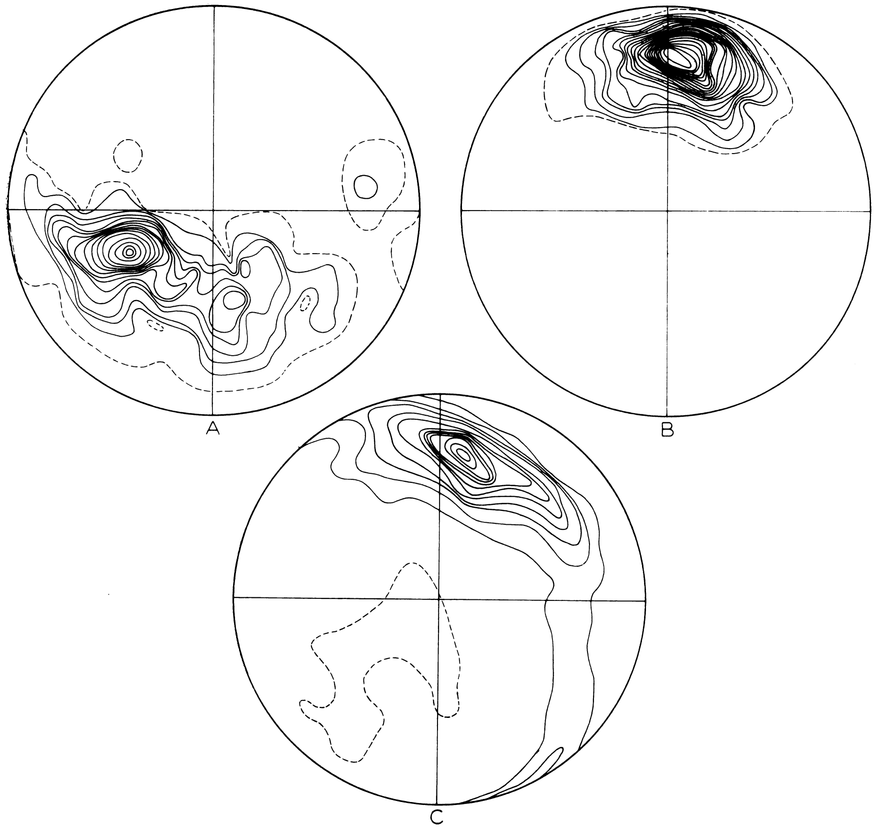

Fig. 5. Sector 2.
A. Poles of 162 bedding and bedding foliation planes. Contours: 0% (dashed), 1-15%.
B. 138 lineations and minor fold axes. Contours: 0% (dashed), 1-21%.
C. Computer-prepared beta diagram of 162 planes. Contours: 0% (dashed), 1-12%.

sions in the program were detected and corrected. These 25 plane results can also be compared with runs on a larger number of planes from the same sector (compare figs. 4C,D. and 9C,D).

For each sector the poles of measured bedding and bedding foliation planes, and the lineations and minor fold axes are shown. These have been contoured manually using the same set of 1 percent subdivisions used by the computer. Wherever possible, contour intervals of 1 percent per 1 percent area have been used. This has been done, not because of a belief that detail is important, but because the difference in intensity of concentration on any two diagrams may be contrasted at a glance. In figures 1, 4, and 9 some contour lines are at 10 percent intervals with dashed contours every other 5 percent. A 0 percent contour is shown dashed on all diagrams in which more than 100 points are contoured.

Sector 1 (*fig. 4*).—The fold in sector 1 is a tight overturned isoclinal syncline plunging gently north, the axial plane of which dips 45° E. From the pro-

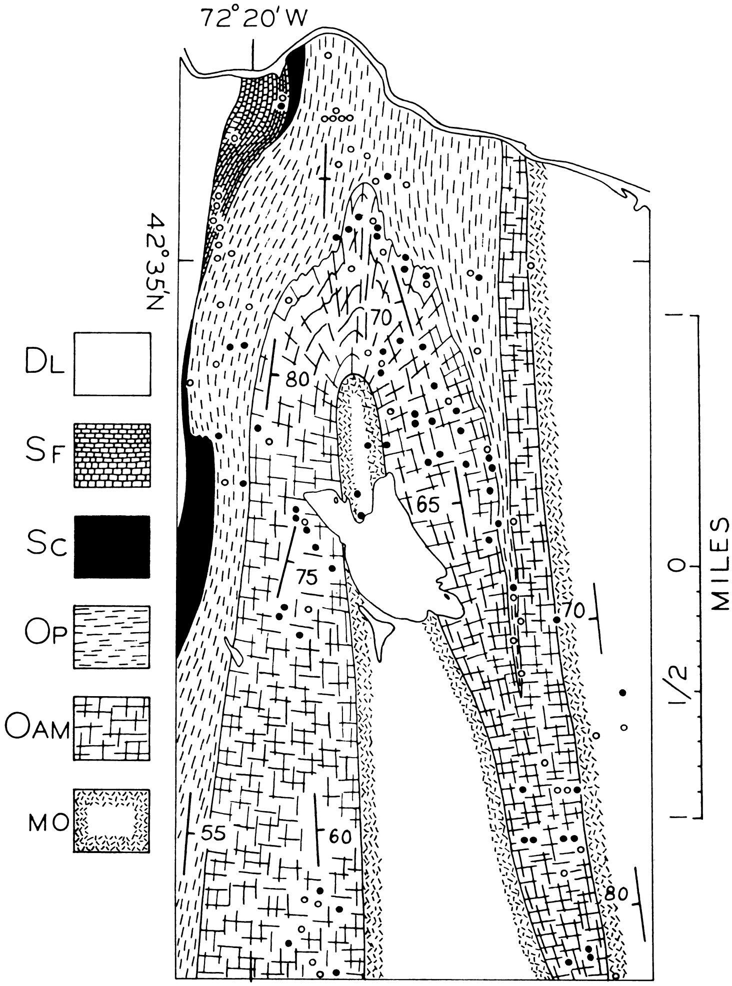

Fig. 6. Geologic map of sector 3, Orange area, Massachusetts. North boundary of sector is Millers River. Lake Mattawa in center. Dip symbols show generalized attitude of both normal and inverted bedding.

Open circle: Measurement of bedding or bedding foliation.

Closed circle: Measurement of bedding or bedding foliation, and associated lineation or minor fold axis.

Closed circle with cross bar: Measurement of lineation or minor fold axis.

Rock Units:

DL: Lower member of the Littleton formation—mica schist with sillimanite, garnet, and staurolite (lower Devonian).

SF: Fitch formation—calc-silicate granulite and sulfidic schist (middle Silurian).

SC: Clough formation—quartzite and quartz conglomerate (lower Silurian).

OP: Partridge formation—sulfidic mica schist, amphibolite, and minor calc-silicate rock (middle Ordovician).

OAM: Ammonoosuc volcanics—amphibolite, quartz-feldspar gneiss, and quartz-plagioclase-anthophyllite gneiss (middle Ordovician).

MO: Monson gneiss—dominantly layered quartz-feldspar gneiss with interbedded amphibolite.

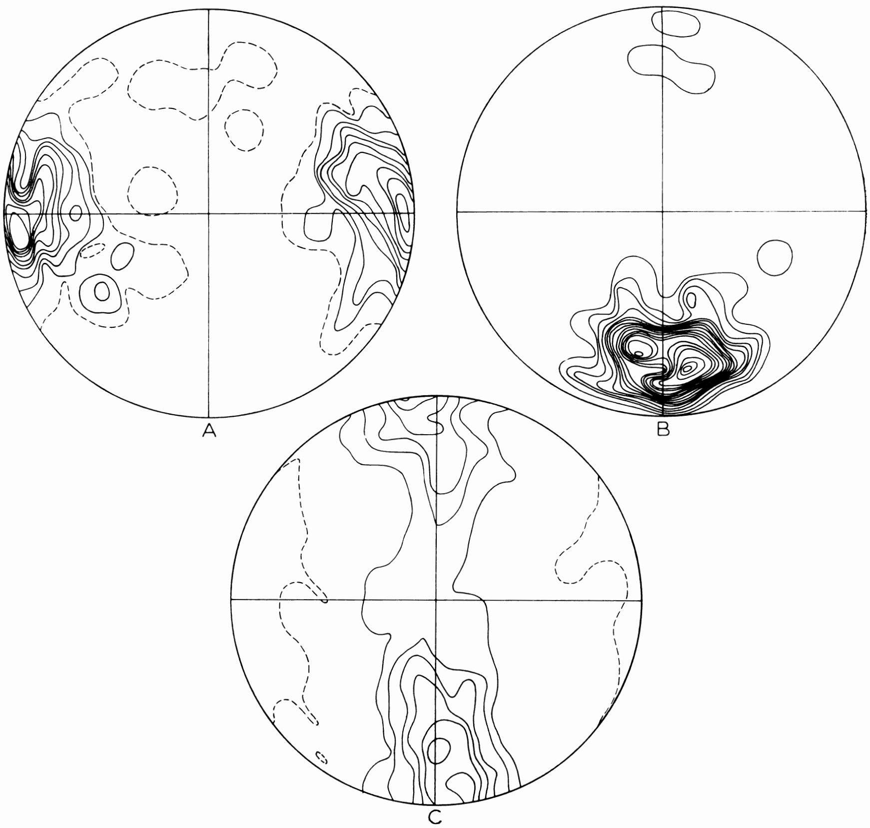

Fig. 7. Sector 3.
A. Poles of 120 bedding and bedding foliation planes. Contours: 0% (dashed), 1-10%.
B. 65 lineations and minor fold axes. Contours: 1-18%.
C. Computer-prepared beta diagram of 120 planes. Contours: 0% (dashed), 1-6%.

nounced girdle of poles in figure 4A the fold axis can be readily determined without recourse to the beta diagram. The beta diagram (fig. 4D) shows a very strong maximum striking north and plunging 12° N, which coincides exactly with the concentration of lineations and minor fold axes (fig. 4B). The same strong maximum shows up even with intersections of only 25 randomly selected planes (fig. 4C).

Sector 2 (*fig. 5*).—The fold in sector 2 consists of the northern extension of the syncline in sector 1 plus the less tightly folded rocks of the overturned south end of the Warwick dome. The poles of bedding and bedding foliation (fig. 5A) show a much less pronounced girdle which may, nevertheless, be used successfully to predict the greatest concentration on the beta diagram (fig. 5C). The girdle seen in the beta diagram represents low angle intersections between nearly parallel planes, all of which are roughly parallel to the axial plane of the major isoclinal syncline. In this case the girdle in effect gives the general attitude of the axial plane, and the maximum representing the fold

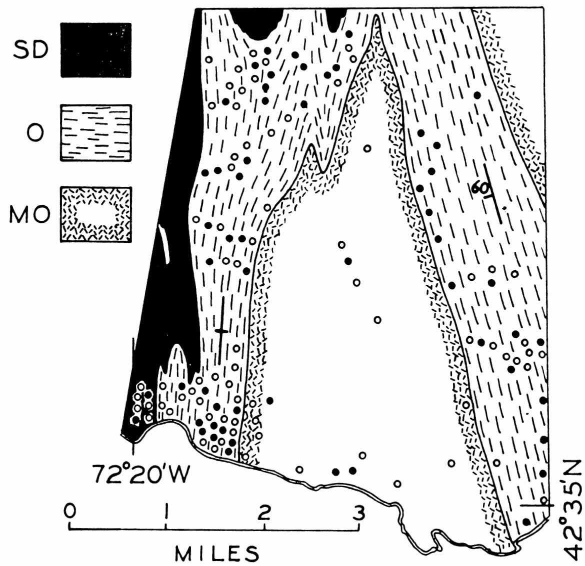

Fig. 8. Generalized geologic map of sector 4, Orange area, Massachusetts. Millers River forms southern border of sector. Dip symbols show generalized attitude of normal or inverted bedding.

Open circle: Measurement of bedding or bedding foliation.

Closed circle: Measurement of bedding or bedding foliation and associated lineation or minor fold axis.

Rock Units:

SD: Silurian Clough and Fitch formations, and Devonian Littleton formation.

O: Ordovician Partridge and Ammonoosuc formations.

MO: Monson gneiss—main Monson body in center and Tully body to northeast.

axis (strike N 10° E, plunge 30° N) lies in it. Measured lineations and fold axes (fig. 5B) agree closely with this maximum.

Sector 3 (*fig. 7*).—Sector 3 contains two isoclinal folds, an anticline on the east and a syncline on the west. The bedding and bedding foliation planes have steep dips, and the diagram of their poles (fig. 7A) gives little hint of the plunge direction of the folds. The beta diagram (fig. 7C) shows a concentration (centered at strike N 5° W, plunge 20° S) that agrees closely with the concentration of lineations and fold axes (fig. 7B). In this case the computer produced new information not otherwise obtainable. Again, as in the case of sector 2, the girdle of intersections in figure 7C gives a rough indication of the attitude of the axial planes of the isoclinal folds which are essentially vertical.

Sector 4 (*fig. 9*).—Sector 4 was studied as a means of reconnaissance and represents a series of early measurements taken over the whole area of the northern end of the Monson gneiss and surrounding formations (see fig. 8). The intersections of 106 planes (fig. 9D) give a maximum with center at strike N 5° W, plunge 20° S which agrees well with the concentration of measured lineations and fold axes (fig. 9B). A beta diagram of 25 randomly selected planes from sector 4 (fig. 9C) gives unsatisfactory results which hint at the fold direction inaccurately. Comparison of figures 9C and 9D illustrates graphically

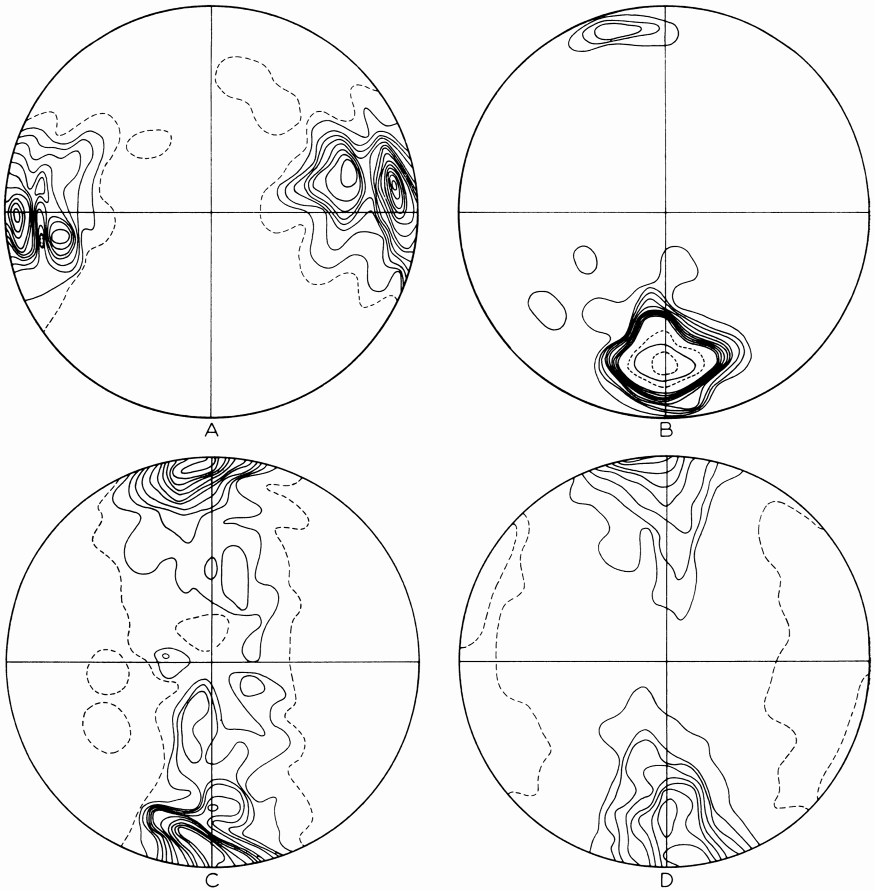

Fig. 9. Sector 4.
A. Poles of 106 planes. Contours: 0% (dashed), 1-14%.
B. 62 lineations and minor fold axes. Contours: 1-10%, 15% (dashed), 20%, 25% (dashed).
C. Computer-prepared beta diagram of 25 randomly selected planes. Contours: 0% (dashed), 1-11%.
D. Computer-prepared beta diagram of 106 planes. Contours: 0% (dashed), 1-8%.

the need to employ a large number of planes in preparing beta diagrams of sectors where fold directions are not obvious. The results on sectors 3 and 4 as well as a large body of other evidence, which will not be cited here, indicate that the Monson gneiss occupies a synclinal position overlying surrounding younger formations.

SUMMARY OF CONCLUSIONS

(1) Beta diagrams are useful tools in the study of folded regions, but graphical preparation of such diagrams is practically limited to 25 or 30 planes. The use of a much larger number gives a fairer sample and reduces the effect of anomalous or erroneous field measurements.

(2) A program has been devised for the preparation of beta diagrams using a digital computer so that as many as 150 planes may be utilized conveniently.

(3) In a portion of the Orange area, Massachusetts-New Hampshire, maxima on computer-prepared beta diagrams agree very closely with concentrations of measured lineations and minor fold axes, indicating that these are "b lineations" and suggesting that these minor structures give a reliable indication of the attitude of major fold axes.

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