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ART. IX.—*An Experimental Investigation on the Reflection of Light at Certain Metal-Liquid Surfaces*; by LYNDE P. WHEELER, Assistant Professor of Physics, Sheffield Scientific School of Yale University.

Introduction.

WHEN light is incident on a transparent substance at an angle whose tangent has the numerical value of the index of refraction of the material, theory, as embodied in the Fresnel equations, demands that the reflected light should be plane polarized in the plane of incidence. Experience teaches, however, that but for a very few substances, solid or liquid, is this true. Most substances show at this angle an elliptic polarization of small ellipticity. This ellipticity (defined as the ratio of the amplitudes of the components of the vibration parallel and perpendicular to the plane of incidence) is found to be sometimes positive and sometimes negative. (Positive ellipticity corresponds to a counter-clockwise description of the ellipse when viewed from the side of the incident light.)

That this well-nigh universal though small discrepancy between theory and experiment cannot be entirely assigned to contamination of the reflecting surface seems to have been definitely settled by the experiments of Lord Rayleigh on very clean water surfaces.* He found that as the surface was made progressively cleaner, the negative ellipticity previously observed became numerically smaller, passed through a zero value, and for the cleanest surfaces obtainable, assumed a very small positive value. On the other hand, Drude, working with the fresh cleavage surfaces of transparent crystals where the chance of surface contamination is very remote, found that within the limits of error of observation the ellipticity van-

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* Phil. Mag. (5), xxxiii, p. 1, 1892.

ished.* It would thus seem to be established that the theory stands in need of correction in the case of reflection from liquids, while remaining entirely competent in the case of solids.

Now the boundary conditions from which the equations of Fresnel are deduced assume an abrupt, discontinuous change in physical properties as one passes through the surface separating the two media. As such physical discontinuity is, a priori, highly improbable, it is natural to seek the source of the observed discrepancies in a thin transition layer where the two media interpenetrate, and the physical properties change continuously though rapidly between the constant values they possess beyond the limits of the layer on either side. On this hypothesis it is natural to expect a thinner transition layer and hence a closer approximation to the conditions of a discontinuous change in the case of solids than of liquids. Thus admitting that such layers must exist in all cases, we must conclude from the experiments mentioned that it is only in the case of liquids that they attain a sufficient thickness to be appreciable.

But while the existence of the transition layer seems a fairly certain inference from the phenomena of reflection, it is equally certain that the greater part of the divergences of experiment from the predictions of theory are to be ascribed to films of surface contamination due to dirt or polisher. Such films may be exceedingly thin—of the same order of magnitude in fact as the transition layer. In any given case both the layer and the film may be present and the observed ellipticity be due to their combined action. The principal difference in the effect produced by these two causes would be due to the fact that in the case of the transition layer its index of refraction must vary between those of the two media; while in the case of the film of surface contamination its index should be approximately constant and its value might be greater or less than or intermediate between those of the two media.

Mathematically the effects to be expected from either the transition layer or the film can be thrown into identical form, the only difference in the two cases being that imposed on the interpretation of the results by the possible values of the index of refraction, as indicated above. The mathematical theory of such layers or films has been worked out in great detail by Drude,† who has shown that on the assumption that the thickness of the film is small compared with the wave length

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* Wied. Ann., xxxvi, p. 532, 1889.

† Wied. Ann., xxxvi, p. 865, 1889. A brief development of the theory of the transition layer is given in the *Theory of Optics*, by Drude, translated by Mann and Millikan, 1902, p. 287. Also in the *Physical Optics*, by Wood, 1905, p. 296; and in Winklemann, *Handbuch der Physik*, 2d Auf. 7, 1906, vol. vi, p. 1256.

of the light employed, its effect can be expressed as a correction term to the ordinary Fresnel equations. The form of this correction term shows that when the reflection takes place from the surface of a substance of greater index than that in which the light is incident, we should expect a positive ellipticity at the polarizing angle for any film which has an index lying between those of the two media, and a negative ellipticity for a film with a greater index than that of either medium. Thus a real transition layer can theoretically produce only a positive ellipticity; while a negative ellipticity must always be attributed to films of surface contamination with indices greater than that of the reflecting substance. Of course *all* of an observed positive ellipticity may not be due to a transition layer, for the matter contaminating the surface may have an index intermediate between those of the two media; but a negative ellipticity is theoretically a certain criterion of a film of contamination. Lord Rayleigh's experiments mentioned above are thus in striking accord with the theory.

The same general considerations hold in the case of the reflection from metals. Here, however, since metals naturally produce an elliptic (or a circular) polarization at all angles of incidence, the effect of the transition layer or film of contamination will be to change the ellipticity and the phase difference of the two components of the vibration from the values they would have if the film were not present. The theory, based on the same assumption as in the case of the reflection from transparent substances,* shows that, providing the index of refraction of the film or layer is greater than unity, the ellipticity should be slightly increased and the phase difference markedly decreased by the presence of such a film or layer. Expressed in terms of the principal incidence and azimuth instead of the phase difference and ellipticity, the theory predicts that the effect of the film should be to decrease the principal incidence markedly and to increase the principal azimuth but a very little. Unlike the result for the reflection from transparent substances the sign of the effect is independent of the magnitude of the index of refraction of the reflecting medium unless that should be less than unity.

We have here then no such criterion as before to enable us to distinguish between films of surface contamination and a real transition layer. A possible exception to this statement is the case of those metals which have an index of refraction less than unity. That some metals, notably gold, silver, and copper, do possess such small values of the index seems certain, since the same result is yielded by the reflection and the direct transmission methods. With these substances it might be

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* Wied. Ann., xxxvi, p. 865, 1889.

expected that the transition layer would have an index less than unity and that consequently according to the theory the phase difference would be increased and the ellipticity decreased over the values they would have if no such layer were present. However, from general considerations as to the degree to which it would be possible for the two media to interpenetrate and produce a transition layer, it would seem unlikely that we could obtain any evidence as to its existence; exactly as any such evidence fails in the case of reflection from transparent solids. And in confirmation of this expectation it is found that for all metals, increasing the cleanliness of the surface shows uniformly an increase of the phase difference and a decrease of the ellipticity; there is never any indication of any reversal of the sign of the effect. So that it is fair to assume that when perfect cleanliness has been attained, the correct values of the phase difference and the ellipticity are yielded by experiment, the effect of the transition layer being negligible.

Further light on the question of the transition layer is afforded by experiments on the reflection from metal mirrors immersed in transparent liquids. Here the uncorrected theory leads to an expression for the index of refraction of the liquid (n') in terms of the observed phase differences and ellipticities in air and in the liquid respectively.* An examination of the recorded observations of this nature shows, however, very considerable discrepancies between the values so calculated and those given by the direct refraction methods (n_0). In the following table are given the values of n' and n_0 as calculated from the experimental results. It includes all of the observations on record.†

	n'	n_0	Observer
Gold in water	1·30	1·33	Conroy
“ “ CS_2	1·54	1·64	“
Silver in water	1·25	1·34	Quincke
“ “ “	1·25	1·33	Conroy
“ “ “	1·43	1·34	Sissingh
“ “ turpentine	1·44	1·47	Quincke
“ “ CCl_4	1·35	1·46	Conroy
Copper in water	1·33	1·33	Drude
“ “ alcohol	1·32	1·37	“
“ “ cassia oil	1·60	1·61	“
“ “ CS_2	1·52	1·63	“
Mercury in water	1·24	1·33	Des Coudres
“ “ alcohol	1·28	1·36	“

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* See below, under heading, “Theory.”

† The values are taken from a paper by Drude, Wied. Ann., xxxix, p. 539, 1891. The calculations have been verified except in the case of the experiments of Sissingh and Des Coudres, to whose original papers I have not had access.

	n'	n_0	Observer
Mercury in HCl (dilute)	1·44	1·39	Des Coudres
“ “ “ (conc.)	1·30	1·39	“
“ “ $Na_2S_2O_3$ (conc.)	1·32	1·42	“
“ “ “ (sup. sat.)	1·28	1·43	“
“ “ petroleum	1·38	1·44	“
“ “ chloroform	1·37	1·44	“
“ “ olive oil	1·47	1·47	“
“ “ turpentine	1·25	1·47	“
“ “ “	1·32	1·47	“
“ “ CS_2	1·59	1·63	“
“ “ “	1·48	1·63	“

Now it is impossible to ascribe these discrepancies in every case to films of surface contamination; for they occur for those metals (notably copper and mercury) for which entirely satisfactory results in air have been obtained, and under circumstances apparently precluding the formation of any chemical film. Hence, as we have seen that it is highly probable that a perfectly clean metal possesses no appreciable transition layer when in contact with air, it is only possible to ascribe these discrepancies to a transition layer due to the liquid. Now an inspection of the above table shows that the index calculated from the reflection from the metal is, in all but four cases, less than that deduced by the direct refraction methods. On the theory as developed by Drude,* this result must be interpreted as due to a film or layer having an index less than that of the liquid. This suggests that the effect is due to a film of air or other gas caught between the liquid and the mirror, since the transition layer of a liquid-air surface would have an index less than that of the liquid. It is, moreover, natural to expect the presence of such films due either to entrainment in the act of immersion or to dissolved gas in the liquid, more probably the former.

On the other hand, if it is assumed that no such gaseous film is present, the observed discrepancies must be attributed to a transition layer due to the liquid. In view of the fact that the effects of the transition layer in the case of transparent solids in air are inappreciable, it would seem that the layer in question here cannot be a region of interpenetration of the metal and the liquid. Consequently we must conclude (if we reject the possibility of the gaseous film) that the observed discrepancies are due to a layer at the surface of the liquid which is independent of the medium with which it may be in contact and which has optical properties different from those in the interior. According to the theory, the index of such a layer will be less than that of the interior if n' is less than n_0;

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* Wied. Ann., xxxix, p. 539, 1891. See also, below, under heading, “Theory.”

while its index should be greater when n' is greater than n_0. Such effects might possibly be expected as a result of surface tension.

There are thus two possible explanations of the observed discrepancies. It would seem worth while, therefore, to attempt to eliminate the possibility of gaseous films between the mirror and the liquid. The desirability of doing this has been pointed out by Drude,[*] but so far as I can ascertain, it has not up to the present time been attempted. Drude has expressed the opinion that there would still remain evidence of a transition layer when such gaseous films have been removed.[†] But this opinion is supported by no evidence which cannot be interpreted, as we have seen, in another manner. Hence I have attempted, in the investigation reported in this paper, to see if any residual effect of a transition layer remains, when the possibility of the presence of a gaseous film between the mirror and the liquid is reduced to a minimum.

Theory.

The theory of metallic reflection in transparent liquids is not developed in any of the ordinary works of reference, though it is, of course, accessible in the original memoirs. So it may not be out of place to give a brief résumé of the theory here. We start with the equation

$$\frac{1 + \operatorname{tg}\psi e^{i\Delta}}{1 - \operatorname{tg}\psi e^{i\Delta}} = \frac{\sin\phi \sin\chi}{\cos\phi\cos\chi},\ddagger \qquad (1)$$

where ψ is the azimuth of the restored polarization (the angle whose tangent gives the ratio of the amplitudes of the two components of the reflected vibration when the incident vibration is polarized in a plane making an angle of 45° with the plane of incidence; $\operatorname{tg}\psi$ is what we have called the "ellipticity" in the introduction); e is the Naperian base; $i = \sqrt{-1}$, ϕ and χ are the angles of incidence and refraction respectively; while Δ is the phase difference between the two components of the reflected vibration.

In the case of the reflection in a vacuum (or air) we have the relation

$$\frac{\sin\phi}{\sin\chi} = \sqrt{K}, \qquad (2)$$

where K is the dielectric constant of the reflecting medium.

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[*] Wied. Ann., xxxix, p. 545, 1891. Also, Winklemann, Handbuch der Physik, 2te Aufl., 1906, vol. vi, p. 1308.

[†] Winklemann, loc. cit.

[‡] The derivation of this equation may be found in any standard text on Optics; e. g., those of Drude, Schuster, Wood.

In the case of a metal this must be supposed complex and the real part of $\sqrt{K}$ is the index of refraction. If we substitute for χ in (1) from (2), replace the exponential by its equivalent trigonometrical expression, and rationalize the denominator of the left-hand side of (1), we obtain

$$\frac{\cos 2\psi(1+i\sin\Delta\,\mathrm{tg}\,2\psi)}{1-\cos\Delta\sin 2\psi\cdot}=\frac{\sin\phi\,\mathrm{tg}\,\phi}{\sqrt{K-\sin^2\phi}}$$

This may be simplified by making the following substitutions:

$$\begin{aligned}&\sin\Delta\,\mathrm{tg}\,2\psi=\mathrm{tg}\,Q\,;\ \cos\Delta\,\sin\,2\psi=\cos P\,;\\&\cos\,2\psi=\cos Q\sin P;\ S=\mathrm{tg}\frac{P}{2}\sin\phi\,\mathrm{tg}\phi;\end{aligned}\tag{3}$$

which yield

$$e^{iQ}=\frac{S}{\sqrt{K-\sin^2\phi}}.\tag{4}$$

In the case of the reflection in a transparent medium of index of refraction n_0, we have in place of (2) the relation

$$n_0\frac{\sin\phi}{\sin\chi}=\sqrt{K}.$$

Substituting from this in (1) and performing the same operations and making the same substitutions as before; we get

$$e^{iQ_0}=\frac{n_0 S_0}{\sqrt{K-n_0^2\sin^2\phi}}.\tag{5}$$

Dividing (4) by (5) and substituting for the radical in the denominator its value from (4) we get

$$n_0\frac{S_0}{S}e^{i(Q-Q_0)}=\frac{e^{iQ}}{S}\sqrt{K-n_0^2\sin^2\phi}.$$

Using in this the value of K from (4),

$$K=\frac{S}{e^{i2Q}}+\sin^2\phi,$$

we obtain

$$n_0\frac{S_0}{S}e^{i(Q-Q_0)}=\sqrt{1-\frac{\sin^2\phi}{S^2}(n_0^2-1)e^{i2Q}}.$$

Expanding the radical and retaining no terms higher than the second in $\frac{\sin\phi}{S}$,* and substituting

$$\delta = -\tfrac{1}{2}\frac{\sin^2\phi}{S^2}(n_0^2-1)\cos 2Q \quad (6)\,;$$

$$\epsilon = \tfrac{1}{2}\frac{\sin^2\phi}{S^2}(n_0^2-1)\sin 2Q\,; \quad (7)$$

we have, on equating the real and imaginary parts of each side, approximately,*

$$n_0 = \frac{S}{S_0}(1+\delta) \quad (8)\,; \quad Q-Q_0 = -\epsilon. \quad (9)$$

Thus from the observed values of Δ and 2ψ we can calculate, by means of equations (3) and (6) the right-hand side of equation (8). Calling the value so calculated n', we have, $n_0 = n'$; or, the index of refraction of the liquid calculated by refraction methods should be the same as that deduced from the change of phase and the ellipticity produced by the metal.

If there are transition layers or films of surface contamination present, their effect may be expressed as a correction term to equation (5) as follows:

$$e^{iQ_0} = \frac{n_0 S_0}{\sqrt{K-n_0^2\sin^2\phi}}(1-i\sqrt{K}A).\dagger \quad (10)$$

If we assume that there is but one transition layer, that between the liquid and the gas film, the value of A will be given by

$$A = \frac{2\pi}{\lambda}\int_0^L\left(\frac{n_0^2}{n_1^2}-1\right)dl,\dagger \quad (11)$$

where λ is the wave length of the light employed, L the thickness, and n_1, the index of the layer. Of course a similar correction term to equation (4) can be written down, but under the assumption that a metal in air possesses no appreciable transition layer, that may be omitted.

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* These approximations are justified by the magnitudes of S and Q as found from experiment.

† Since the results of the experiments I have to report in this paper seem to show that A=0 (for the liquids used, at least), it seems scarcely worth while in this place to devote the very considerable amount of space which would be required, to the derivation of equations (10) and (11). The derivation is given in full in the papers of Drude in Wied. Ann. already cited.

Proceeding now as before, we obtain the equation

$$n_0 \frac{S_0}{S} e^{i(Q-Q_0)} = \sqrt{1 - \frac{\sin^2\phi}{S^2}(n_0^2-1)\, e^{i2Q}}\,(1+i\sqrt{K}\,A).$$

Substituting in the correction term the approximate value of $\sqrt{K}$ from (4) we have on expanding and making the substitutions (6) and (7),

$$n_0 = \frac{S_0}{S}(1+\delta+AS\sin Q) \quad (12)\,; \quad Q-Q_0 = -\epsilon + AS\cos Q. \quad (13)$$

If we set as before, $n' = S/S_0\,(1+\delta)$, equation (12) can be written

$$n' = n_0(1-AS\sin Q), \tag{14}$$

since both δ and $AS\sin Q$ are small quantities.

By means of this equation and equation (11) we can compare the results of experiment with theory. Thus from the latter we see that if n_1 is less than n_0, then A will be positive and consequently by equation (14) n' should be less than n_0. This then is the result we should expect in case there is a gaseous film between the mirror and the liquid; or, providing that is eliminated, it is the result to be expected in the event that a real transition layer (belonging to the liquid alone, as we have seen) exists. Similarly, if n_1 is greater than n_0, we see from equation (11) that A will be negative, and consequently from equation (14) that n' should be greater than n_0. Such a result would mean a film of greater index than that of the liquid and hence must be attributed either to films of surface contamination or to a real transition layer, no gaseous film being possible under these conditions. Finally, we see that if $A = 0$, $n' = n_0$, and this is the result to be expected if no films of any kind are present.

Thus if we can make sure that there is no gaseous film nor any film of surface contamination present, a value of n' less n_0 would indicate that at its surface a liquid has an index of refraction less than in the interior; while a value of n' greater than n_0 would be evidence showing that the index of the liquid at the surface is greater than in the interior.

Experimental Arrangements.

In repeating these experiments which have been performed so many times and by such competent observers, it was sought to improve upon their results in two directions: first, in securing a mirror which should be more free from surface contamination than those used before; and second, in reducing the possibility of a gaseous film between the mirror and the liquid to a minimum. On account of the difficulties in the way of certainly getting rid of films of surface contamination in solid mirrors, it was decided to attempt to attain the first object by using a mercury mirror. The advantages of such a mirror are, first, it admits of obtaining an extraordinarily clean surface much more easily than any other metal; second, exactly similar surfaces can be easily and quickly reproduced; and third, the degree of cleanliness attained can be estimated better than with mirrors of solid metal. The method of cleaning the surface by stretching the dirt to the edges (first suggested, in the case of water, by Lord Rayleigh)* was adopted in the form recommended by Röntgen.† This is described below.

It was feared at first that the attainment of the second object would present formidable if not insuperable difficulties. Drude has suggested the possibility of removing the film of gas by heating;‡ but this would involve obvious complications and difficulties. As mentioned in the introduction, it seems probable to expect that the gas film is introduced by entrainment in the act of immersion. Hence it was decided to produce the clean mercury surface *while* the mirror was immersed. The method of Röntgen for producing the clean surface lends itself very readily to this purpose, and it would seem, from the results obtained, that complete success in this important object has been attained. The form which the apparatus for producing the mirrors finally took is shown in the figure.

A glass tube of about 4^{cm} diameter was drawn down to form a funnel-shaped vessel (A), into the neck of which was sealed a small tube (B), having its end drawn out to a capillary point and projecting into the funnel. This tube, passing through a cork at the bottom of the cylindrical iron vessel (C) which surrounded the funnel, was supported so that the mouth of the funnel (ground to a plane) came slightly above the top of the cylinder. Another tube (D) through the cork (which was provided with a suitable stopcock, not shown in the figure) carried away the mercury which overflowed the funnel. The iron cylinder passed through the bottom of an open rectangular iron box (E) and projected to within about $1{\cdot}5^{cm}$ of its top. This box served to hold the liquids under which the reflection was observed.

This surface-cleaning apparatus was mounted on a stand furnished with levelling screws to enable one to adjust the mouth of the funnel accurately to a horizontal plane. The

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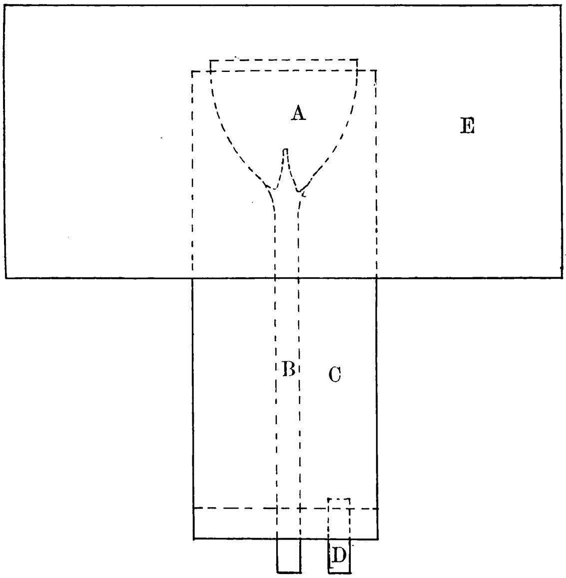
* Phil. Mag. (5), xxx, p. 392, 1890.
† Wied. Ann., xlvi, p. 152, 1892.
‡ Wied. Ann., xxxix, p. 545, 1891.

stand was also provided with a rack and pinion to give the vertical adjustment necessary to bring the reflecting surface to the center of the spectrometer table. A reservoir of mercury hung at a higher level than that of the funnel was connected with the tube (B) by a flexible tube provided with a suitable pinchcock. In order to secure a mirror sufficiently free from vibration, it was found necessary to mount the stand in a Julius suspension.

The operation of producing a clean surface was simply to allow the mercury to overflow the edge of the funnel rapidly for a few seconds. If care is taken to make the overflow uniform all around the edge (accomplished by adjusting the levelling screws), surfaces of extreme cleanliness can easily be obtained. The test of the cleanliness is the behavior of a drop of water placed on the surface. In making the test it is essential that the drop itself be clean. This is most easily done in practice by dipping a piece of platinum wire (cleaned by heating) beneath the surface of a beaker of distilled water, and removing the adhering drop quickly to the mercury surface. If the surface is really clean, the drop is immediately stretched out to the edge in a film of invisible thickness. The appearance of a mercury surface to the eye is no test at all of its cleanliness; it may seem to be a perfect mirror and yet the drop remain in an hemi-



spherical form when placed on it, unless it has been recently cleaned in the manner described. It is not necessary to start with especially clean mercury,—it cleans itself in a very short time. Nor is it necessary to test the cleanliness of the surface except after each fresh filling of the reservoir. A good surface gets contaminated in the air in a brief time, but retains its cleanliness long enough to permit the necessary observations to be taken comfortably; while a fresh surface can be obtained easily and quickly.

Thus it is reasonably certain that the reflecting surfaces used were free from surface contamination. Of course it is not so certain, a priori, that the film of gas was removed when the mirror was submerged. It is natural to expect, however, that the rapid overflowing of the mercury would tend to carry away any such film and leave a more perfect contact between the two media. And from the final result of these experiments it would seem that this expectation is fully justified. During the course of a set of observations under a liquid it was of course necessary to close the overflow tube (D), consequently the cylinder (C) was made large enough to hold all of the mercury which it was necessary to allow to overflow in the course of such a set. The liquid thus displaced from the box (E) was made to overflow at one corner and was caught in a suitable vessel.

In order to avoid any deviation of the beam of light by refraction at the free surface of the liquid, glass tubes terminated by plane-parallel glass ends were carried in suitable brass holders by both the collimator and telescope arms; when the mirror was in position the ends of these tubes were just submerged in the liquid in (E). The length of the box (E) was made as great as the space between the tube-holders would allow, in order to obtain as large an angle of incidence as possible. The space thus available was, however, too small to permit the use of angles of incidence greater than 60°, which, though lessening the precision attainable in this kind of experiment, still yielded ample accuracy for the ends in view.

The apparatus for measuring the phase difference and azimuth of restored polarization was of a familiar type and needs but brief description. The spectrometer* was one by the Société Genevôise, arranged so that the table could be swung to a vertical position to accommodate a horizontal mirror. The collimator arm carried the polarizing nicol behind the lens, the Babinet compensator was mounted before the objective of the telescope, while the analyzing nicol was carried at the other end of the telescope tube. The nicol and compensator came with the spectrometer and were of good quality. By means of

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* This was obtained by the aid of a grant from the Bache fund of the National Academy of Science.

a second lens which could be inserted behind the analyzer, and an extra draw tube, the eyepiece could be focused on the compensator fringes. Since these are not definite objects on which to focus, a small circular diaphragm was inserted behind the last lens of the ocular in order to avoid a small residual parallax which was otherwise necessarily present. These modifications of the apparatus as originally received from the Société Genevôise, and also the holders for the glass tubes and the surface-cleaning apparatus described above, were made in the instrument shop of the Sheffield Scientific School. The divided circle of the spectrometer table could be read to 20″, and those of the nicols to 6′ of arc. The source of light used was that obtained from the sodium flame of a small blast lamp fed with oxygen.

Measurements.

The preliminary adjustments of the apparatus, i. e., making the axes of collimator and telescope to intersect the axis of the spectrometer table orthogonally, and the adjustment of the axes of the nicols to parallelism with those of the collimator and telescope, were made in the usual manner. The positions of the polarizer giving light polarized in the plane of incidence were determined by reflection from water at the polarizing angle. The water surface was cleaned in the same manner as described above for the mercury. This plane was determined several times in the course of the investigation and with several minor modifications of method. Thus, sometimes the extinction position of the analyzer was determined with the reflection taking place at the polarizing angle, and then that of the polarizer by getting its "crossed" position with reference to the analyzer when the two were put in line; or, sometimes the extinction position of the polarizer for the reflection at the polarizing angle was determined directly, the beam of light being sent through the apparatus in the reverse direction: another modification was the use of glass of known index in place of water. None of these various determinations yielded a position of the polarizer differing from that of any other determination by an amount as large as the probable error of the readings. The result of all these determinations gives a value of 156·58° for the position of the polarizer. The probable error is less than 0·003°.

The calibration of the compensator was made with the polarizer in all four of the positions yielding light vibrating in a plane making an angle of 45° with the plane of incidence. For each position the reading for the center fringe and the first fringe on either side of the center was determined as the mean of fifteen settings. Thus each position of the polarizer yields two values of the displacement of the compensator corresponding to a change of phase of 360°. The results, in terms of turns of the compensator screw, follow :

Polarizer set to -----	21·58°	201·58°	111·58°	291·58°	
Compensator (left) --	8·988	8·967	8·982	8·962	
" (right)-	8·971	8·982	8·974	8·979	
" (av.)---	8·980	8·975	8·978	8·970	8·9756 ± 0·0021

Compensator left or right means that the first fringe to the left or right of the central one was under the cross wires. On two occasions in the course of the investigation it became necessary to take the compensator to pieces, once to clean it and the second time to replace the cross wires. On each occasion it was re-calibrated and both times with the same result within the probable error of the determinations. The constant of the compensator, i. e., the factor by which its readings must be multiplied in order to reduce them to degrees, deduced from the above readings is 40·11. The position of the central fringe (zero point of the compensator) was found to vary slightly with temperature but otherwise was constant. The maximum variation observed was 0·035 turns. The scale on the compensator is so placed that its direct readings yield the supplement of the angle of phase difference rather than the angle itself.

The observations of the phase difference and the angle of restored polarization were made in the following manner in every case : (1) With both the telescope and collimator arms level and the surface-cleaning apparatus lowered out of the way, the zero point of the compensator was determined as the mean of at least twelve settings taken coming up from both directions. (2) With the two arms set to give an angle of incidence $\phi = 60°$, the mirror was brought into position, adjusted and cleaned in the manner which has been described. (3) With the polarizer set to give light vibrating in a plane inclined at an angle of 45° to the plane of incidence, the displaced position of the central fringe was observed (again as the mean of at least twelve settings from each direction), and the position of the analyzer making the fringes blackest was determined. For this last at least twenty-four settings were made in each case. (4) With the polarizer set to give light vibrating in the other 45° plane, the compensator and analyzer positions were again determined and with the same number of readings for each. (5) Finally, with the arms brought back into line, the zero point was again determined as in (1), so as to be able to take account of any temperature change in the compensator. The reflecting surface was renewed, of course, as often as might be necessary in the course of a set of observations.

The phase difference is obtained from the difference of the compensator readings of (3) and (4) and those of (1) and (5). The double angle of restored polarization is the difference of the positions of the analyzer in (3) and (4).

There were made, in all, nine complete sets for Δ and 2ψ as described above: four in air, three in water, and one each in the HCl and turpentine. The results are shown in the following table. In the first column are given the observed values of the displacement of the center fringe in terms of turns of the compensator screw, and in the second the directly observed values of 2ψ (or $2\psi_0$).

	Comp.	2ψ or $2\psi_0$
Air	0·7538 ± 0·0041	81·04° ± 0·35°
"	0·7549 ± 0·0018	81·54 ± 0·19
"	0·7550 ± 0·0017	81·32 ± 0·54
"	0·7548 ± 0·0052	81·46 ± 0·23
" (av.)	0·7546 ± 0·0017	81·34° ± 0·18°
Water	0·9975 ± 0·0035	81·65 ± 0·48°
"	0·9935 ± 0·0010	82·05 ± 0·35
"	0·9934 ± 0·0039	81·74 ± 0·20
" (av.)	0·9948 ± 0·0020	81·81° ± 0·16°
HCl	1·0030 ± 0·0099	81·91° ± 0·35°
Turpentine	1·0035 ± 0·0069	82·08° ± 0·34

From the average values above, the values of $\pi-\Delta$ (or $\pi-\Delta_0$) are computed by multiplication with the compensator constant, while the values of Q (or Q_0), P (or P_0), S (or S_0), δ, and n' are calculated by equations (3), (6), and (8). The results of the computations are given in the next table.

	Δ or Δ_0	Q or Q_0	P or P_0	S or S_0	δ	n'
Air	149·73° ± 0·07°	73·19°	148·63°	5·343		
Water	140·10° ± 0·08°	77·38°	139·40°	4·055	0·0084	1·329 ± 0·005
HCl	139·73° ± 0·40°	77·58°	139·10°	4·024	0·0086	1·340 ± 0·015
Turpentine	136·35° ± 0·28°	78·61°	135·78°	3·693	0·0127	1·465 ± 0·011

Finally, samples of the liquids used were put in a hollow prism and their indices determined in the usual way from the angle of the prism and the angle of minimum deviation. The values thus obtained for n_0 were:

	n_0
Water	1·3320
HCl	1·3375
Turpentine	1·4690

These values are certainly correct to the third place.

Discussion of the Results.

An inspection of the tables just given shows that in the case of each of the liquids examined $n' = n_0$ within the limits of error of the observations. In view of what has been said in the introduction and in the discussion of the theory of the experiments, this result indicates that there was present no appreciable film or layer of any description; and consequently, for these liquids at least, the transition layer which has been inferred from the phenomena of the reflection in air is not independent of the medium with which it is in contact. In other words, the transition layer is to be regarded as a true region of interpenetration of two media; and the fact that it is appreciable in liquids and not in solids must be attributed to the greater freedom of molecular motions in the case of the former, and not to any change in physical properties at the surface brought about by the force of surface tension.

The liquids chosen for this investigation include those for which the previous observations have yielded values of n' less than n_0, and one for which a value of n' greater than n_0 has been found. As the result of this work goes to show that both of these sorts of discrepancy vanish when all possible extraneous films are removed, it would seem a fair inference that all of the discrepancies which have been found previously are to be attributed to the same causes, and that the conclusion deduced from these experiments is a general one.

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New Haven, Conn., April, 1911.