THE FOUCAULT PENDULUM OF THE PEABODY MUSEUM OF YALE UNIVERSITY.

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INTRODUCTION.

The demonstration of the rotation of the earth by purely physical means, as devised by Foucault in 1858, has always exercised a fascination upon intelligent people. For this reason it has long been recognized as a desirable addition to museums and to physical laboratories; but the prevalent opinion that for successful action a great length of pendulum is requisite, which demands a free space possessed by few buildings, has restricted the number of examples. One of the purposes of this article is to show that the importance of this condition is overestimated; another is to prove the extraordinary attainable accuracy of the pendulum when properly designed and constructed, ranking it indeed among instruments of precision.

The principle guiding Foucault in his invention is that a pendulum, absolutely free from constraint, will maintain its plane of vibration irrespective of the rotation of its supporting member. Therefore, if suspended over a fixed horizontal table which must revolve with the earth, its plane will appear to rotate at a constant angular velocity in a determinate direction in the northern hemisphere, which is reversed south of the equator. The hourly angle of apparent revolution of the plane is 15° sin Lat., since the earth makes a complete rotation of 360° in twenty-four hours. The chief practical difficulties in executing the experiment are two; first, the frictional resistance of the air and of the suspension may bring the pendulum to rest too quickly, and, second, the inevitable tendency of the bob to move in a varying ellipse instead of remaining in a vertical plane. Evidently both of these defects are reduced by increase of the length and weight of the pendulum, whence the inclination to look for success only in such pendulums.

In the case under discussion great length is precluded since, although the tower of the museum is high, it has a free opening only through the second floor, so that above the table a vertical height of 35 feet only is at command. The accepted methods of suspension are in general, either the familiar gimbals, consisting of two knife edges at right angles such as employed in the mariner's compass, or a hook of which the point rests on a hard steel plate while the wire which carries the bob depends from the lower part of the hook. The former method has been adopted in the massive pendulum, weighing 1,300 pounds, which swings in the Palmer Physical Laboratory at Princeton University, and the latter in the famous 105 ft. pendulum at Washington. A third method, which the writer used more than forty years ago in his regular lectures on experimental physics, was by means of an elastic wire firmly clamped at the top. This was found to be far better than the elaborate knife-edge suspension supplied by the German instrument makers and which was at that time, perhaps, quite novel. This experience was taken as a guide in our design. Another departure from customary practice was the use of a far less massive bob. This was, of course, at a sacrifice of a more sustained amplitude, but it possessed a certain convenience and, as will appear below, the inevitable decrement in amplitude does not impair its value for the end in view.

DESCRIPTION OF PENDULUM.

The bob is a lenticular mass of lead weighing 28 pounds suspended by a highly elastic piano wire about 35 feet in length over a cement table. The form of a horizontal double convex lens is chosen so as to reduce the air resistance and the wire is as fine as it was thought safe to use; it breaks under a stress equivalent to twice the weight of the bob. The bob has a pointer below at its center. Upon the table rests a circle of paper, its center being vertically under the point of suspension and having a plate of glass lying upon it to keep it in place. The first and third quadrants are divided by diameters spaced 9.9° apart and numbered with the hours of the clock commencing with IX when the pendulum is released every morning. The radius of the circle is 11 inches, which is the amplitude of the swing when starting. The table bears an octagonal lantern 53½ inches across and 32 in height which serves to protect the pendulum from air currents; this is an essential precaution because its locality is just inside the main entrance doors. The release is accomplished by the ordinary method of burning a thread which initially fixes the bob at the edge of the disk.

When this pendulum was first mounted it was found that the path of the pendulum changed slowly from an apparent straight line to a very elongated ellipse with an increasing excentricity. This fault seems practically characteristic of all such systems. The defect would not be very objectionable since the direction of the major axis of the ellipse is determinable with almost as great accuracy as rectilinear oscillations, were it not for the fact that such elliptic paths carry with them also a variation in the azimuth of the path which contradicts the assumed law of rotations. The causes of this troublesome defect are various. Any variation in the moment of inertia of the oscillating system with respect to axes in different directions; minute differences in position of knife-edge suspension or even of different degrees of sharpness; or, in case of hook suspension, departure from a true surface of revolution in the point of the hook. In our own case the cause is demonstrably due to different degrees of stiffness of the wire in different directions. The difference may be due to minute lack of roundness or to strain in its structure. The fact that when unconstrained the wire lies in a coil seven or eight inches in diameter inclines me to the latter explanation. To eliminate it we may seek a better wire, or, which is far from impossible, compensate by a proper change in effective diameter at the upper portion of the wire, neither of which methods have we followed. It is easy to describe what the general effect of such disturbances would be although a rigid solution of the problem may be troublesome on account of the large number of constants involved. For example, if the pendulum were subject to the elasticity of the wire alone it would swing with an enormously long period and in a path which may be described as a Lissajous figure of a false unison; as it is, we must think of this modification impressed upon the simple pendulum motion with the important restriction that, since the wire is revolving with the earth as is the table below, the whole change has a period of about eighteen hours during which it completes all of its phases since the wire makes a half revolution in that time. It is this property of our system which led us to a far simpler adjustment. The top suspension was first turned so that the growth of ellipticity was thought to be the greatest attainable; it chanced that the circulation in this ellipse was left-handed as seen from above, indicating that the initial release was in the quadrant following the plane of minimum rigidity. Then the upper suspension was adjusted until the growth of ellipticity in the path was thought to be equal but with a circulation in the opposite direction. Finally the upper holder was turned to a position midway between these. With this adjustment the pendulum meets all our requirements; it followed the division of the dial with a precision which shows no obvious errors for much more than the eight hours during which the Museum is open to the public. The time can be read from the dial with greater precision than from a sun dial.

THE PENDULUM AS AN INSTRUMENT OF PRECISION.

The instrument is so satisfactory and repeats so exactly its performance day after day that we were prompted to test its ultimate capacity. To this end Mr. Herpich and his assistant made for me a series of observations by a very simple method. A sheet of cardboard with a centered circle of 22 inches diameter was placed upon the table and over this was laid a thin wire straightened by a bow. At nine o'clock, when the pendulum was released, and at every successive hour until four o'clock in the afternoon, the wire was adjusted so as to lie as accurately as possible in the plane of oscillation as determined by the pointer at the bottom of the bob, and the points where the wire crossed the circle were marked with a pencil. This process was repeated on four different days so that we secured a total of 64 such points. From these cards the angles of rotation of the table for each interval were found from the measured chords, thus eliminating errors of excentricity. The accuracy of the records proved highly satisfactory as appears below in the final analysis.

The mean values of the rotation angles are given in the first column of the table below under the heading θ, and the dif-

θ	h	δ	H	v
°	°	°	°	°
5.00	9.875	+0.074	9.949	+.018
14.79	9.816	0.119	9.935	+.004
24.50	9.774	0.161	9.935	+.004
34.12	9.718	0.198	9.916	—.015
43.68	9.692	0.228	9.920	—.011
53.27	9.674	0.254	9.928	—.003
62.87	9.663	0.272	9.935	+.004
			9.931	±.002

ferences in the next column, under h, are the successive hourly angles of rotation. The latter angles are subject to accidental errors of observation and to periodic errors due to the imperfection of the wire. Assuming the latter to follow the law

$$\delta = a \sin (\theta + \alpha)$$

we have to find such values of a and of α as will make the sum of the apparent errors, under v, equal to zero and the sum of their squares a minimum. This process yields $a = 0^\circ.284$, $\alpha = 10^\circ$, whence we conclude that the rotation per hour of a horizontal plane at the latitude of the pendulum is $9^\circ.931 \pm .002$. To reduce this to sidereal time by which we measure the rotation of the earth, it is necessary to multiply by the ratio of 23 hours and 56 minutes to 24 hours, which reduction gives $H_0 = 9^\circ.903 \pm .002$ for the required hourly rotation.

Perhaps the readiest way to give a definite idea of the accuracy of our measurement is to deduce from it the latitude of the pendulum. This is derived from the equation, given above,

$$\sin \text{Lat.} = H_0 / 15^\circ$$

whence Lat. $= 41^\circ.312 \pm .016$; the true latitude, as measured directly from the Yale Observatory, is $41^\circ.311$. Thus the final conclusion is that the pendulum has determined the latitude with an error of only 0.4 of a statute mile with an expectation of an error not more than twice as great. This is far more accurate than ordinary sextant determinations at sea, and it could obviously be improved by better, readily devised methods of measuring.

CONCLUSION.

The sharp distinction between mean solar time and sidereal time so plainly betrayed by the pendulum suggests some interesting reflections as to how far our accumulation of knowledge derived from astronomical observations might be obtained were we cut off entirely from such observations—in short, how far the physicist with his methods could replace the astronomer and his older science. Suppose, for example, that the earth were surrounded by a layer of clouds, as are probably all the other planets with a single exception, so that no celestial body could be observed, then by means of such pendulums the physicists could prove that we were living on the surface of a rotating globe having a determined period. Moreover, they could fix accurately the position of the equator and of the latitude of any accessible place; by measurements between two determined latitudes the size, and, in view of the lengthening of the degrees at higher latitudes, even the spheroidal shape of the earth would reveal themselves. By observing the intervals and durations of successive periods of light and darkness, aided by photometry, the length of the solar year in terms of the period of the earth's revolution could be determined accurately in a very few years. Such careful photometric observations would be certain to fix the apparent orbit of the external source of light including its declination and excentricity; the data attaching to the sun would be confirmed by analysis of the tides and the inevitable discovery of the existence of the moon, with the approximate constants of its orbit, would follow such analysis.

In closing this description of our pendulum and its capacities it gives me pleasure to record our obligations to Mr. F. C. Herpich, the mechanician of the Museum, for his sustained and highly effective interest in the experiment.

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