Art. XIV.—*On the Drawing of Crystals from Stereographic and Gnomonic Projections;* by S. L. Penfield.

In a previous communication by the present writer,* some methods for drawing crystal forms from a stereographic projection were described, and, after the publication of the paper, there was observed in a recent volume by Professor C. Viola of Rome,† a method, based upon different principles, which is so simple and ingenious that it seems wise to give a brief description of it, for the benefit of those readers of this journal who may be interested in the subject: this paper may also serve as a supplement to the writer's earlier article, referred to above.

In explaining the method, a general example has been chosen; the construction of a drawing of a crystal of axinite, of the tricilinic system. Figure 1 represents a stereographic projection of the ordinary forms of axinite, *m* (110), *a* (100), *M* (1$\bar{1}$0), *p* (111), *r* (1$\bar{1}$1) and *s* (201). As shown by the figure, the *first meridian*, locating the position of 010, has been chosen at 20° from the horizontal direction *SS'*: This is wholly arbitrary, but it makes a good starting point for the construction of a stereographic projection.

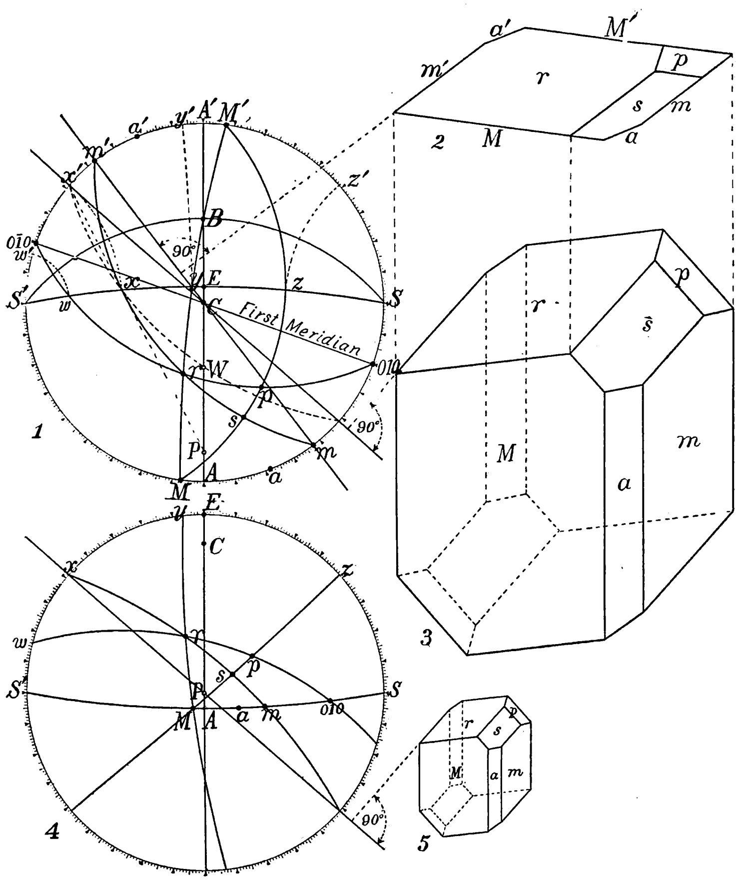
Figure 2 is a *plan*, or an orthographic projection of an axinite crystal, as it appears when looked at in the direction of the vertical axis. It may be derived from the stereographic projection in a simple manner, as follows:—The direction of the parallel edges made by the intersections of the faces in the zone *m*, *s*, *r*, *m'*, figure 1, is parallel to a tangent at either *m* or *m'*, and this direction may be had most easily by laying a straight edge from *m* to *m'* and, by means of a 90° triangle, transposing the direction to figure 2, as shown by the construction.

The construction of figure 3, which is called by Viola a *parallel-perspective* view, may next be explained: It is not a clinographic projection like the usual crystal drawings from axes, but an orthographic projection, made on a plane intersecting the sphere, represented by the stereographic projection, figure 1, along the great circle *SES'*; the distance *EC* being 10°. The plane on which a drawing is to be made may, of course, have any desired inclination or position, but by making the distance *CE* equal 10° and taking the first meridian at 20° from *S*, almost the same effects of plan and parallel-perspective are produced as in the conventional method of drawing from axes, where the eye is raised 9° 28' and the crystal turned

* This Journal (4), xix, p. 39, 1905.

† Grundzüge der Krystallographie, p. 29, W. Engelmann, Leipzig, 1904.

18° 26′*; in fact, even when drawn on quite a large scale, the plan and parallel-perspective views, figures 2 and 3, are so nearly identical with corresponding figures of axinite, page 73 of the writer's earlier paper just referred to, that the eye can



FIGURES 1 to 5. Development of a plan and parallel-perspective figure of axinite, triclinic system, from a stereographic projection.

scarcely detect any difference between them, even when placed one above the other.

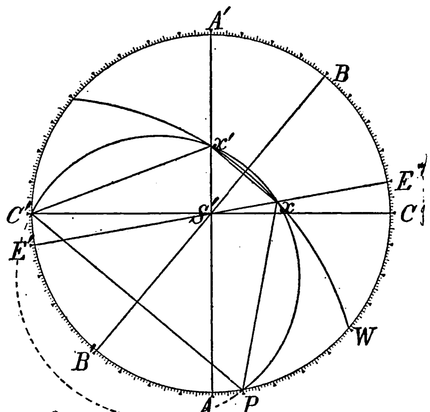
The easiest way to explain the construction of figure 3 from figure 1 is to imagine the sphere, represented by the stereographic projection, as revolved 80° about an axis joining *S* and *S*′, or until the great circle *SES*′ becomes horizontal. After such a revolution, the stereographic projection shown

* This Journal (4), xix, p. 40, 1905.

in figure 1 would appear as in figure 4, and the parallel-perspective drawing, figure 5, could then be derived from figure 4 in exactly the same manner as figure 2 was derived from figure 1. This is, for example, because the great circle through m, s and r, figure 4, intersects the graduated circle at x, where the pole of a vertical plane in the same zone would fall, provided one (artificially constructed or otherwise) were present; hence the intersection of such a surface with the horizontal plane, and, consequently, the direction of the edges of the zone, would be parallel to a tangent at x: In other words, figure 5 is a *plan* of a crystal in the position represented by the stereographic projection, figure 4. Although not a difficult matter to transpose the poles of a stereographic projection so as to derive figure 4 from figure 1, it takes both time and skill to do the work with accuracy, and it is not at all necessary to go through the operation. To find the direction of the edges of any zone in figure 3, for example $m\ s\ r$, note first in figure 1 the point x, where the great circles $m\ s\ r$ and SES' cross. During the supposed revolution of 80° about the axis SS', the pole x follows the arc of a small circle and falls finally at x' (the same position as x of figure 4) and a line at right angles to a diameter through x', as shown by the construction, is the desired direction for figure 3. Similarly for the zones pr, MrM' and $MspM'$, their intersections with SES' at w, y and z are transposed by the revolution of 80° to w', y' and z'. The transposition of the poles w, x, y and z, figure 1, to w', x', y' and z' may easily be accomplished in the following ways:—(1) By means of the transparent, small-circle protractor described by the writer,* the distances of w, x, y and z from either S or S' may be determined and the corresponding number of degrees counted off on the graduated circle. (2) Find first the pole P of the great circle SES', where P is 90° from E or 80° from C, and is located by means of a stereographic scale or protractor: A straight line drawn through P and x will so intersect the graduated circle at x', that $S'x$ and $S'x'$ are equal in degrees. The reason for this is not easily comprehended from figure 1, but if it is imagined that the projection is revolved 90° about an axis AA', so as to bring S' at the center, the important poles and great circles to be considered will appear as in figure 6, where P and C' are the poles, respectively, of the great circles $ES'E'$ and $AS'A'$, and x is $41\frac{1}{4}$° from S' as in figure 1. It is evident from the symmetry of figure 6 that a plane surface touching at C', P and x will so intersect the great circle $AS'A'$ that the distances $S'x$ and $S'x'$ are equal. Now a plane passing through C', P, x and x', if extended, would intersect the sphere as a

* This Journal (4), xi, p. 17, plate I, 1901.

small circle, shown in the figure, but since this circle passes through C'', which in figure 1 is the pole of the stereographic projection (antipodal to C), it follows that it will be projected in figure 1 as a straight line, drawn through P and x.* (3) In figure 6, B is located midway between E and A', $BS'B'$ is a great circle, and W, 40° from C, is its pole: It is now evident from the symmetry of the figure that a great circle through W and x so intersects the great circle $AS'A'$, that the



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distances $S'x$ and $S'x'$ are equal. Transferring the foregoing relations to figure 1, W, 40° from C, is the pole of the great circle SBS', and a great circle drawn through W and x falls at x'. However, it is not necessary to draw the great circle through W and x to locate the point x' on the graduated circle: By centering the great circle protractor, described by the writer,† at C, and turning it so that W and x fall on the same great circle, the point x may be transposed to x', and other points, w', y' and z', would be found in like manner.

The three foregoing methods of transposing x to x', z to z', etc., are about equally simple, and it may be pointed out that, supplied with transparent stereographic protractors, and having the poles of a crystal plotted in stereographic projection, it is only necessary to draw the great circle SES' and to locate one point, either W or P, in order to find the directions needed for a parallel-perspective drawing, corresponding to figure 3. Thus, with only a great circle protractor, the great circle through the poles of any zone may be traced, and its intersection with SES' noted and spaced off with dividers

* This Journal (4), xiii, pp. 247–249 and 269–271, 1902.
† Ibid. (4), xi, pp. 21–22, 1901.

from either *S* or *S′*; then the great circle through the intersection just found and *W* is determined, and where it falls on the divided circle noted, when the desired direction may be had by means of a straight edge and 90° triangle, as already explained.

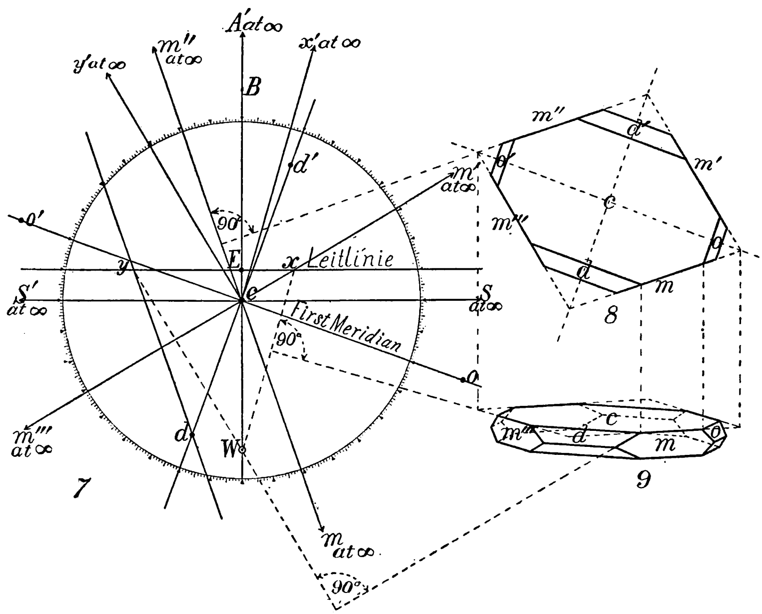
The gnomonic projection is preferred by many for representing crystallographic relations, and it seems best, therefore, to indicate how readily the methods just explained may be adapted to this kind of projection. This subject has received careful treatment from Goldschmidt * and G. F. H. Smith, † hence what follows might seem somewhat superfluous; but although the final results of the crystal drawings are essentially the same as those of the authors just mentioned, the presentation and explanations here given are somewhat different, and it is hoped that some of the suggestions may be of service to students of crystallography.

As an illustration, the method of drawing a simple combination of barite has been chosen. The forms shown in figures 7, 8 and 9 are *c* (001), *m* (110), *o* (011) and *d* (102). The location of the poles in the gnomonic projection is shown in figure 7, where, as in figure 1, the *first meridian* is taken at 20° from the horizontal direction *SS′*. A simple method for locating the poles *o* and *d* on their respective meridians is by means of the stereographic scale No. 3, described by the writer,‡ by laying off with this scale double the angle $c \wedge o$ and $c \wedge d$; for both stereographic and gnomonic scales are derived from a table of natural tangents, 2° of the former being equal to 1° of the latter. The poles of the prism *m* and the locations of *S* and *S′* (compare figure 1) fall in the gnomonic projection at infinity. In any plan, such as figure 8, the direction of an edge made by the meeting of two faces is at right angles to a line joining the poles of the faces, shown in figures 7 and 8 by the direction at 90° to the line joining *m″* and *c*.

The parallel-perspective view, figure 9, is an orthographic projection (compare figures 1 and 3) drawn on a plane passing through *S* and *S′*, and intersecting the sphere on which the gnomonic projection is based as a great circle,§ passing through *E*, figure 7, and drawn parallel to *SS′*, the distance *cE* being 10°: This great circle is called by Goldschmidt the *Leitlinie*. To find such intersections as between *m‴* and *c*, and *m* and *d*, figure 9, note, as in figure 1, where the great circles through the poles of the faces intersect the *Leitlinie;* thus, the one

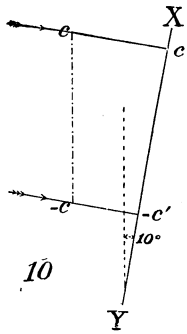
* Zeitschr. Kryst., xix, p. 352, 1891.
† Min. Mag., xiii, p. 309, 1903.
‡ This Journal (4), xi, p. 7, 1901.
§ All great circles in the gnomonic projection are represented by straight lines.

through m''' and c at x, and that through m and d (through d parallel to $m\ m''$, since m and m'' are at infinity) at y. Next imagine the points x and y transposed as in figure 1 to x' and y', which latter points, however, are located at infinity: This transposition is done by locating first the so-called *Winkelpunkt*, W, of Goldschmidt, 40° from c in figure 7, and as in figure 1, 90° from a point B, which is an equal number of



FIGURES 7 to 9. Development of a plan and parallel-perspective figure of barite, orthorhombic system, from a gnomonic projection.

degrees from E and A' (compare figure 6). Of the three methods given on pages 208 and 209 for transposing x and y to x' and y', the third may be easily applied in the gnomonic projection. Great circles, or straight lines, through W and x and W and y, figure 7, if continued to infinity, would determine x' and y', which is accomplished by drawing lines parallel to Wx and Wy through the center. It is not necessary, however, to draw the lines Wx and Wy, nor the parallel lines through the center; all that is needed to find the directions of the edges $m''' \wedge c$ and $m \wedge d$ is to lay a straight edge from W to x, respectively W to y, and with a 90° triangle transpose the directions to figure 9, as indicated in the drawings. The principles are exactly the same as worked out for the interrelations of figures 1 and 3. As in the case of the stereographic projection, it is evident that, given the poles of a crystal plotted in the gnomonic projection, it would be necessary to draw only one line, the *Leitlinie*, and to locate one point, the *Winkelpunkt*, *W*, in order to find all possible directions for a plan and parallel-perspective views, corresponding to figures 8 and 9.



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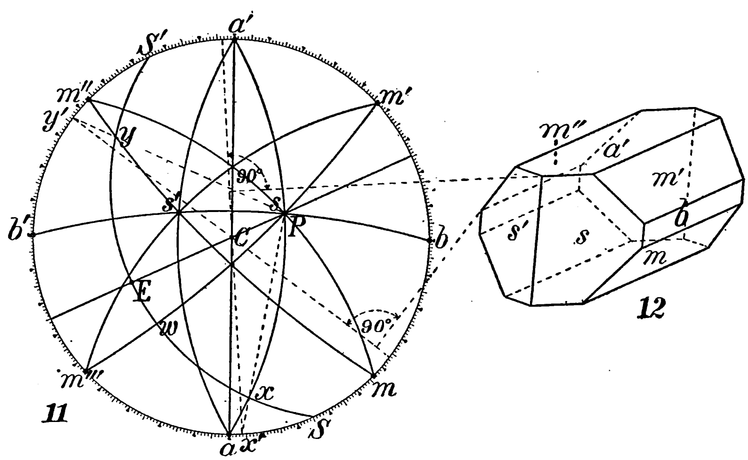
In any parallel-perspective drawing corresponding to figures 3 and 9, it is important to keep in mind that, since the projection is orthographic and made on an inclined surface, there will be some fore-shortening of vertical lengths. Thus, if one has in mind a certain height of a crystal, or the length of a vertical axis $c, -c$, figure 10, and if XY is the trace of the plane on which the projection is made, the length $c, -c$ would become fore-shortened to $c', -c'$.* The fore-shortening is best done graphically, or it would be the length $c, -c$ times the sine of 10°, provided E, as in figures 1 and 7, is 10° from the center.

The methods of drawing, as developed in the foregoing pages, have been such as to yield parallel-perspective figures essentially like the conventional ones found in treatises on crystallography and mineralogy; but, as already stated, the plane on which a drawing is to be made may have any desired position, and it may not be out of place to indicate briefly, by an example, how easily the methods may be modified to suit varying requirements. Figure 11 represents a stereographic projection of augite, the forms being a, b, m and s ($\bar{1}11$), and it is desired to draw a parallel-perspective on a plane parallel to the pyramid face s, $\bar{1}11$. Through s draw a diameter, and on it locate E, 90° from s; then draw the great circle SES': Under the conditions, the pole corresponding to P of figure 1 is s in figure 11. In the parallel-perspective, figure 12, such directions as the edges $a' \wedge s$ and $m'' \wedge s'$ are found by noting in figure 11 where the great circles $a' s$ and $m'' s'$ cross SES' (at x and y) and then following out the construction indicated by the figure, as previously explained.

In figure 12, the plane angles of s are the same as those of an actual crystal, and the angles, or their supplements, may always be measured on the great circle SES' of the stereographic projection. To measure the angle made by the edges $a's$ and $m's$, figure 12, the great circles through the poles, figure 11, are at right angles, respectively, to the two edges, and their intersections with SES' are at x and w, hence the angular distance x to w, measured with a stereographic protractor as 47°, is the supplement of the desired angle, or 133°.

* Compare figure 5, page 41 ; this Journal (4), xix, 1905.

Certainly the methods of getting both a plan and a parallel-perspective from either a stereographic or gnomonic projection appeal strongly to one at first, both because of their simplicity and the doing away with the multiplicity of construction lines which frequently are needed in drawing from axes. To convince himself of the relative advantages of the different methods of drawing crystals, the writer has taken special pains to experiment with those explained in this paper, and it is his belief that most persons, especially beginners, will find it easier to draw from axes, while at the same time finished



FIGURES 11 and 12. Development of a parallel-perspective of augite, drawn on a plane parallel to *s*, $\bar{1}11$.

drawings will in most cases be completed more quickly and, probably, with greater accuracy. It certainly would seem as though with the axes constantly before one, they must be of value to a student in developing the symmetry of a crystal figure during the process of drawing. When it comes to a complicated problem, such as one in the triclinic system, it may be questioned whether it is easier to incline the axes and draw from them, or to make either a stereographic or gnomonic projection and draw from it: The determination of such a question would depend somewhat upon the data at hand, and largely upon one's familiarity or facility with either the one or the other method. Certainly, as is often the case, having made either a stereographic or gnomonic projection for the purpose of study, it would be easier to draw from the projection than to plot the inclined axes and draw from them. Every one who is at all interested in crystal drawing would do well to become familiar with the methods based on the use of the stereographic and gnomonic projections, for they may be employed to advantage when drawing from axes becomes difficult or impossible. For example, in some twin crystals, when drawing from axes, serious difficulties are at times encountered in finding the intersections of interpenetrating surfaces; difficulties readily overcome, however, by drawing from the projections, if the poles of the twin crystal have been plotted; or again, the projections may be employed in drawing some odd shape or some obscure crystal, the planes of which cannot be referred to axes.

To any one desiring to make much use of the methods of drawing from either of the two projections, it is recommended to employ a T-square and, in connection with it, special triangles, figure 13, similar to those previously described by the writer*, as follows:—The drawing paper is fastened to a board so that a T-square gives the direction *SS'*, figures 1 and 7. A 20°, 90° triangle, *I*, gives the direction of the first meridian of the two projections, and, in any plan, the direction of the front-to-back, or $a, -a$ axis, and, triclinic system excepted, the right-to-left, or $b, -b$ axis. A 90°, 25° 30′ triangle, *II*, is used

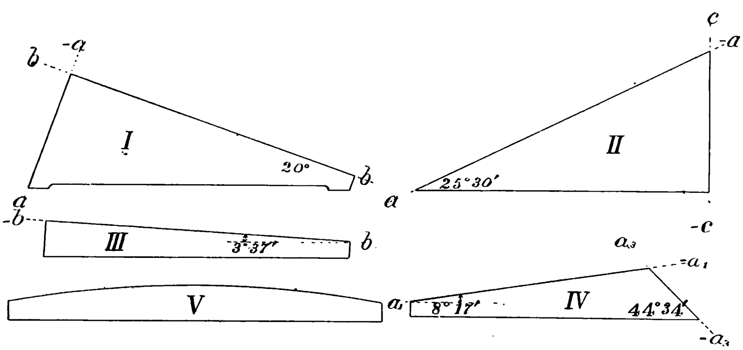


FIGURE 13. Special triangles to be used in connection with a T-square when drawing from the stereographic or gnomonic projections.

for uniting corresponding points of plan and parallel-perspective figures, parallel to the $c, -c$ axis, and it also gives the front-to-back or $a, -a$ axis in any parallel-perspective figure, when the axis is 90° to the vertical. A truncated 3° 37′ triangle, *III*, gives the direction of the right-to-left or $b, -b$ axis of parallel-perspective figures, provided the system is not triclinic. An 8° 17′, 44° 34′ triangle, *IV*, gives the directions of two of the horizontal axes in the hexagonal system, triangle *III* giving the third. Lastly, a circular arc, *V*, is used for

* This Journal (4), xix, p. 53, 1905.

drawing the great circle *SES'*, figure 1. The angles of the triangles, figure 13, are based upon the data chosen for constructing figures 1 and 7; namely, the first meridian 20° from *S*, and *E* 10° from the center of the projections: For any desired variation from these data the angles could be calculated readily. The writer has found triangles made from heavy bristol-board in every respect serviceable for pencil work, and if cut, for example, as shown in figure 13–*I*, so that only two extremities of the base line touch the T-square, they may first be made approximately right and then accurately adjusted by taking off a little from one end or the other by rubbing against sandpaper. Referring to figure 8; having determined the direction $c \wedge m'''$, and knowing the orthorhombic symmetry of the crystal, the remaining lines could all be determined by means of triangle *I*. From figure 8; and by use of the triangles *II* and *III*, figure 9 could be constructed by finding only one direction, for example, $d \wedge m$. The triangles thus serve as time-savers to any one engaged in this kind of crystal drawing, and likewise insure increased accuracy in the work.

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